FROM RIEMANN HYPOTHESIS APPROACH VIA THE THEORY OF CATEGORY TO MODERN MONETARY THEORY

Alain Le MEHAUTE*, Philippe RIOT**, Dmitrii TAYURSKII**

Abstract. *The Riemann and Goldbach hypotheses are explicit respectively since 1859 and 1742. The proof of the validity of these hypotheses is today unknown because the mathematicians face a major difficulty due to entanglement, – insurmountable in the frame of usual arithmetic – between addition and multiplication included inside the zeta function expressions. A progression, step by step upon the different pillars of the theory of category highlights nevertheless a fundamental decoupling between both concepts within 2 distinct monoids* $(N+)$ *and* $(N\times)$ *upon the field of natural numbers. Then we can show the consequences of putting them in coincidence. This coincidence brings to the emergence a renormalization group which, applied to the Riemann zeta function, provides, via the theorems of Voronin and Bagchi, mathematical proofs of the validity of both hypothesis.*

For fundamental reasons the authors assert the existence of a relationships between zéta function and the dynamics of complex systems. The economic system being among others, an example of complex systems, the authors show that the Adam Smith assumption concerning the invisible hand of the market is only relevant if the concept of equilibrium and/or steady state are relevant, that is to say, if the set of the zeros of zéta function is able to represent the spectrum of the economy. Nevertheless, in this case the economy is static, it behaves like a close-system, and evolution is no other than the result of strictly random processes.

The Riemann hypothesis is solely associated to the zeros of the zeta function; therefore, this hypothesis excludes any consideration about the environment of the set of zeros. It tells nothing concerning the functional relationships that characterize the fundamental symmetries of zeta functions. But, these functions staying partly determined by the renormalization group, this group becomes the source of freedom that is hidden when focusing on the field of the zeros. Outside of the zeros field, the splitting (the decomposition into a product of primes, or in terms of physics, in the characteristic time of the each specific action) leads, to a symmetry breaking and to the emergence of entropic factors. These factors invalidate the market efficiency hypotheses and fundamentally accredit the role of decisions and axiological choices.

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The zeta splitting explains the requirement for currency in economic exchanges. We show why the monetarist hypothesis can be associated with the characteristics of the zeta function and why the symmetry breaking induces the importance of the role of a monetary policy in economics.

Keywords: *Riemann Hypothesis, Categories Theory, Fractal and Hyperbolic Geometry, Path Integrals, Irreversibility, Management, Modern Monetary Theory.*

1. Introduction

For any beotians the mathematics differs from physics, in that the foundations of this science concerns natural objects, and not abstract entities. But a deep analysis of this topic shows that this point of view is not so simple. For instance, the mathematical physics is the science of axioms and logical consistencies from which 'scientific objects', for instance 'atoms', emerge. These 'beings' are obviously 'technical beings' but they contribute to our representation of the real world because they raise our capability of experimentation or of falsification. This capability results mainly from the checking of invariances in algebraic relations, whose archetype is the symmetry relations (Boson *versus* Fermion; Matter *versus* Antimatter, etc). In category theory, the equivalents of these relations are named homomorphisms [21, 22]. The ambiguity of the concept of 'scientific objects' can then be expressed through the relations between operators (fonctors). It is a specific mathematical object. As Dana Mackenzie notes in the book about the morphisms of algebra [30], the implementation of a couple of operations within a field, – if the second operation is managed through a neutral element –, leads to what we name a group of transformations. For instance, the operations $(+,-)$ *or* $(\times,\div)$ - define a Group. It differs upstream from morphisms on a field characterized by 4 algebraic operations with two neutral elements $(+,-,\times,\div)$. They also differ from unitary rings which compute 3 operations with only a sole neutral element, the unit $(+,\times,\div)$. They are also distinguished downstream from morphisms upon the associatif unitary magma. Magma is associated with the concept of half-group. If, this last is regular, the magma is called monoid. The concept of monoid is defined by a single operation -on a certain field, N hereafter. Monoids can be associated as contravariant operations. For instance, we can define a product $(N\times)$ and its co product $(N+)$ respectively inductive and projective covariant structures. The aim of this note is to address the emergence of one-parameter groups resulting from the coincidence between $(N\times)$ and

