# AN ECONOPHYSICS APPROACH AND MODEL FOR THE KEYNES'S MULTIPLIER OF INVESTMENTS

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***Abstract.*** *In this paper are emphasized two main directions of development of modern econophysics, namely statistical econophysics and phenomenological econophysics.*
*Unlike statistical econophysics, using mainly methods from statistical physics and mathematical statistics, phenomenological econophysics is using the similarity between economic concepts and laws processes and the phenomena and laws from diferent areas of physics, other than statistical physics.*
*In this paper it is shown equivalence between investments multiplier introduced by J. M. Keynes and the economic development factor introduced by the econophysics model named "Electronic Amplifier", created based on the similarity between the amplification phenomenon of electronic physics and the economic amplification process based on investments in various economic sectors and units.*

***Keywords:*** *investments multiplier, statistical econophysics, phenomenological econophysics, economic amplifier, economic development factor*

## 1. Introduction

To study economic phenomena or processes, researchers have turned to laws and methods of other sciences, especially from the exact sciences (mathematics, physics) or nature sciences (physics, biology, chemistry).

The papers published during the last two decades (after 1995) have shown that Physics is the most suitable science for modeling the economic phenomena or processes, and thus was founded the new interdisciplinary border science named *Econophysics,* which was developed in several directions or domains [3,4]. Most scientists have been used for this purpose statistical physics, especially for the study of economic and financial processes or distribution of money, companies, individual welfare processes etc. in a framework of a distinct direction named *Statistical Econophysics* [5,13].

In the last two decades have been proposed several econophysics models for the study of economic phenomena and processes, using the

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similarity between economic phenomena or processes and phenomena and laws from other areas of physics (other than statistical physics), in the framework of so-called *Phenomenological Econophysics,* resorting to the phenomena studied in various chapters of fundamental physics (Electricity, Mechanic, Solid State Physics, Optics etc.).

In this paper, for substantiating of economic concepts and in particular of the J. M. Keynes's multiplier, we propose an Econophysics model inspired by the amplification phenomenon from electronic physics namely economic amplifier belonging to the *Phenomenological Econophysics* [4, 14].

## 2. Investments multiplier introduced by J. M. Keynes

J. M. Keynes (1883-1946) was one of the great thinkers and creators of economic theory in 20th century. Although Keynes has sometimes been challenged, many of the concepts of his theory are always invoked and used especially during or after various economic or financial crises as was the case of the recent financial crisis in the years 2007-2008. The criticisms of the Keynes's theory refer mainly to the theses or concepts which upheld the State intervention in economy during periods of crisis or economic recession. Remain however in force the purely economic concepts such as those relating to economic equilibrium law in the market economy or those related to economic development through investments and savings for further investments etc.

The main concepts of J. M. Keynes' theory are based on the action of fundamental psychological laws, such as: propensity for consumption law, impetus towards investment law and propensity towards saving law. The author explains these laws as follows: increased use of labor leads to income growth, which will ensure increase in consumption, but not to the same degree in which the incomes increased. So, at a specific size of consumption, called the **community's propensity for consumption**, the equilibrium level will depend on the current investments, and they will depend on the **investments stimulus,** which in turn depends on the marginal efficiency of capital. The **marginal efficiency** of capital means the amount of **future capital effect** obtained from an **investments effort unit** (replacement cost), the future effect can be measured by the **profit** that the investor will get the entire period of investments' production.

In Keynes' conception the income is viewed in terms of spending for consumption and investments, where usually the persons or companies are inclined to increase consumption with increasing income, but not proportionally. So, from an increase in investments and the use of labor, there is a income growth $\Delta Y$ and hence an increase in the consumption $\Delta C$, but not to the same degree in which income increased, because individuals and companies tend to make savings $E$ for future development (through new investments); we can conclude that although $\Delta C$ and $\Delta Y$ have the same positive sign of growth, almost always $\Delta C < \Delta Y$ and hence results that:

$$\frac{\Delta C}{\Delta Y} < 1 \tag{1}$$

i.e. always the report between the two quantities $\Delta C$ and $\Delta Y$ it is also positive but subunitary [1, 2].

