THE CHAOTIC INFLATION GROWTH MODEL

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Abstract. *This paper examines inflation dynamics in the United States since 2000. The basic aims of this paper are: firstly, to provide a relatively simple chaotic inflation rate growth model that is capable of generating stable equilibria, cycles, or chaos; and secondly, to analyze the inflation rate growth stability in the period 2000-2017 in the U.S. economy. This paper confirms the existence of the convergent fluctuations of the inflation rate in the U.S. economy in the observed period.*

Keywords: *Single equation model, Inflation, Unemployment.*

JEL classification numbers: C2, E31, J64

Introduction

Global output growth is estimated at about 3 percent for the third quarter of 2016. Economic activity rebounded strongly in the United States and the economy is approaching full employment. Long-term nominal and real interest rates have risen substantially in the United States. Also, U.S. fiscal policy is projected to become more expansionary, with stronger future demand implying more inflationary pressure. The U.S. dollar has appreciated in real effective terms by over 6 percent since August 2016. (www.imf.org)

In the short run, the fall in aggregate demand leads to falling output and price level and rising unemployment. What should policymakers do when faced with such a recession? One possibility is to take action to increase aggregate demand. An increase in government spending or an increase in money supply would increase aggregate demand. According to the Phillips curve, when aggregate demand is low, then unemployment is high and inflation is low. On the other hand, Okun's law is a relationship between changes in the unemployment rate and economic growth. In this sense, this relationship predicts that growth slowdowns coincide with rising unemployment. (see Fig. 1.)

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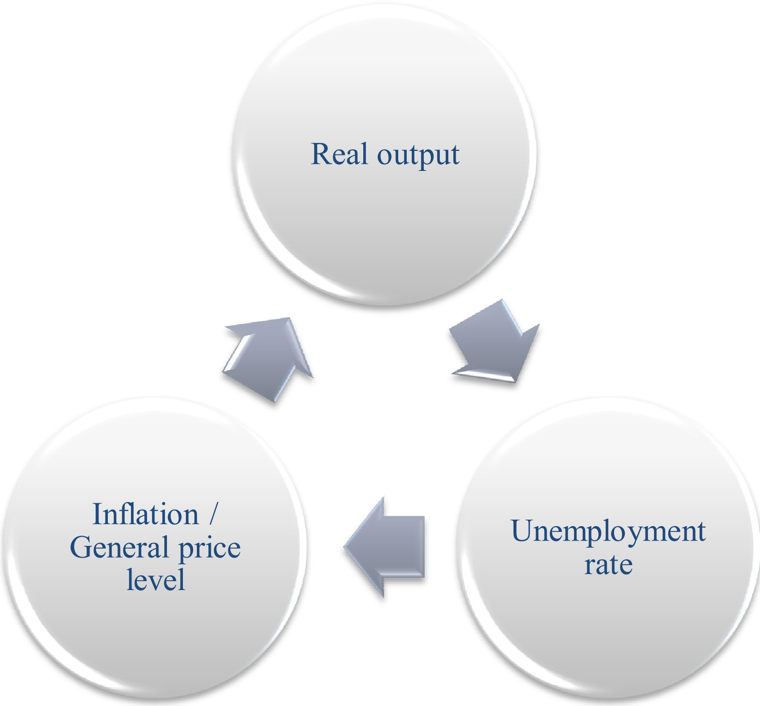

Figure 1. Relations between real output, unemployment rate and inflation / general price level.

Okun's (1962) paper regarding the unemployment – output relationship considers the measurement of potential output. Okun believed that the potential output should not be defined as the maximum output the economy could produce. Instead, he argued that the potential should be measured at full employment, which he characterized as the level of employment absent inflationary pressures. In accordance with Okun's Law, the rising GDP growth rates were accompanied by declining unemployment rates, while declining growth rates went hand in hand with higher unemployment rates in the United States in the period 2000-2017. (see Fig. 2. and Fig. 3.)