$(N+)$. According to an historically vision, we must note that in his treaty titled monadology [29,32], Leibniz concluded – against Newton whose aim was to reach the 'scientific object' through its universality by taking the time away-that the links between a couple of monads, leads the emergence of a fundamental pattern (represented by a pond in his masterpiece) which is named renormalization group, in our current mathematical language (Monadologie § 67). This last is a group having a sole degree of freedom [5,45]. This degree may be reducible to the usual temporality [5]. This 'historical' parameter might justify the increasing of complexity observed through the phylogenetic tree. The purpose of this note is to highlight the internal dynamic meaning of the word 'complexity'[28]. The analysis will be built from a specific categorical approach of the resolution of the Riemann conjecture (1859) [3, 8, 9, 17, 35, 41]. The Goldbach hypothesis (1752), will be left for other publications [37].

2. First puzzle elements

The Riemann hypothesis concerns the properties of the zeros of the series $\zeta_D(s) = \sum_n b_n(n)^{-s}$ respectively Riemann and Dirichlet series if $b_n = 1$ or $b_n \neq 1$. These series depend of a complex variable $s = \alpha + it$, with as usual $i^2 = -1$. These functions are also expressed using primes with, for example $b_n = 1$, $\zeta(s) = \prod_{p \in P_1} 1/(1 - p^{-s})$. More precisely $\zeta(s)$ is the only analytic function upon C the integer complex set, private of the unit, in coincidence with the series for $\mathrm{Re}(s) > 1$. The Riemann hypothesis also relates the link between the Riemann functions and analytical functions of the form $\sum_n a_n(s)^{-n}$. In the field of analysis this connection operates through the Voronin theorem [44] which asserts that it is possible to approximate any holomorphic function by using an 'universal function', the Riemann function being precisely a member of this class of functions.

By involving the complex variable in a power expression, the analytic functions acquires a remarkable property that is no less than the one-unit increase in the dimension of freedom of any real variable function. This conceptual step radically changes the usual physical standpoint because it overcomes the role of singularities – that must be removed in physics. This feature gives to holomorphic functions the complex parametrization of any hyperbolic manifold [7, 11, 12, 14-16, 25]. The time parameter, usually used to represent status change and the dynamic state of a 'scientific object', is then split within two parts. This new class of representation increases considerably the possibilities for physical treatment of the dynamics by replacing a sole geodesic, like a serie of 'steady states' depending on time, by a manifold seen as a functional series. Initiate a study on analytic functions is therefore equivalent to add a freedom in the dynamic of any processing. This freedom allows for bypassing eventual catastrophic situations when many steady states are in competition. Another aspect of the complex extension is related to the peculiar duality rising between analytic and Riemann function, this last function inverting the role of the power law with respect to the analytic function with $(n)^{-s}$ while the analytic function is expressed using a polynomial form, in which the complex number s is a variable to a certain power $a_n(s)^{-n}$.

This duality raises a major difficulty when it is carried through an algebraic sum as it is usual in measure theory. The function $\sum_n a_n(s)^{-n}$ is then a function in the form of a series expansion, the traditional meaning being in this case a Taylor series or a Fourier transformation, so that, the zeta Riemann function $\zeta(s)=\sum_n (n)^{-s}$ and its equivalents characterized by b_n, raises difficult mathematical and physical problems [34] requiring new concepts such as the concept of 'virtual-set', concept introduced by André Joyal in the framework of category theory. The physical meaning of this concept is an embedment of a singular set into a continuous matrix, afterward cut into infinitely thin slices [38] and analysed from the state.

Starting from Voronin theorem [44], the Indian mathematician Bagchi [3] managed to demonstrate that Riemann hypothesis, -asserting that the zeros of the function $\zeta(s)=0$ are located in the complex plan strictly upon the line $\alpha = 1/2$ $(s = 1/2 + it)$– is true if and only if $\zeta(s)$ is self similar that is to say if it verifies $\zeta(at,s) = a^{\alpha}\zeta(t,s)$ This feature is qualified by the word renormalization [45]. Like on fractal curves a similitude then exists between every part of its diagram and the whole of the function.