As a result, for the production of goods, the correlation between the three indicators: aggregate income of population, $Y$, population aggregate consumption (at the macroeconomic level), $C$ and its savings, $E$, is as follows:

$$Y = C + E. \tag{2}$$

Since household savings are usually transformed in investments, $I$, the equation (2) can be written as [3,7][1]:

$$Y = C + I. \tag{3}$$

The equation (3), written as above, refers to the entire analyzed phenomenon, i.e. to the **entire** earned income of population, to the **entire** consumption of the population and to **all** investments, which means that it is performed an integral analysis [3]. If we are considering an increase $\Delta Y$ of the income compared with its level in previous period, as well as the variations $\Delta C$ and $\Delta I$ during a given period, then we can write:

$$\Delta Y = \Delta C + \Delta I. \tag{4}$$

which represents a marginal analysis of income and consumption.

J. M. Keynes defines the **investments multiplier, $k$,** as being given by the supplementary income requirements (the increase of income, $\Delta Y$), achieved through increased investments by one unit [1-3,7]:

$$k = \frac{\Delta Y}{\Delta I}. \tag{5}$$

According to equation (5), we can see that the income growth $\Delta Y$ is conditioned by supplementary investments $\Delta I$ and it can be said that the investments multiplier expresses the efficiency of these investments,

[1] The equation (5) is generally accepted, although it has a certain degree of approximation: it is not mandatory, especially in the case of individuals, that all the value of savings $E$ to be used for developing or increasing the investments $I$.

respectively the income growth achieved when the investments volume increased by one unit.

From the equation (5) it is shown that when there is an increase in investments $\Delta I$, the income will increase by an amount $\Delta Y$, which is $k$ times higher than the investments' growth [1,2].

Dividing both members of equation (4) by $\Delta Y$ we obtain:

$$\frac{\Delta Y}{\Delta Y} = \frac{\Delta C}{\Delta Y} + \frac{\Delta I}{\Delta Y}; \tag{6}$$

$$1 = q + \frac{1}{k}$$

or

$$1 - q = \frac{1}{k}; \tag{7}$$

and, hence, [3]:

$$k = \frac{1}{1-q} = \frac{1}{1 - \frac{\Delta C}{\Delta Y}} \tag{8}$$

where $q = \frac{\Delta C}{\Delta Y}$ represents the marginal propensity for consumption of the population [1,2,7].

## 3. An econophysics approach for Keynes's investments multiplier

### *3.1. Statistical Econophysics and Phenomenological Econophysics*

As it was shown in our previously papers [4-7] in order to study the economic phenomena and processes, in addition to specific methods of economic science, researchers used the methods and theories borrowed from other sciences, especially from the exact sciences (mathematics, physics) or natural sciences (physics, biology, chemistry) whose laws and concepts have an exact character [5-7].

The researchers in economics are working with the economic data series and mathematical methods in order to develop statistical models of economy and to elaborate econometric models or econometric equations etc. Using also the methods of statistical physics including quantum statistics has led to the emergence of **statistical econophysics.** This name comes from the fact that we are dealing with a large number of events, states or variable parameters from the economic practice, which can be considered to have a collective-type behavior, and therefore they can be described and analyzed with the laws of statistical physics or statistical mathematics in the frame of *Statistical Econophysics*.

The studies in the domain of statistical econophysics had a great development, especially in the last two decades, especially in the domain of stock markets analysis. But within the frame of the statistical econophysics, there have also been and are being studied other economic processes or subjects such as: global or individual income distributions [8,9], companies distributions by size [11,12], social phenomena (demographics etc.), the distribution of money [10], the distribution of resources (gold, oil etc.) or products etc.

On the other hand, as we pointed out in a previous papers [4-6,13], econophysics also implies the inclusion in its area of research of other domains of physics (not only statistical physics), especially from phenomenological or fundamental physics, as chapters of electricity, solid state physics, thermodynamics, nuclear or optical physics etc. Indeed many researchers included in their papers some econophysics models based on analogies between the economic phenomena or processes and laws or phenomena from different domains of physics other than statistical physics (see for example [4-7,13]). In this way we can talk about a new econophysics' direction named **phenomenological econophysics** [5,12].

Both the econophysics types and directions are equally useful for modeling of the economic processes or phenomena, resorting to the laws of general (phenomenological) physics or especially to the statistical physics in the frame of the statistical econophysics [5,13]. In the next section we give as example, a phenomenological econophysics model named Economic Amplifier, which we have early proposed and analyzed in our previously papers [5, 13].