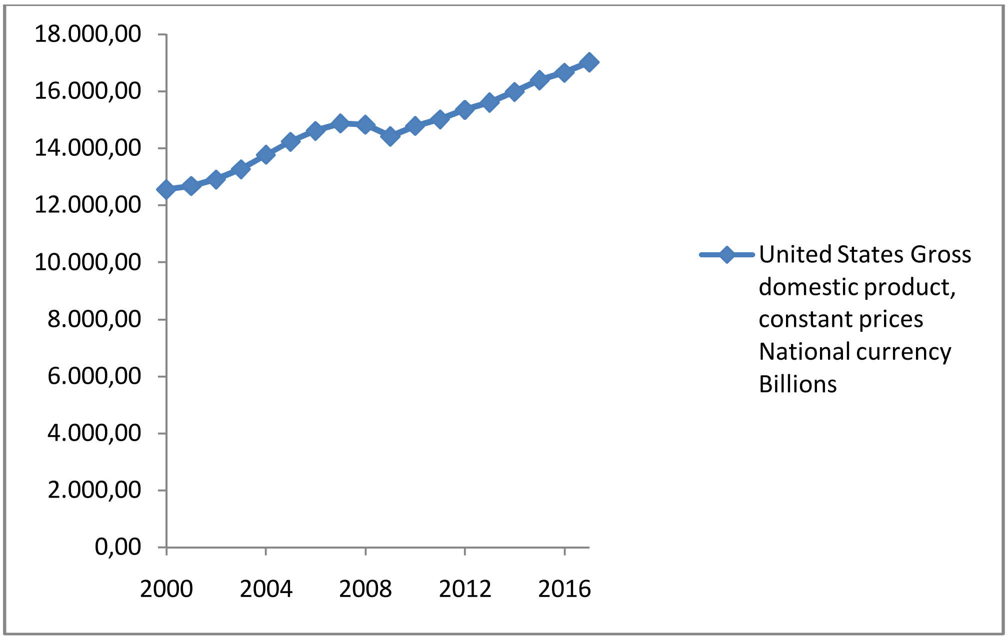

Figure 2. Gross domestic product, constant prices,national currency, billions: U.S. 2000-2017 (www.imf.org).

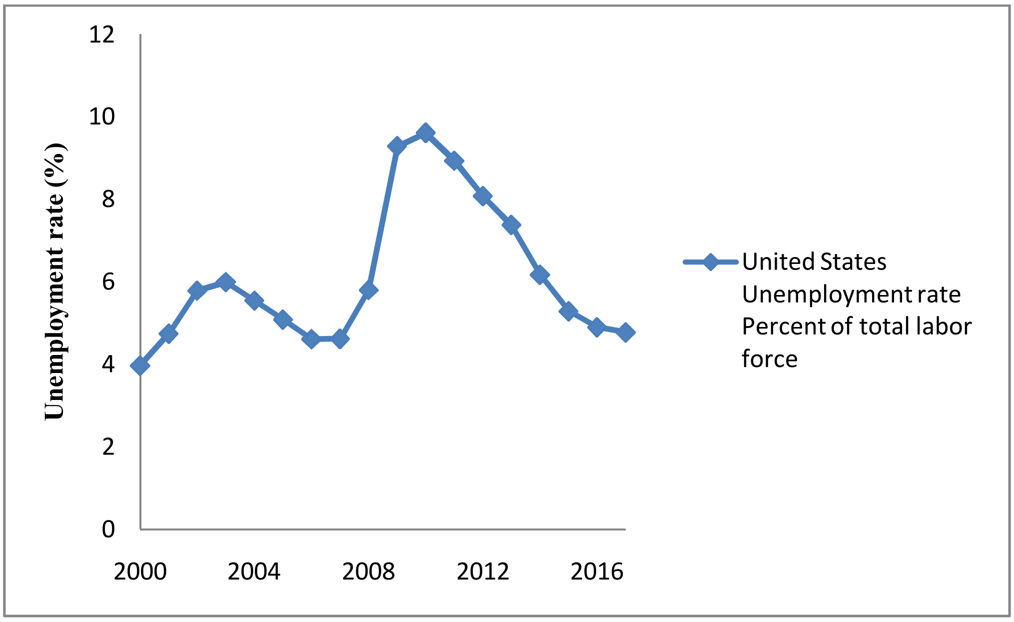

Figure 3. Unemployment rate (% of total labor force): U.S. 2000-2017 (www.imf.org).

The Phillips curve (1958) represents the negative short-run relationship between the rate of inflation and the unemployment rate. The Phillips curve shows that cyclical unemployment is related to unexpected movements in the inflation rate.