The authors are not only concerned by solving the cited hypothesis (Riemann and Goldbach), which establish the conditions of validity of the classes of zeros and their consequences in arithmetic, but also by an another question of practical importance in physics, which is the following [7,]: *if the conjecture is true, (i) what meaning does it give the*

Riemann function outside its zeros and then (ii) what meaning can we attribute to the well-known functional relationship $\xi(s) = \xi(1-s)$ *that impose an ad hoc 'symmetry' on either side of the critical affix ½* ? We have to remind that the parity is expressed through the function $\xi(s) = \frac{1}{2}s(s-1)\pi^{-s/2}\Gamma(s/2)\zeta(s)$ that is to say [9] via some projections involving the combinatorial gamma function. The parity explicitly links the reversibility of a variable looking like a time parameter (associated with complexe part of s (see Voronin theorem)) and the zeros of the zeta function. At this step, we do not know through what kind of artefact the link may be established. The development that follows will help to understand that this constraint is related to an epistemological issue that could be summed up as followed: except within a few number of specific cases (example in quantum mechanics [12]), the natural state of a complex system can not be distributed (and therefore vectorially represented) upon a set of stable states; therefore it cannot be reduced to an 'object of science' [28, 37]. This last was thought to be solely associated with a pure reversible time-space, hypothesis obviously false herein. In almost all cases the vectorial system of representation based upon stable states is incomplete and the human action is necessarily faced with a duality (more generally at a singularity of choices). This last must be transcendental and must choose a reference outside of the system of axioms [29, 32].

This theoretical constraint undermines any will of computation of an 'optimal human action'. Except in very special cases, it is indeed not possible to determine intuitively or rationally, a self-coherent solution of an issue into a complex problematic. So the question of our ability to understand (or to close) a functorial problematic is inextricably linked with mathematical questions, which are still widely misunderstood (except by few mathematicians), and the probability of what could be understood and solved at short term, concerning the hereinabove hypotheses, seems very low. Among the issues that concern us in the first place, there is the econo-logy. Aside, the insurance, and even the arrogance, that the economic scientism may display [10], the econo-physics could become, if we do not take conceptual precautions, a mere sycophant. Obviously that is not the will, of the experts in charge of the science of complex system. The opinion of the authors is that, in front of the econo-physics, the econo-logy should pave the way for a categorical rehabilitation of the 'policy'. This 'sciance' [42], developed through structuralist patterns, – instead of answering to a mere will of 'reification' of our representation of the real –, should help us to streamline our relationship to the reality seen as a dynamic research because this reality is now partly the result our technical creations. We will try to show how the new approach of Riemann hypothesis applied in the framework of categories theory [21, 22, 37], can help us to understand the differences aspects of the above comments.

3. Riemann hypothesis in the framework of the theory of category: state of the art

3.1. Categorical introduction

The zeta function $\zeta(s)$ occupies a prominent place in mathematics. In addition to its role in arithmetic (Euler, Riemann), in combinatorial algebra, in the theory of functional operators, or in the mathematical formalization of thermodynamics, this function has appeared more recently in category theory [see 36]. Through renormalization methods [16, 39, 41], this function has aquired an admittedly less ancient role but no less promising in quantum mechanics, in quantum field theory, in the theory of fluctuations, as well as – via the non-commutative geometry [5] – in quantum gravity and loop theories [40]. Recently the physics of heterogeneous media highlighted the close connection between this function and the engineering problems in fractal geometry [20, 24]. The aim of the current works is to suggest a new mathematical approach for solving of the Riemann hypothesis in the framework of category theory. Indeed, the rules upon the functors and the related theorems allow the authors to highlight the presence of a close connection between the Riemann function $\zeta(s)$ and a general principle of coherence inside a couple of contravariant algebraic concepts [2, 28, 36]. Among examples of contravariant couples let us note: algebraic structures versus its coalgébrique structures; limits and colimits diagram; variables versus parameters, etc. Beyond these achievements, let us observe that the general principle of coherence of a couple of contravariant concepts underlies a great number of mathematical methods of resolution. Let us note for instance the Lie-Trotter formula that is the corner stone of the study of semi-groups of operators or as well, the theorem of Dold-Kan-Puppe in homological algebra. In physics and engineering let us note the structure of the formal thermodynamics, the Maxwell and Dirac equations, or the link with the efficiency of batteries. Beyond this brief overview, we also know that this principle is also involved in a systematic way in the construction of self-similar structures, when they are seen as strange attractors, characterized by renormalization groups and by fractal structures [25, 26, 27, 31]. The authors assert that the categoric approach allows approaching the understanding of the famous conjectures still pending and allows solving numerous complex problems that may be written in the frame of a universal class of categorical issues.