## *3.2. An Econophysics Model of Keynes's multiplier*

### ***3.2.1. General Considerations***

Numerous studies and papers have shown that physics is the most suitable for modeling the economic phenomena and processes because allows establishing similarities between the laws or rules determined in the case of economic studies and laws in different domains of physics both for statistical econophysics and phenomenological econophysics.

It follows that for proving certain semi-empirical economic laws or rules, which can have an insufficient degree of approximation due to simplifying assumptions, more indicated appear to be the economic models that can be created based on **natural sciences** such as physics or chemistry, knowing that the laws of nature are stable, being considered “immutable”, always having a rigorous definition, resulted by following the completion of the natural balance in the processes of evolution and development. In this situation, if the economic laws, which sometimes are largely influenced by the human factor through subjective assessments or decisions, determined by personal or collective relationships or interests etc., could be modeled through similarities with models (patterns) derived from natural sciences, especially from physics, this could shape the model of the subjective influences, to be minimized or eliminated as far as possible and the studied economic model to be corrected and optimized [4-7].

In this way, application of models from physics, mathematics or chemistry it has proved particularly useful, which even generated some new disciplines or interdisciplinary research branches [5].

As it was mentioned above the empirical or deterministic economic laws, in many cases, present certain regularities similar to those encountered in physics or other exact sciences. For example, excluding subsistence economy supported by rudimentary means, the modern economy is evolutionary type, i.e. it shows an ascendant shape, presenting an amplification effect of initial investments. So, the amplification phenomena from physics are more suitable to describe the outgrowth of economic processes because the economic development means the amplification of value of utilized and processed resources. Based on this analogy, in our studies was proposed an econophysics model structured on amplification phenomenon concept from applied physics (electronic physics and semiconductor physics etc.) named **economic amplifier** (Fig. 1, a, b) [14].

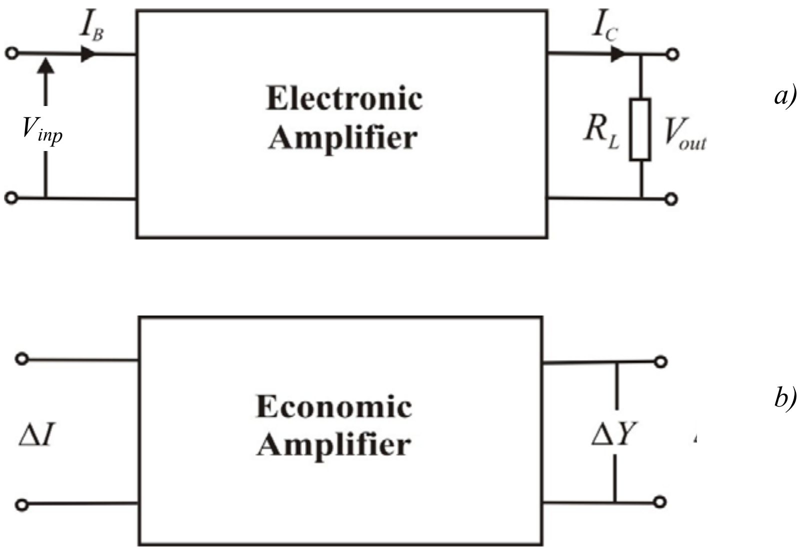



**Figure 1.** a) Simplified representation of an electronic amplifier with transistors or integrated circuits; b) The scheme of economic amplifier.

Schematically, the new econophysics model called the economic amplifier is designed similar to the quadrupole structure of an electronic amplifier from physics, made with semiconductor amplifying devices (transistors or integrated circuits etc.) as shown above by Figure 1, a, b.

The electronic amplifier is designed to amplify electrical signals $v_{inp}$ applied at the input of the amplifier, the amplified signal $v_{out}$ being measured as the electric voltage collected at the terminals of an electrical load resistor $R_L$ (Figure 1, a). Such an amplifier amplifies both the current and the voltage $V_{out}$ or the power obtained at the output of the amplifier.