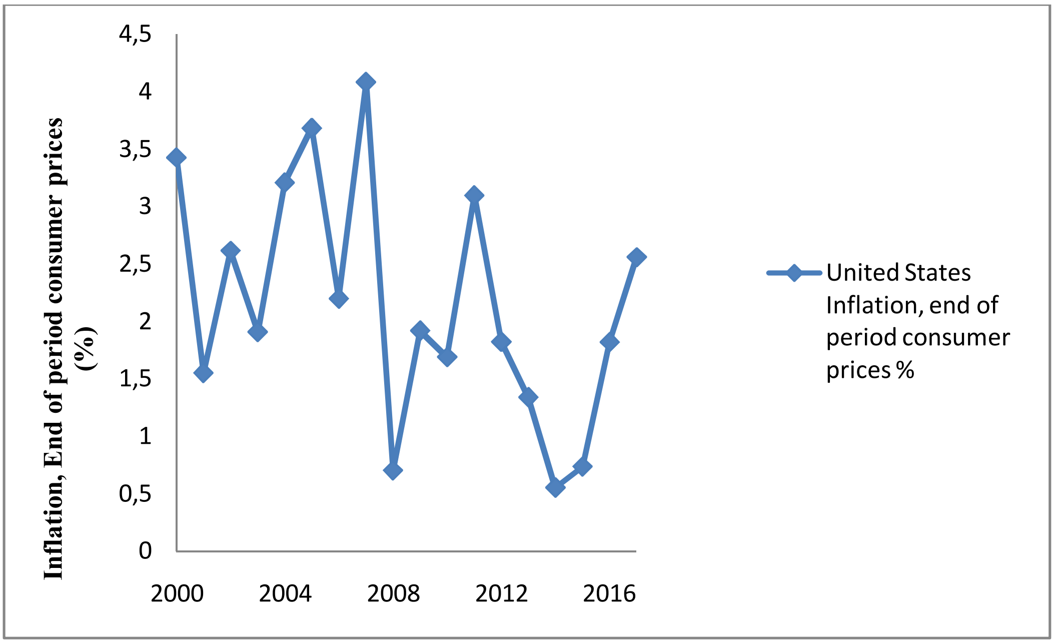

Figure 4. Inflation, end of period consumer prices (%), 2000-2017, U.S. (www.imf.org).

Chaos theory states that small changes can result in large differences. Chaotic system is unpredictable. Namely, a slight difference, in the decimal place, resulted in prediction failure. Chaotic systems exhibit a sensitive dependence on initial conditions: seemingly insignificant changes in the initial conditions produce large differences in outcomes. This is very different from stable dynamic systems in which a small change in one variable produces a small and easily quantifiable systematic change. Chaos theory started with Lorenz's (1963) discovery of complex dynamics arising from three nonlinear differential equations leading to turbulence in the weather system. Li and Yorke (1975) discovered that the simple logistic curve can exibit very complex behaviour. Further, May (1976) described chaos in population biology. Chaos theory has been applied in economics by Benhabib and Day (1981,1982), Day (1982, 1983, 1992, 1997), Grandmont (1985), Goodwin (1990), Medio (1993,1996), Lorenz (1993), Jablanovic (2011,2013,2016), among many others.

This paper considers whether the chaotic inflation growth model can explain the recent behavior of inflation in the United States. The basic aims of this paper are: firstly, to provide a relatively simple chaotic inflation growth model that is capable of generating stable equilibria, cycles, or chaos; and secondly, to analyze the inflation growth stability in the period 2000-2017 in the U.S. economy. This paper confirms convergent fluctuations of the inflation rate in the U.S. economy in the observed period.