Beyond the precautions required for the definition of its field of validity, it is well known that the function $\zeta(s)$ can be considered from two separate formulations given by a couple of series, involving the operator sum Σ with indexes belonging to any integer N, and secondly an infinite product Π, where the subscripts concerned the set of primes. Subject to consider the categorical asymmetry between addition (coproduct) seen as projection and product seen as surjection, any specialist of categories or even expert in formal languages, may have the intuition of the matching of two monoids – the first being additive the second being multiplicative –, as an opportunity for the renormalisation group, to emerge [36,37].

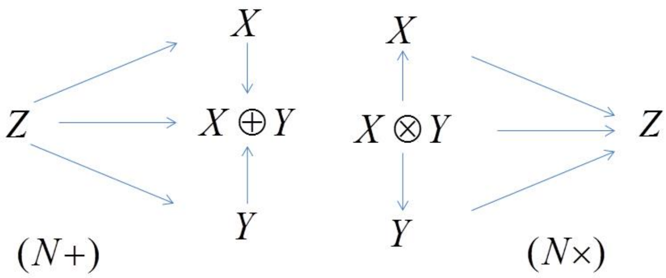

Figure 1. Main categorical algebra distinctions: Product (Partition) *versus* Coproduct (Union). Note the inversion of the arrows to understand the rising of a group through coincidence of the limits. For instance cellular automatons are based on such a structure. Even useful, this diagram is misleading because it supposes a fantasized existence of a final colimit of the cone of the product.

At this stage we cannot neglect a peculiar distinction which has to be considered: if the additive monoid has a clear initial algebra given by $N + 1$ and due to the role of infinity, the multiplicative monoid has not a natural final algebra. This dissymmetry may contribute to render the diagram above, misleading. Obviously, and this point is precisely the main difficulty of the resolution of Riemann hypothesis, this asymmetry must be overcomed to allow to the regular arithmetic consistency. This last is required for reducing the multiplication as to be a sum of additions. Nevertheless, it is obvious that in the case of the zeta function, this putting into coincidence will be complicated by both (*i*) the involvement of primes (atoms of arithmetic) and (*ii*) by the power laws which contribute still more, to the entanglement between the addition and the multiplication hidden into the Riemann hypothesis. Nevertheless, with reference to the usual use in semi-group theory and in the theory of monoids, we may consider the prime numbers such as letters ordered by the lexicographic order of an alphabet of countable infinite size. Any integer *n* is then seen as a word uniquely written in this alphabet with the additional property that the product of two distinct primes remains commutative (the commutative allows moreover an interesting interpretation in terms of automatons or of dependent graphs as well as associated traces). Then, we have for all $n \in N$ $n = \prod_{\{p_i\}} p_i^{r_i}$. It is then useful to suggest a geometric translation using in addition, a linearization of the function $\zeta(s)$ through the relation (*) $\log n = \sum_{\{i\}_{fini}} r_i \log(p_i)$.

3.2. Heuristic approach for solving the Riemann hypothesis [36]

Given a linear space Υ characterized by a dimension $\aleph_0$ where a subspace of dimension 1 is associated with each prime number. Each two dimension sub-area, is then associated with two independent prime numbers. Since it has a non-empty and finite set of coordinates in *N*, if the semi-ring *N* is used as a scalar law, it becomes obvious that, according to the equation (*), each point in the space Υ: $\{r_i\}$ may be enter into a bijection ***B*** defined punctually, by a unique integer. Then let us introduce into the original linear space, an isotropic homothety ***H*** with an amplitude *s* (*s* belonging to a field, typically $s \in C$ the set of complex numbers), then $H_s : \{r_i\} \to \{-s \times r_i\}$. Using the bijection ***B***, this application corresponds to a functional projection $n \to n^{-s}$, defined upon *N* with a co-set ***C***. Note here the introduction of the negative sign for a reason of convergence that will appear naturally in the analysis. Through the above processing, the set *N* is represented by a discontinuous topological set of points whose coordinates are integers. All other coordinates are zero. Therefore the whole set *N* can be 'measured' at different scales *s* through the Riemann function $\zeta(s)$.