One of the most important parameter of an electronic amplifier from physics (or electronics) is the amplification factor, sometime noted with β, which is defined by equation [15,16]:

$$\beta = I_{out} / I_{inp} = Output / Input \tag{9}$$

where $I_{inp}$ and $I_{out}$ represent the input current and respectively output current from active electronic device (transistor or integrated circuit etc.) from final amplifier stage of electronic amplifier structure (Fig. 1, a).

For economic amplifier that models the economic unit's activity (company, factory etc.), we have proposed a similar amplification factor [14]:

$$\beta_{economic} = Income / Investments = \frac{Y}{I}, \quad (10)$$

or for financial organizations (banks, mutual funds, leasing corporations, insurance corporations etc.) as:

$$\beta_{bank} = Financial\ income / Initial\ capital. \quad (11)$$

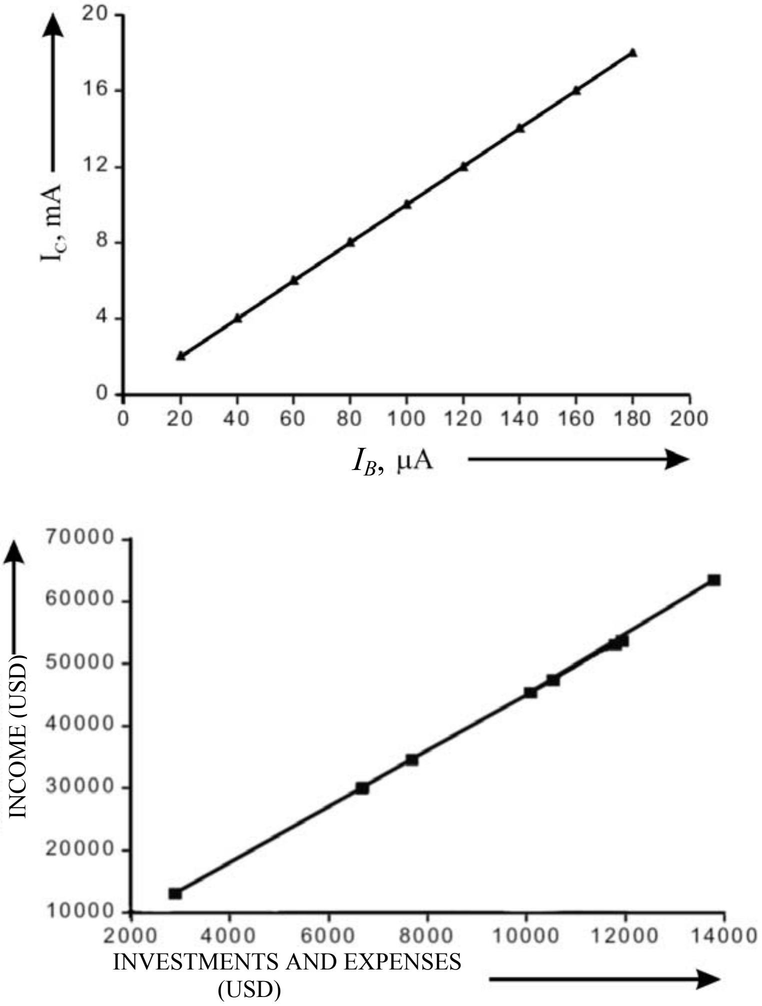
### *3.2.3. Correlation of economic factor amplification with Keynes's multiplier parameter*

As shown in [4-7, 14], the economic amplifier model can be very well applied for modeling the economic development based on the total investments both at the micro and the macroeconomic levels [4,5]. So, for the electronic amplifier with the bipolar transistor in common emitter configuration or with the integrated circuits (Fig. 1, a), the amplification factor β is defined as (see eq. (9)):

$$\beta = \frac{I_{out}}{I_{inp}} = \frac{I_C}{I_B} \quad (12)$$

where $I_C$ represents the collector current of the output transistor and $I_B$ represents the input current through the base of the first transistor from the first stage of electronic amplifier [4-6,14]. To any electronic amplifier for low input current variations $\Delta I_B$ we obtain a greater variation (amplified) $\Delta I_C$ of the current in the output circuit, so an amplification of the input signal [16].