The model

This paper focuses on the inflation rate growth stability in the U.S. economy, combining the output / unemployment relationship (Okun's Law) with the inflation/unemployment relationship (the Phillips Curve), and the Aggregate Demand curve. In this sense, the chaotic inflation growth model is presented by the following equations:

$$u_t - u^n = -\alpha\,(Y_t - Y^n) \qquad \alpha > 0 \qquad (1)$$

$$u^n = \beta\, u_t \qquad \beta > 0 \qquad (2)$$

$$Y^n = \gamma\, Y_t \qquad \gamma > 0 \qquad (3)$$

$$u_t - u^n = -\mu\,(\Pi_t - \Pi^e) \qquad \mu > 0 \qquad (4)$$

$$\Pi^e = \omega\, \Pi_t \qquad \omega > 0 \qquad (5)$$

$$Y_t = C_t + I_t + G_t + Nx_t \qquad (6)$$

$$C_t = \delta\, Y_{t-1}^{\;2} \qquad 0 < \delta < 1 \qquad (7)$$

$$I_t = \lambda\, Y_{t-1} \qquad 0 < \lambda < 1 \qquad (8)$$

$$N_{x,\,t} = n\, Y_t \qquad 0 < n < 1 \qquad (9)$$

$$G_t = g\, Y_t \qquad 0 < g < 1 \qquad (10)$$

with Y – real output, Y^n – the potential output, I – investment, C– consumption, Nx – net exports, G – governmnet spending, Π – actual inflation, Π^e – expected inflation, u – unemployment rate, u_n – the natural rate of unemployment, α – the "Okun's coefficient", δ – the private consumption rate, β, μ and γ – the positive constants, n – the net export rate, g – the government expenditure rate, λ – the investment rate.

(1) shows the Okun' law; the negative correlation between GDP growth and unemployment has been named "Okun's law." The relationship between contemporaneous changes in economic growth and unemployment is often referred to as "Okun's Law". The parameter α is often called "Okun's coefficient." Okun's relationship connected the level of unemployment to the gap between actual output (Y) and potential output (Y^n). Potential output explains how much the economy would produce "under conditions of full employment". (2) shows that the natural rate of unemployment which is known as the non acceleration inflation rate of unemployment (NAIRU) is proportional to the current unemployment rate; (3) shows that the potential output is proportional to the actual output; (4) the form of the short-run Phillips curve; μ is a parameter which shows the responsiveness of unemployment to inflation; (5) shows expected rate of inflation; (6) shows GDP (Y) as the sum of consumption (C), investment (I), government spending (G) and net exports; (7) In this model, the consumption function displays the quadratic relationship between consumption (C_t) and real output of the previous period (Y_{t-1}). Real output is multiplied by the coefficient δ, „the marginal propensity to consume" (MPC). The MPC coefficient can be between zero and one. (8) shows the investment function; (9) shows the relation between net export (Nx) and real output (Y); and (10) shows the relation between government spending (G) and real output (Y).

Now, putting (1), (2), (3), (4), (5) , (6), (7), (8), (9) and (10) together we immediately get:

$$\Pi_t = \left[\frac{\lambda}{(1-n-g)}\right]\Pi_{t-1} - \left[\frac{\mu(1-\omega)\delta}{\alpha(\gamma-1)(1-n-g)}\right]\Pi_{t-1}{}^2 \qquad (11)$$

Further, it is assumed that the current value of the inflation rate is restricted by its maximal value in its time series. This premise requires a modification of the growth law. Now, the inflation rate growth rate depends on the actual value of the inflation rate, Π, relative to its maximal size in its time series Π^m. We introduce π as $\pi = \Pi/\Pi^m$. Thus π range between 0 and 1. Again we index π by t, i.e., write π_t to refer to the size at time steps t = 0,1,2,3,... Now the inflation growth rate is measured as

$$\pi_t = \left[\frac{\lambda}{(1-n-g)}\right]\pi_{t-1} - \left[\frac{\mu(1-\omega)\delta}{\alpha(\gamma-1)(1-n-g)}\right]\pi_{t-1}{}^2 \qquad (12)$$

This model given by equation (12) is called the logistic model. For most choices of α, γ, μ, δ, λ, n, ω, and g there is no explicit solution for (12). Namely, knowing α, ω, γ, μ, δ, λ, n, and g and measuring π_0 would not suffice to predict π_t for any point in time, as was previously possible. This is at the heart of the presence of chaos in deterministic feedback processes. Lorenz (1963) discovered this effect – the lack of predictability in deterministic systems. Sensitive dependence on initial conditions is one of the central ingredients of what is called deterministic chaos.