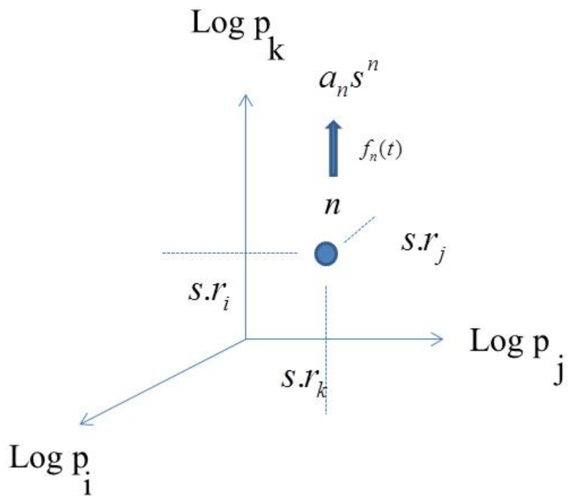

Figure 2. Original 'Zeta-Primes' diagram [36] based upon primes number and used to built analytical function. The total measure of the infinite set of points has zeta function as reference. The parameter *s*, in the relevant compact complex domain, is no more than the stretching factor of the axis, that is to say, the scaling factor of the Riemann Measure (zeta function). Self similarity appears obvious in the framework of this representation.

The measure of Υ is hereafter identified with $\zeta(s)$ seen as a sum of unit through n^{-s} in bijection ***B*** with *N*. In other words $\zeta(s)$ must be understood as the total measure of a discontinuous topological space seen at different scales *s*. Surprisingly, to our knowledge – except communications already issued by the authors of this note [36] –, this interpretation of $\zeta(s)$ as a measure of this type of space does not appear in the mathematical or physical literature.

In the same space Υ we can also consider a second operator defined by two sets of parameters: A global parameter $t \in C$. In practice this field *C* is the same as that which sets the scaling parameter *s* for zeta. A mathematical operator noted *F* establishes a link between an analytical function and each point with integer coordinates $\{r_i\}$. Obviously *t* may be match with *s* if necessary because a collection parameters $\{a_n\}_{n \in N}$ indexed upon *N* is given so that an analytical weight $a_n s^n$ can be given if *n* is defined by (*). Indeed as before, we may consider the total measure of the operator. The measure can define by (**) $f(\{a_n\},t) = \sum_{n \in N} a_n t^n$ (**). For both classes of operator: (*i*) the amplitude of scaling is *s* and (*ii*) the operators that assign a weight $a_n s^n$ *to* the corresponding point via the bijection ***B*** with the integer *n*, (that occurs in the filed considered for *s* =1), is given by the point corresponding to the translated obtained by amplitude of scaling $s + it$. In other words, the point corresponding to the integer *n* by the formula (*) considered in the expanded space with an amplitude *s*, is associated the weight $a_n s^n$. Geometrically, we can consider the graph of the operator *F*, then stretch the coordinate space with the factor *s*. If the functional weighting is such that its initial measure given by (**) is bounded, the coefficients evolve in an appropriate field, and the total measure of functional is given by (***) $\xi(s+it) - f(\{a_n\}_{n\in N}, s)$.

