THE CHAOTIC PUBLIC DEBT GROWTH MODEL: GREECE

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Abstract. *Chaos theory is used to prove that chaotic fluctuations can indeed arise in completely deterministic models. Chaotic systems exhibit a sensitive dependence on initial conditions: seemingly insignificant changes in the initial conditions produce large differences in outcomes.*
The basic aim of this analysis is to provide a relatively simple chaotic public debt growth model that is capable of generating stable equilibria, cycles, or chaos. It is important to analyze the stability of the public debt growth in Greece in the period 2000-2014. This paper confirms stable growth of the public debt in Greece in the observed period.

Keywords: *Public debt, Growth, Chaos O4, H62.*

1. Introduction

The global financial crisis developed with remarkable speed starting in the late summer of 2008. This crisis has damaged a large part of the world's financial system. A typical financial crisis is described by declines in asset prices. Government budget deficit raises real interest rates. When the government spends more money than it spends, then the government runs a budget deficit and public saving becomes a negative number. Further, the higher interest rate reduces net foreign investment (net capital outflow). Reduced net foreign investment (net capital outflow), in turn, reduces the supply of domestic currency in the market for foreign-currency exchange, which causes the real exchange rate of domestic currency to appreciate.

In an open economy, government budget deficit raises real interest rates, crowds out domestic investment, decreases net capital outflow, decreases the level of asset prices, and causes the domestic currency to appreciate. However this appreciation makes domestic goods and services more expensive compared to foreign goods and services. In this case, exports fall, and imports rise. A decrease in aggregate demand causes output and prices to fall. The recession may further increase budget deficit and public debt.

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The European debt crisis is a debt crisis that has taken place in several eurozone member states since the end of 2009. The European debt crisis was characterized by an environment of overly high government structural deficits and accelerating debt levels. The states faced a strong rise of interest rate spreads for government bonds. The Greek government-debt crisis is generated by strong increase in government debt levels along with continued existence of high structural deficits.

Deterministic chaos refers to irregular or chaotic motion that is generated by nonlinear systems evolving according to dynamical laws that uniquely determine the state of the system at all times from a knowledge of the system's previous history. Chaos embodies three important principles: (*i*) extreme sensitivity to initial conditions; (*ii*) cause and effect are not proportional; and (*iii*) nonlinearity

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2. Methodology

Chaos theory is used to prove that erratic and chaotic fluctuations can indeed arise in completely deterministic models. Chaos theory reveals structure in aperiodic, dynamic systems. The number of nonlinear business cycle models use chaos theory to explain complex motion of the economy. Chaotic systems exhibit a sensitive dependence on initial conditions: seemingly insignificant changes in the initial conditions produce large differences in outcomes. This is very different from stable dynamic systems in which a small change in one variable produces a small and easily quantifiable systematic change.

Chaos theory started with Lorenz's [1963] discovery of complex dynamics arising from three nonlinear differential equations leading to turbulence in the weather system. Li and Yorke [1975] discovered that the simple logistic curve can exhibit very complex behavior. Further, May [1976] described chaos in population biology. Chaos theory has been applied in economics by Benhabib and Day [1981, 1982], Day [1982, 1983], Baumol W. & Benhabib, J. [1989], Grandmont [1985], Goodwin [1990], Medio [1993], Lorenz [1993], Jablanovic [2010, 2012a, 2012b], Puu, T. [2003], Zhang W. B. [2006], among many others.

3. The model

The chaotic public debt growth model can be described by the following equations:

$$S_t = I_t + NCO_t + rB_{t-1} \tag{1}$$

$$S_t = sY_t \tag{2}$$

$$k_m = \frac{\Delta K}{\Delta Y} \tag{3}$$

$$NCO_t = nY_t \tag{4}$$

$$I_t = K_{t+1} \tag{5}$$

$$B_{t-1} = dY_t \tag{6}$$

$$Y_t = K_t^{1/2}. \tag{7}$$

Where: S – national saving; Y – the real gross domestic product; NCO – net capital outflow (net foreign investment); I – investment; B – public debt; n – net capital outflow as a percent of the real gross domestic product; d – public debt as a percent of the real gross domestic product; K – capital; K_m – the marginal capital coefficient; ΔK – the increment in capital; ΔY – the increase in output (*GDP*); r – real interest rate; s – saving rate.

By substitution one derives:

$$B_t = \left[\frac{(s - n - r\ d + k_m)}{k_m}\right] B_{t-1} - \left(\frac{1}{d\ k_m}\right) B_{t-1}{}^2. \tag{8}$$

Further, it is assumed that the current value of the public debt is restricted by its maximal value in its time series. This premise requires a modification of the growth law. Now, the public debt growth rate depends on the current size of the public debt, B, relative to its maximal size in its time series B^m. We introduce b as $b = B / B^m$. Thus b range between 0 and 1. Again we index b by t, *i.e.*, write b_t to refer to the size at time steps $t = 0,1,2,3,...$ Now growth rate of the public debt is measured as:

$$b_t = \left[\frac{(s - n - r\ d + k_m)}{k_m}\right] b_{t-1} - \left(\frac{1}{d\ k_m}\right) b_{t-1}{}^2. \tag{9}$$

This model given by equation (9) is called the logistic model. For most choices of s, n, r, d, and K_m there is no explicit solution for (9). This is at the heart of the presence of chaos in deterministic feedback processes. Lorenz (1963) discovered this effect – the lack of predictability in deterministic systems. Sensitive dependence on initial conditions is one of the central ingredients of what is called deterministic chaos.