This kind of difference equation (12) can lead to very interesting dynamic behavior, such as cycles that repeat themselves every two or more periods, and even chaos, in which there is no apparent regularity in the behavior of π_t. This difference equation (12) will posses a chaotic region. Two properties of the chaotic solution are important: firstly, given a starting point π_0 the solution is highly sensitive to variations of the parameters α, ω, γ, μ, δ, λ, n, and g; secondly, given the parameters α, ω, γ, μ, δ, λ, n, and g, the solution is highly sensitive to variations of the initial point π_0. In both cases the two solutions are for the first few periods rather close to each other, but later on they behave in a chaotic manner.

The Logistic Equation

It is possible to show that iteration process for the logistic equation (see Fig. 3.)

$$z_t = \eta\, z_{t-1}\,(1 - z_{t-1}), \quad \eta \in [0, 4], \quad z_t \in [0, 1] \tag{13}$$

is equivalent to the iteration of growth model (12) when we use the identification

$$z_{t-1} = \left[\frac{\mu\ \delta\,(1-\omega)}{\alpha\,\lambda\,(\gamma-1)}\right]\pi_{t-1} \quad \text{and} \quad \eta = \left[\frac{\lambda}{(1-n-g)}\right] \tag{14}$$

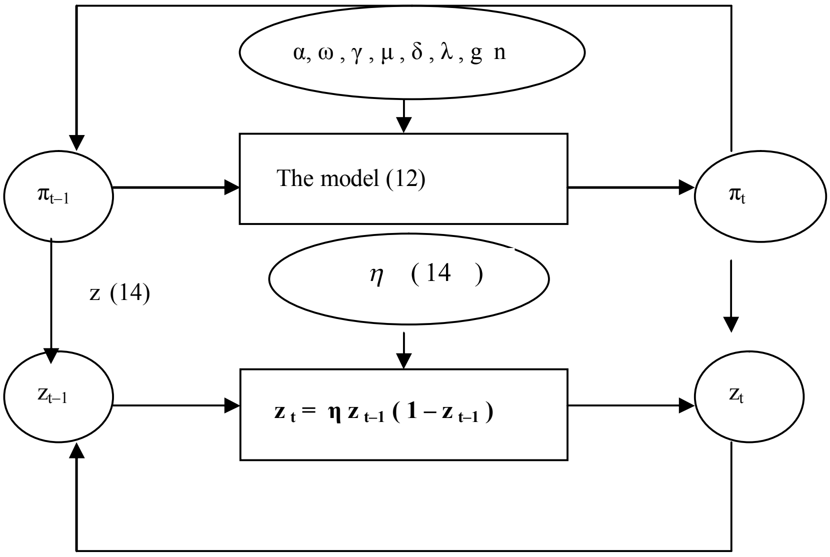

Figure 5. Two quadratic iteratiors running in phase are tightly coupled by the transformations indicated

Using (12) and (14) we obtain:

$$z_t = \left[\frac{\mu\ \delta\,(1-\omega)}{\alpha\ \lambda\,(\gamma-1)}\right]\pi_t =$$

$$= \left[\frac{\mu\,\delta\,(1-\omega)}{\alpha\ \lambda\,(\gamma-1)}\right]\left\{\left[\frac{\lambda}{(1-n-g)}\right]\pi_{t-1} - \left[\frac{\mu\,(1-\omega)\,\delta}{\alpha\,(\gamma-1)(1-n-g)}\right]\pi_{t-1}{}^2\right\} =$$

$$= \left[\frac{\mu\,\delta\,(1-\omega)}{\alpha\ (\gamma-1)(1-n-g)}\right]\pi_{t-1} - \left[\frac{\mu^2\,\delta^2\,(1-\omega)^2}{\alpha^2\,\lambda\,(\gamma-1)^2(1-n-g)}\right]\pi_{t-1}{}^2.$$

On the other hand, using (13) and (14) we obtain:

$$z_t = \eta\, z_{t-1}\,(1 - z_{t-1}) =$$

$$= \left[\frac{\lambda}{(1-n-g)}\right]\left[\frac{\mu\ \delta\,(1-\omega)}{\alpha\ \lambda\,(\gamma-1)}\right]\pi_{t-1}\left\{1 - \left[\frac{\mu\ \delta\,(1-\omega)}{\alpha\ \lambda\,(\gamma-1)}\right]\pi_{t-1}\right\} =$$

$$= \left[\frac{\mu\,\delta\,(1-\omega)}{\alpha\ (\gamma-1)(1-n-g)}\right]\pi_{t-1} - \left[\frac{\mu^2\,\delta^2\,(1-\omega)^2}{\alpha^2\,\lambda\,(\gamma-1)^2(1-n-g)}\right]\pi_{t-1}{}^2.$$

Thus we have that iterating (12) is really the same as iterating (13) using (14). It is important because the dynamic properties of the logistic equation (13) have been widely analyzed (Li and Yorke (1975), May (1976)).