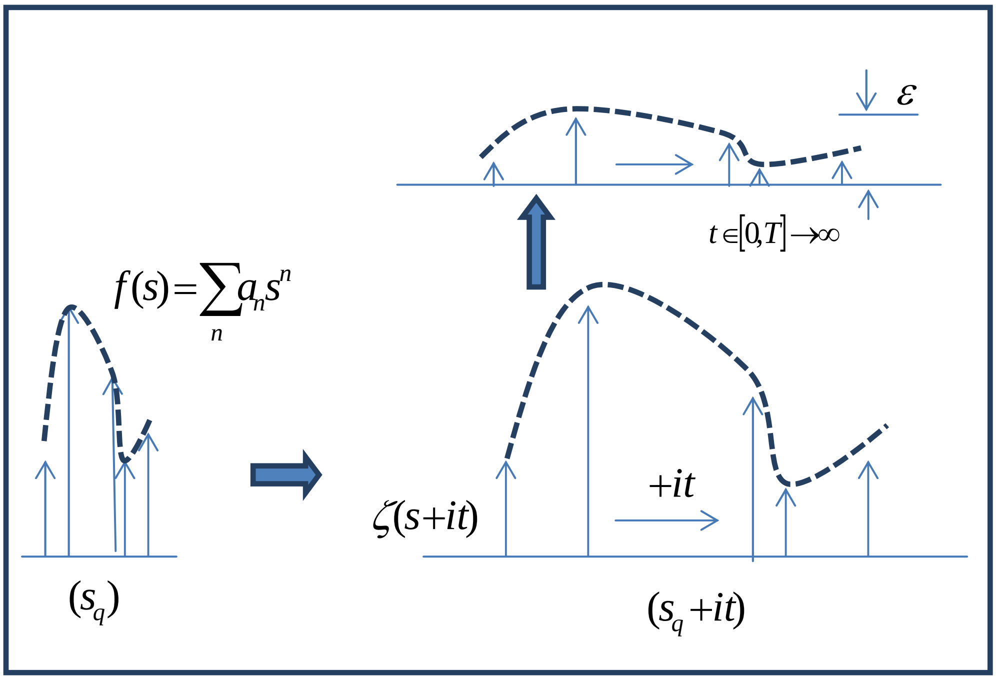

Figure 3. Schematical significance of the universality of the Voronin theorem [44] in terms of stretching of the Zeta-Primes Diagram. The basic line refers to the Riemann measure (zeta function).

Let us suppose that *t* increases to the infinite, the graph of the function *f* tends to 'flatten' upon the coordinate space. This property can be interpreted by stating that the measure of $f(s)$ is infinitely close to the measure of $\zeta(s+it)$ upon a set having a zero measure. This statement is none other than theVoronin' point of view ([44]****):

$$\forall \varepsilon > 0, \liminf_{T\to\infty} (1/T) mes\{t \in [0,T]; \max |\zeta(s+it) - f(s)| < \varepsilon\}.$$

This explanation may sound far from mathematical formalism, but it is justified because both mathematical and intuitive approaches, give a deep understanding concerning the Riemann function $\zeta(s)$ through the analysis which was established in 1975 by Voronin [44] and whose precise statement is the following: *For K a compact set in a critical field* ½<*s*<1 *with a connex environment and for f(s) continuous function with out zero in K and analytic inside,* (****) *is true,* which means in other words that any analytic function can be infinitely approximated by the zeta function, that is universal in the sense of Voronin. The combination of these couple of approaches is a serious indication that the understanding of the Riemann hypothesis using the discrete topological spaceΥ and the categorical operators are probably able to restore the essential of the properties of $\zeta(s)$ functions.

The step used to interpret the Voronin theorem is important for at least two reasons: (*i*) it allows to introduce and interpret the Bagchi's theorem [3], which autorize the connection between the scaling properties of zeta function and the Riemann hypothesis. Indeed, shortly after Voronin had established his theorem (1975), Bagchi proves (1982) that the relevance of the Riemann conjecture is strictly equivalent to the asymptotic condition which would concern the measure contained in the Voronin theorem for the limit case $f(s) = \zeta(s)$. On the other hand (*ii*) the geometric representation of discrete space we designed to build the function $\zeta(s)$ as a distribution upon *N* is satisfied by a simple scaling of the discrete space in bijection with *N* in relation to himself. Being acquired elsewhere, that this space is patently self similar, the approach leads naturally to base the validity of the Riemann conjecture by confirming the Bagchi hypothesis.