This kind of difference equation (9) can lead to very interesting dynamic behavior, such as cycles that repeat themselves every two or more periods, and even chaos, in which there is no apparent regularity in the behavior of b_t. This difference equation (9) will possess a chaotic region. Two properties of the chaotic solution are important: firstly, given a starting point b_0 the solution is highly sensitive to variations of the parameters *s*, *n*, *r*, *d*, and K_m; secondly, given the parameters *s*, *n*, *r*, *d*, and K_m, the solution is highly sensitive to variations of the initial point b_0. In both cases the two solutions are for the first few periods rather close to each other, but later on they behave in a chaotic manner.

4. The logistic equation

The logistic map is often cited as an example of how complex, chaotic behavior can arise from very simple non-linear dynamical equations. The logistic model was originally introduced as a demographic model by Pierre François Verhulst, a Belgian mathematician interested in the modeling of human populations. He applied his logistic equation for population growth to demographic studies. The model was created to produce solutions for a single population that grows from a small initial number of individuals to a limited population with an upper bound implicitly set by factors such as food supply, living space or pollution (the effect of a "carrying capacity" that would limit growth). His formula is called the "logistic model" or the Verhulst model.

It is possible to show that iteration process for the logistic equation

$$z_t = \pi \, z_{t-1}(1 - z_{t-1}), \quad \pi \in [0,4], \quad z_t \in [0,1] \tag{10}$$

is equivalent to the iteration of growth model (9) when we use the identification

$$z_t = \frac{1}{d\,(s - n - r\,d + k_m)}\, b_t \tag{11}$$

and

$$\pi = \left[\frac{(s - n - r\,d + k_m)}{k_m} \right]. \tag{12}$$

Using (9) and (11) we obtain

$$z_t = \frac{1}{d\ (s-n-r\ d+k_m)}\ b_t =$$

$$= \left[\frac{1}{d\ (s-n-rd+k_m)}\right]\left\{\left[\frac{(s-n-r\ d+k_m)}{k_m}\right] b_{t-1} - \left(\frac{1}{d\ k_m}\right) b_{t-1}^{\ 2}\right\}$$

$$= \left\{\left(\frac{1}{dk_m}\right) b_{t-1} - \left(\frac{1}{d^2 k_m (s-n-rd+k_m)}\right) b_{t-1}^{\ 2}\right\}.$$

On the other hand, using (10), (11) and (12) we obtain

$$z_t = \pi\ z_{t-1}(1-z_{t-1}) =$$

$$= \left[\frac{(s-n-r\ d+k_m)}{k_m}\right]\left[\frac{1}{d\ (s-n-rd+k_m)}\right] b_{t-1}\left\{1-\left[\frac{1}{d\ (s-n-rd+k_m)}\right] b_{t-1}\right\}$$

$$= \left\{\left(\frac{1}{dk_m}\right) b_{t-1} - \left(\frac{1}{d^2\ k_m\ (s-n-rd+k_m)}\right) b_{t-1}^{\ 2}\right\}.$$

Thus we have that iterating (10) is really the same as iterating $z_{t+1} = \pi\ z_t (1-z_t)$ using (11) and (12). It is important because the dynamic properties of the logistic equation (10) have been widely analyzed (Li & Yorke [1975], May [1976]).

It is obtained that:

- For parameter values $0 < \pi < 1$ all solutions will converge to $z = 0$;
- (ii) For $1 < \pi < 3.57$ there exist fixed points the number of which depends on π;
- (iii) For $1 < \pi < 2$ all solutions monotonically increase to $z = (\pi - 1)/\pi$;
- (iv) For $2 < \pi < 3$ fluctuations will converge to $z = (\pi - 1)/\pi$;
- (v) For $3 < \pi < 4$ all solutions will continuously fluctuate;
- (vi) For $3{,}57 < \pi < 4$ the solution become "chaotic" which means that there exist totally aperiodic solutions or periodic solutions with a very large, complicated period. This means that the path of z_t fluctuates in an apparently random fashion over time, not settling down into any regular pattern whatsoever.

