ANALYTIC APPROACH IN STUDY OF THE SLOW-WAVE STRUCTURES

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Abstract. *Electromagnetic wave propagation in the slow-wave structures is described by the specific case of differential Maxwell equations in the Cartesian coordinate system. Mathematical simulation of the concrete engineering processes is proposed by the boundary value problems whose solutions are got explicitly.*

Keywords: *electromagnetic wave propagation, differential Maxwell equations, boundary value problems.*

1. Introduction

$$\begin{cases} \mathrm{rot}\vec{H} = \partial_0\vec{D} + \vec{i}, \ \mathrm{rot}\vec{E} = -\partial_0\vec{B}, \vec{i} = \sigma\vec{E}, \\ div\vec{D} = \rho, \ \vec{D} = \varepsilon_a\vec{E}, \ div\vec{B} = 0, \ \vec{B} = \mu\vec{H}. \end{cases} \quad (1)$$

$$\vec{E}, \vec{H}, \vec{D}, \vec{B} = \vec{E}, \vec{H}, \vec{D}, \vec{B}(x, y, z, t);$$

$$\vec{i} = \vec{i}(x, y, z, t), \quad \rho = \rho(x, y, z, t); \quad \partial_0 = \partial / \partial t.$$

(1) for the slow-guided structures [1]: those are metallic, without either magnetics, or dielectrics, or charges inside $(\vec{i}, \rho = 0)$; metal as the ideal conductor $(\sigma = \infty)$; electromagnetic field vector intensities are harmonic regarding the time argument, i.e.

$$\vec{E}, \vec{H} = \vec{E}, \vec{H}(x, y, z)\exp(i\omega t), i = \sqrt{-1},$$

ω is the vibration frequency.

The aim of the present article is the detailed analytic solution and partial numerical implementation of (1) with/without aforesaid restrictions.

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2. Supporting results

$$(\Delta + \varepsilon_a \mu_a \omega^2) F_{kj} = 0; \ (k = 1, 2; \ \ j = \overline{1, 3}), \tag{2}$$

with

$$F_{1j} = E_j = E_j(x, y, z), \ \ F_{2j} = H_j = H_j(x, y, z), \ \ (j = \overline{1, 3});$$

$$\Delta = \sum_{j=1}^{3} \partial_j^2, \ \partial_1 = \partial/\partial x, \ \partial_2 = \partial/\partial y, \ \partial_3 = \partial/\partial z. \tag{3}$$

(1) $\Leftrightarrow$ (2), (3) by [2].

Mathematical modeling of electromagnetic field study for the flat rectangular resonator is based on the boundary value problem with the general wave equation (2), (3) in $\mathbf{R}_2$ and boundary conditions

$$g_{1kj}(y) = F_{kj}(0, y), \ \ g_{2kj}(y) = F_{kj}(p, y);$$

$$h_{1kj}(x) = F_{kj}(x, 0), \ \ \ h_{2kj}(x) = F_{kj}(x, q), \ \ \ (k, \ j = 1, 2). \tag{4}$$

In (4): g…; h… are the given continuous functions; $x \in [0, p], \ \ y \in [0, q]$ and p, q are the sizes of resonator.

The unified explicit solution of (2)-(4) using integral transform method [3], and irrespectively to specific boundary conditions, was shown briefly in [4] by the formula

$$F_{kj}(x, y) = \left(\frac{2}{\pi^2}\right)^2 pq \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} {}_{tr}F_{kj} \sin\left(\frac{\pi n}{p} x\right) \sin\left(\frac{\pi m}{q} y\right), \ (k, \ j = 1, 2) \tag{5}$$

with the corresponding found transforms

$${}_{tr}F_{kj} = \left(\frac{\pi n}{p}({}_{tr}h_{1kj} + (-1)^{n+1}\,{}_{tr}h_{2kj}) + \frac{\pi m}{q}({}_{tr}g_{1kj} + (-1)^{m+1}\,{}_{tr}g_{2kj})\right) \times$$