In addition, as expresses by the summation formula $\sum_{n_1 \times n_2 = n} f(n_1)g(n_2)$ to describe the process in Υ(2D) extent, let us observe that the function $\zeta(s)$ introduced by scaling operator is defined by series which implicitly involves natural convolution products. In contrast, the second functional *f*(*s*) (**) constructed for the need of the case, is structurally combined with an additive monoid (*N*+). But as suggested by the universality of the function $\zeta(s)$, the conjunction of both algebraic monoids – we shall show what requires a matching between a co initial algebra associated with *N* associated with $(N+)$ and constructing of a specific final algebra associable with $(N\times)$ –, should lead the emergence of a group, which can only be a group to a single free parameter upon ***C***. Due to the initial scaling properties, it can be shown that this is a renormalization group, confirming the self-similarity of $\zeta(s)$. The diagram below provides synthetic approach of the demonstration of the Riemann hypothesis based upon monoids coincidence. Subject to passing to the limit, the two distinct monoidal structures can be combined, involving a categorical compactification. Nevertheless, due to the role of infinity, this path to a limit is not so easier for the multiplicative monoid. We shall see, in the framework of categories theory, which kind of 'projection' is required, to make this transition in a mathematical relevant manner, and how an ad hoc projection causes a natural emergence of self similar features.

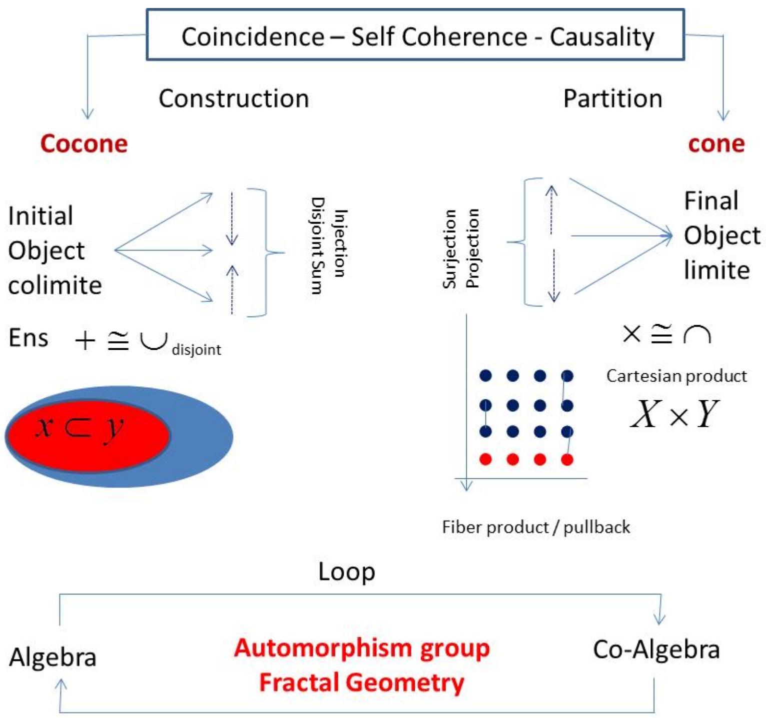

Figure 4. Diagram giving – through the coincidence of algebra with coalgebra and of the count with the measure – the principle of the self coherence of both points of view: construction and partition. The action of forcing the coherence causes the emergence of renormalisation group also characterized by the emergence of time irreversible factor.

4. Yoneda lemma, Kleisli category: physical meaning and applications

In the frame of universal algebra, the connection between the final inductive algebra associated with the product monoïde $(N\times)$ and the initial projective coalgebra associated with the monoïde $(N+)$ has to lead to the emergence of a group. At this stage, we assume that this group is a group of renormalization characterized by one degree of freedom but we have to prove it. Downstream the arguing, this group must validate the demonstration of Bagchi's theorem upon the Riemann hypothesis [3]. The closure (Figure 4) requires a 'practical con-fusion' hence, in physical terms, the emergence of an entropic factor in the sense of thermodynamic theory. This factor led to the assimilation of the parameter of order of the renormalization (spatial in Υ) with a parameter usualy named the 'time', expressed in the field of complex numbers. Due to ambiguity factor, we can then assert that this parameter contains at least one dynamical irreversible component. Given the universality of the Riemann function and by arguing from the theorem of Voronin, it should be possible to find, in the physic of complex systems, some analytical functions related to fractal properties pointing the irreversibility. In this case they should be approximated by part of Riemann functions or extensions of them. Well before the current analysis, this is exactly the fact that one of the authors highlighted by pointing out in 2008 [26,27], a link between the Riemann series and a self-similar canonical distributions associated with transfer functions in heterogeneous media qualified 'Cole and Cole' transfer functions [23, 24, 25]. These functions confirm the link between self similar characteristics, which is the geometrical background of the fractional derivative operators [24, 25], the basis of the Cole and Cole transfert functions and of the zeta series. Nevertheless, the issues resulting from the physical publications need for being mathematically justified, of at least two additional ingredients which can be extracted from the theory of categories [36, 37]: The first ingredient is the Yoneda lemma. It must justify the rôle of a certain phase angle use in the physical approach of the conjecture [23, 26, 27]. The second ingredient is a peculiar class of categories named the Kleisli categories [36, 37]. They are required to assure the construction of a limit for the multiplicative monoïd and hence to achieve a compactification of the class of inductive functors into a multiplicative final algebra.