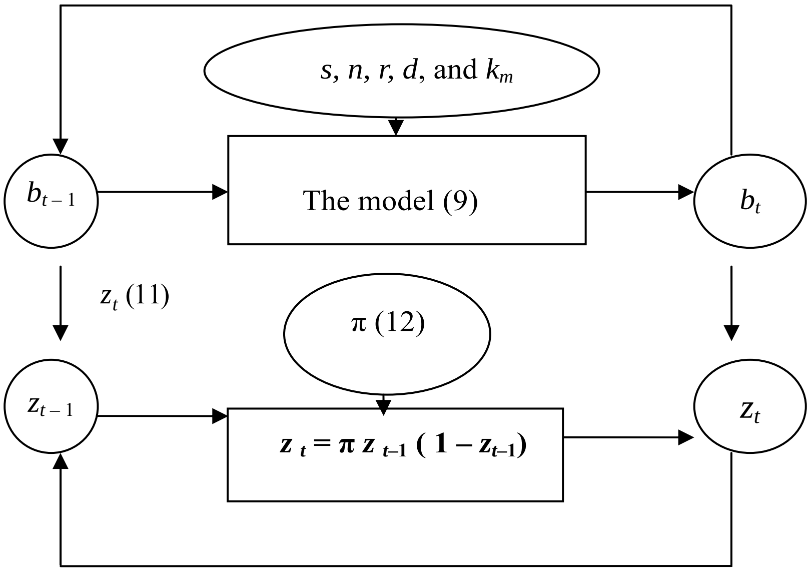

Figure 1. Two quadratic iterations running in phase are tightly coupled by the transformations indicated.

5. Empirical Evidence

The main aim of this paper is to analyze the public debt growth stability in the period 2000-2014, in Greece, by using the presented non-linear, chaotic public debt growth model (13):

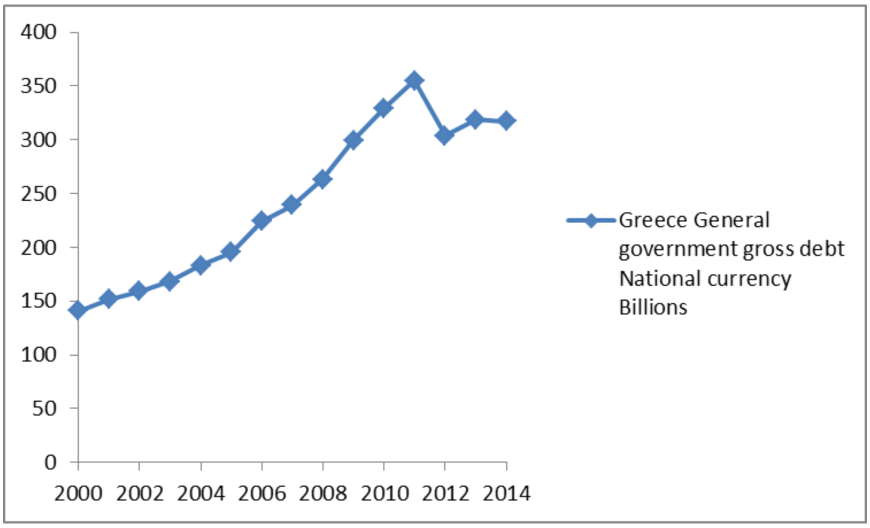

Figure 2. General government gross debt (billions Euros): Greece 2000-2014 (www.imf.org)

$$b_t = \pi\, b_{t-1} - \theta\ b_{t-1}^2 \qquad (13)$$

where:

b – the public debt,

$\pi = \left[\dfrac{(s - n - r\, d + k_m)}{k_m} \right]$,

$\theta = \left(\dfrac{1}{dk_m} \right)$.

Firstly, data on the public debt are transformed (www.imf.org) from 0 to 1, according to our supposition that actual value of the public debt, *B*, is restricted by its highest value in the time-series, B_m. Further, we obtain time-series of $b = B / B^m$.

Table 1.

The estimated model (13): Greece, 2000-2014 (Variance explained: 93.194%)

	π θ
Estimate	1.25214 .268663
Std.Err.	.9171 .114257
***t*(12)**	13.65368 2.351395
***p*-level**	0.0000 0.036620

(www.imf.org)

6. Conclusion

This paper creates the simple chaotic public debt growth model. The model (9) has to rely on specified parameters *s*, *n*, *r*, *d*, and k_m, and initial value of the public debt, b_0. This difference equation (9) will possess a chaotic region. Two properties of the chaotic solution are important: firstly, given a starting point b_0 the solution is highly sensitive to variations of the parameters *s*, *n*, *r*, *d*, and k_m; secondly, given the parameters *s*, *n*, *r*, *d*, and k_m, the solution is highly sensitive to variations of the initial point b_0. In both cases the two solutions are for the first few periods rather close to each other, but later on they behave in a chaotic manner. A key hypothesis of this work is based on the idea that the coefficient $\pi = (s - n - r\, d + k_m) / k_m$, where: *r* – real interest rate, *s* – saving rate,

n – net capital outflow as a percent of the real gross domestic product, d – public debt as a percent of the real gross domestic product, k_m – the marginal capital coefficient.

The estimated value of the coefficient π is greater than 1. This result confirms stable public debt growth in Greece in the period 2000-2014. In accordance with the obtained results, fiscal policy reforms are recommended.

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