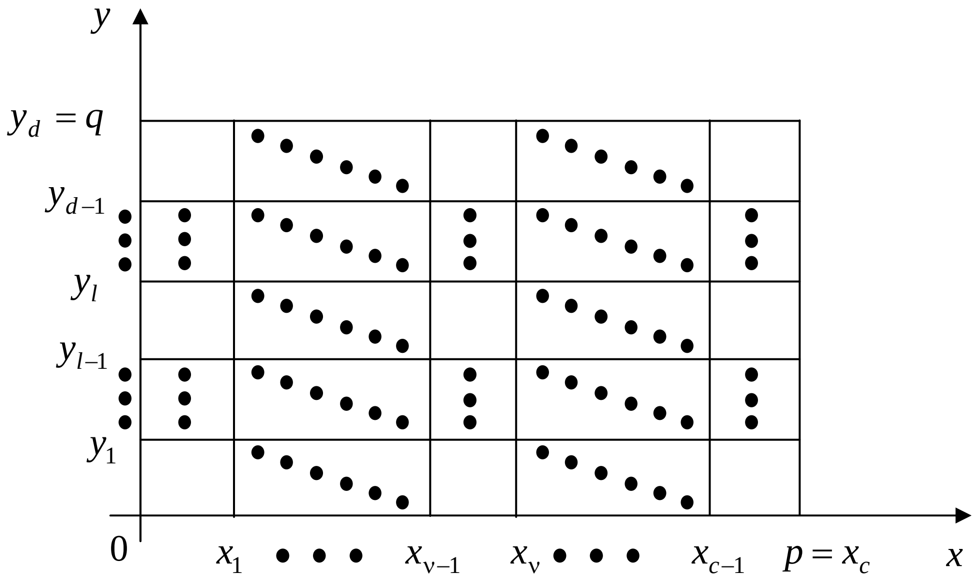
$$\times \left(\left(\frac{\pi}{p} n\right)^2 + \left(\frac{\pi}{q} m\right)^2 - \varepsilon_a \mu_a \omega^2\right)^{-1}; \tag{6}$$

$$
{}_{tr}F_{kj} = \frac{pq}{\pi^2} \int_0^p \int_0^q F_{kj}(x, y) \sin\left(\frac{\pi n}{p} x\right) \sin\left(\frac{\pi m}{q} y\right) \mathrm{d}\, x \, \mathrm{d}\, y;
$$

$$
{}_{tr}g_{skj} = \frac{q}{\pi} \int_0^q g_{skj}(y) \sin\left(\frac{\pi m}{q} y\right) \mathrm{d}\, y, \qquad (7)
$$

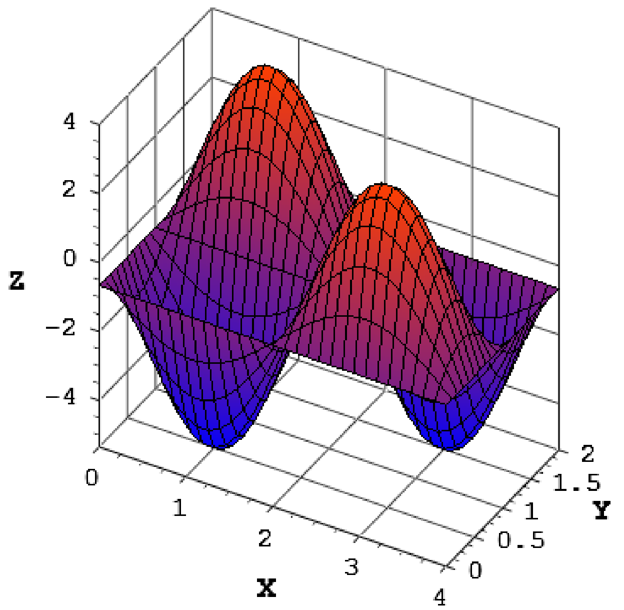
$$
{}_{tr}h_{skj} = \frac{p}{\pi} \int_0^p h_{skj}(x) \sin\left(\frac{\pi n}{p} x\right) \mathrm{d}\, x, \ \ (s, k, \ j = 1, 2).
$$