The dependence of the output or collector current $I_C(\text{mA}) = I_{out}$ as a function of input current, $I_B(\mu\text{A}) = I_{inp}$ of an electronic amplifier, therefore, β, given by Eq.(12), have a linear shape resulting a constant value for β (Fig. 2,a).




a)

b)

**Figure 2.** a) $I_C = f(I_B)$ dependence represents a straight line; b) Income dependence of investments and expenses for a small manufacture [5,14].

Indeed, the Eq. (9 or 12) can be written as:

$$I_C = \beta I_B \tag{13}$$

representing a straight line (when $\beta = \text{const}$) intersecting the origin of the coordinate axes (Fig. 2,a).

Under normal economic conditions and policies, by applying the economic amplifier model for the investments analysis, one may see that the output's income as a function of input total expenses (investments, raw materials etc.) has also a linear or near linear dependence (Fig. 2,b) justifying the econophysics model we have proposed [4-6].

From equation (10) that defines $\beta_{ec}$ we can see that an increase $\Delta I$ of investments is followed by a corresponding increase of income $\Delta Y$ i.e. we also have:

$$\beta_{ec} = \frac{\Delta Y}{\Delta I}. \tag{14}$$

Equation (14) coincides with the investments multiplication factor $k$ from the famous model developed by Keynes [1] given by the equation (5):

$$k = \frac{\Delta Y}{\Delta I} \tag{15}$$

where $\Delta Y$ represents the increase of income and $\Delta I$ the increase of investments.

From a comparison between equations 14 and 15, results that $\beta_{ec}$ represents the econophysics equivalent of Keynes multiplier $k$, namely:

$$\beta_{ec} = k. \tag{16}$$

From those shown above it is found that for $\beta_{ec}$ = constant, the equation (10) can be written as:

$$V = \beta_{ec} I \tag{17}$$

that represents a linear ascending dependence confirmed in economic practice, both at the microeconomic and macroeconomic level [4-6, 14].

## 4. Conclusions

In the last 2-3 decades were developed several econophysics models for studying economic phenomena and processes both in the frame of statistical econophysics (in particular for the study of financial and banking processes), as well as in the frame of phenomenological econophysics based on the similarity between the economic laws, processes or phenomena and the laws of different domains of physics (other than statistical physics) as electricity, solid state physics, thermodynamics, nuclear or optical physics etc.

The use of economic models created based on simplifying assumptions or axioms (in the case of econometric models) or based on some semi-empirical laws do not always confer accuracy or certainty to the various practical applications in economics. More reliable and accurate are the econophysics models because they are based on the analogy or similarity between economics phenomena or processes and the laws and phenomena of physics or chemistry, which are exact sciences and also natural sciences, knowing that the laws of nature are accurate and stable, having a rigorous form, resulting from achieving the natural balance of the development processes.

As a result, if we introduce and apply an econophysics model based on laws and phenomena of physics (that is an exact and a natural science) the following possibilities appear:

a) the econophysics model does not confirm the uncertain existing economic models or the semi-empirical laws previously determined for the analyzed economic process; in this case shall take account and shall be used the econophysics model whose characteristics, assumptions or laws (expressed by equations or mathematical relations etc.) were deduced based on some analogies or similarities with laws of physics, which as we already mentioned, are exact laws that describe natural processes or phenomena, so which cannot be questioned, are immutable and describe accurately the respective phenomena.

b) the econophysics model does not have a corresponding economic model with which to compare, such latter model wasn't proposed until the econophysics model has been applied. In this case shall be applied the only existing model i.e. the econophysics model with the conclusions and rules (equations or mathematical relations etc.) determined by the respective model.

c) the proposed econophysics model confirms the semi-empirical laws found in the case of a previously proposed economic model, laws that may continue to be used, both by the old economic model and by the new econophysics model developed for that process.

In this last category can be included Keynes multiplier $k$, this being confirmed by the econophysics model of economic development i.e. by the economic amplifier proposed by the authors since 2007 [14].

In conclusion, in this paper has been shown that the amplification factor of economic development process $\beta_{economic}$ is identical to Keynes multiplier $k$ [1,2].

Thus, the "Economic Amplifier" model being based on the amplification laws of physics confirms Keynes Multiplier Model proposed for economic development through investments and efficient use of labor force.

From those shown above results the great importance of econophysics for economic theory and practice: the econophysics models can validate or not the use of economic models proposed so far, for the study of different economic laws, processes or phenomena.

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