It is obtained that:

(i) For parameter values $0 < \eta < 1$ all solutions will converge to $z = 0$;

(ii) For $1 < \eta < 3,57$ there exist fixed points the number of which depends on η;

(iii) For $1 < \eta < 2$ all solutions monotnically increase to $z = (\eta - 1) / \eta$;

(iv) For $2 < \eta < 3$ fluctuations will converge to $z = (\eta - 1) / \eta$;

(v) For $3 < \eta < 4$ all solutions will continously fluctuate;

(vi) For $3,57 < \eta < 4$ the solution become "chaotic" wihch means that there exist totally aperiodic solution or periodic solutions with a very large, complicated period. This means that the path of z_t fluctuates in an apparently random fashion over time, not settling down into any regular pattern whatsoever.

Important parameter η values "0, 1 , 1, 2, 3" are part of the Fibonacci sequence. The Fibonacci Sequence is the series of numbers: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, ... There is an interesting pattern: The Fibonacci Sequence is found by adding the two numbers before it together. The is found by adding the two numbers before it (0+1). The 2 is found by adding the two numbers before it (1+1). The 3 is found by adding the two numbers before it (1+2). Namely, each number is the sum of the two numbers before it. If we make squares with those widths, we get a nice spiral. Also, if we take any two successive, important values of parameter π, (" 2, 3 ") , their ratio is very close to the Golden ratio which is approximately 1.618034... The adjacent numbers divided yield the Golden Ratio (e.g. 55/34=1.618). For example 3/2 is 1.5. The golden ratio that has approximate value of 1.618. The golden ratio and the golden rectangle are connected. This is because the ratio of the longer side of a golden rectangle to the shorter side is equal to the golden ratio ($1^2 + 1^2 + 2^2 + 3^2 + 5^2 + 8^2 + \dots$) (Jablanovic, 2016., pg. 30)

Empirical Evidence

The main aim of this paper is to analyze the inflation rate growth stability in the period 2000-2017 in the U.S. economy. In this sense, it is important to use the logistic model (15):

$$\pi_t = \eta\, \pi_{t-1} - \upsilon\, \pi_{t-1}^{\;2} \tag{15}$$

where π – inflation rate, $\eta = \left[\dfrac{\lambda}{(1-n-g)}\right]$, $\vartheta = \left[\dfrac{\mu\ \delta(1-\omega)}{\alpha\ (\gamma-1)(1-n-g)}\right]$.

Now, the model (15) is estimated (see Table 1).

Table 1.
The estimated model (15): U.S., 2000-2017.

U.S.:	R= 0.53765		
		η	υ
	Estimate	2.238269	1.956742
	Std.Err.	0.325209	0.425380
	t (15)	6.882561	4.585884
	p-level	0 .00000	0.00036

Source: www.imf.org

Conclusion

This paper creates the chaotic unemployment rate growth model. For most choices of α, ω, γ, μ, δ, λ, n, and g there is no explicit solution for (12). Namely, knowing α, ω, γ, μ , δ , λ, n, and g and measuring π_0 would not suffice to predict π_t for any point in time, as was previously possible. But even slight deviations from the values of parameters: α, ω, γ, μ, δ , λ, n, and g and initial value of inflation rate, π_0 show the difficulty of predicting a long-term inflation rate behavior.

A key hypothesis of this work is based on the idea that the coefficient $\eta = \left[\dfrac{\lambda}{(1-n-g)}\right]$ plays a crucial role in explaining the local inflation rate growth stability, where, n – the net export rate, g – the government expenditure rate, λ – the investment rate. An estimated value of the coefficient η (2.238269) confirms convergent fluctuations of the inflation rate in U.S. in the observed period.

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