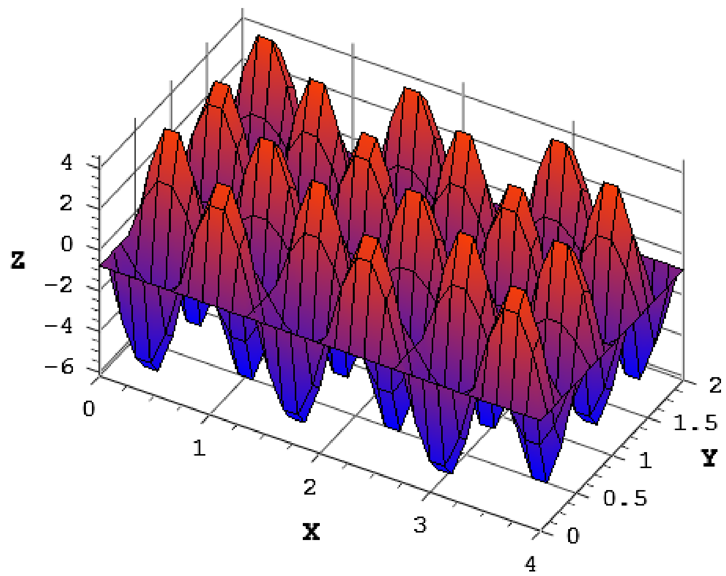
Numerical implementation of (2) – (4) applying simple discretization procedure to (5)-(7) with the knots (x_ν, y_l), $x_\nu = \frac{p}{c}\nu$, $y_l = \frac{q}{d} l$, $(\nu = \overline{1, c}; \ l = \overline{1, d})$; $c, d \in N$



was suggested in [4] with the relevant flow-chart.

Several frames from [4] reflect the electromagnetic field behavior while the medium is an air ($\mu = \mu_a = 12{,}56 \cdot 10^{-7}$ H/m; $\varepsilon = \varepsilon_a = 8{,}85 \cdot 10^{-12}$ F/m), functions (4) obey the exponential law and ω is the microwave frequency.





3. Main results

If $\sigma \neq \infty$, (1) with the same temporal harmonic character of $\vec{E}$, $\vec{H}$ in $\boldsymbol{R}_2$ is reduced to the equivalent wave PDE

$$(\Delta + \mu_a \omega(\varepsilon_a \omega - i\sigma))\vec{F}_k = \vec{f}_k^{\ *}, \quad (k = 1, 2), \tag{8}$$

where

$$\vec{F}_1 = \vec{E}(x, y), \quad \vec{F}_2 = \vec{H}(x, y), \tag{9}$$

$$\vec{f}_1^{\,*} = \frac{1}{\varepsilon}\operatorname{grad}\rho(x, y); \quad \vec{f}_2^{\,*} = \vec{h}^{\,*}(x, y), \quad i = \sqrt{-1}.$$

Technique of [3] gives unified unknown transform of boundary value problem (8), (9), (4)

$$_{tr}F_{kj} = \frac{\left({}_{tr}f_{kj}^{*} + \frac{\pi n}{p}\left({}_{tr}h_{1kj} + (-1)^{n+1}{}_{tr}h_{2kj}\right) + \frac{\pi m}{q}\left({}_{tr}g_{1kj} + (-1)^{m+1}{}_{tr}g_{2kj}\right) \right)}{\left(\left(\frac{\pi}{p}n\right)^2 + \left(\frac{\pi}{q}m\right)^2 - \varepsilon_a\mu_a\omega \right)^2 + (\sigma\mu_a\omega)^2} \times \tag{10}$$

$$\times \left(\left(\left(\frac{\pi}{p}n\right)^2 + \left(\frac{\pi}{q}m\right)^2 - \varepsilon_a\mu_a\omega^2 \right) - i\sigma\mu_a\omega \right), \quad (k,\ j = 1, 2).$$

In (10),

$${}_{tr}f_{11}^{*} = -\frac{qn}{\pi\varepsilon}\int_0^q \left(\int_0^p \rho(x, y)\cos\left(\frac{\pi n}{p}x\right)dx \right) \sin\left(\frac{\pi m}{q}y\right)dy,$$

$${}_{tr}f_{12}^{*} = -\frac{pm}{\pi\varepsilon}\int_0^p \left(\int_0^q \rho(x, y)\cos\left(\frac{\pi m}{q}y\right)dy \right) \sin\left(\frac{\pi n}{p}x\right)dx, \tag{11}$$

$${}_{tr}f_{2j}^{*} = \frac{pq}{\pi^2}\int_0^p \int_0^q h_j^{*}(x, y)\sin\left(\frac{\pi n}{p}x\right)\sin\left(\frac{\pi m}{q}y\right)dxdy,$$

and other notations are from (7).

Formula (5) describes explicit solution of (8), (9), (4) with $_{tr}F_{kj}$ given by(10), (11).

4. Conclusions

Analysis of series (5) as for (2), as for (8) and in the presence of various boundary conditions (4) gives satisfactory conditional convergence at least.

Numerical implementation of the last problem explicit solution is planned for the nearest future.

The aim of the paper is attained.

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