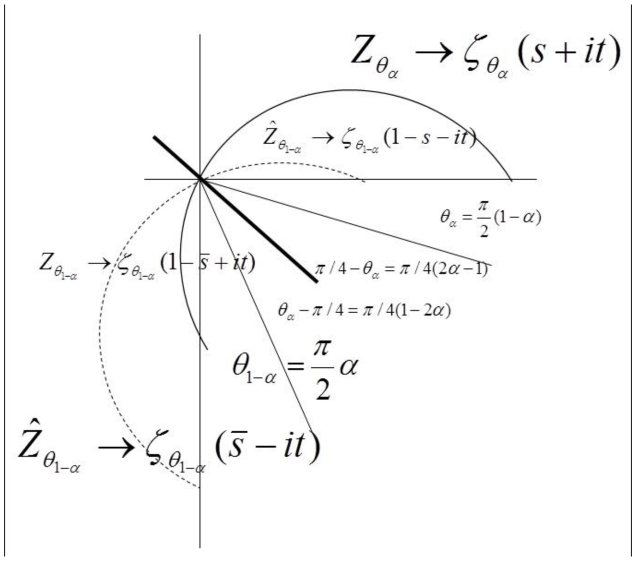

Figure 5. Yonesda basis of zeta Riemann function through Cole et Cole transfer function. This diagram highlights the role of phase angle. The shift of angle with respect to the sign of the rotation founds the relationships with non commutative geometry [5,27].

4.1. Yoneda Lemma

This lemma is related to the embedding of a small category into the category of functors [22]. The objects are identified with the field of representable functors, and the internal morphisms associated with all natural transformations between these functors. It is a vast generalization of Cayley's theorem for groups (seen as small categories with a single object). In other words, if a class of functor is reduced to the different point of views $@\zeta_{\theta_\alpha}(s)$ upon the object, – herein Figure 5 –, the operator @ give its address through the rotation $\pm it_{\theta_\alpha}$ – defined with a phase angle θ_α – Yoneda lemma asserts that the description of an object is equivalent to give the set of all adresses related to the object, as well as their mutual transformations. In practice, the Yoneda lemma justifies the role we have assigned to Cole and Cole canonical transfer functions, like basis (in fact categorical co-limit) of the representation of all possible perspectives (functors) upon $\zeta_{\theta_\alpha}(s)$ parameterized with $\pm it_{\theta_\alpha}$, with the imperative condition of using a relevant tuning factor given by the right phase angle. At this step, our arguing should join another point of view able to be built from Connes noncommutative geometry [5]; in simple words: *the* *commutation change the phase*. But we know starting from all the studies developed upon non integer differential equations that the geometrical basis of the role of the phase angle is none less than the existence of an underlying self-similarity characterizing jointly the Cole and Cole transfer functions, the fractal geometry of the interface of physical transfer in the fourier space, and the path integral upon this fractal geometry [27]. The measure of this path integral upon the dynamics is precisely related to the Riemann zeta function. The problematic is summarized in figure 5 and it is extendable to the functional relationships characterizing the class of functions zeta, according to the bijection established between both following sets $[Z_{\theta_\alpha}(\omega\tau)\; Z_{\theta_{1-\alpha}}(\omega\tau)\; Z_{-\theta_\alpha}(\omega\tau)\; \zeta_{\theta_{\alpha-1}}(\omega\tau)]$ and $[\zeta_{\theta_\alpha}(s)\; \zeta_{\theta_{1-\alpha}}(s)\; \zeta_{\theta_\alpha}(\bar{s})\; \zeta_{\theta_{1-\alpha}}(\bar{s})]$. Of particular note: (i) *nonobstant* the presence or the absence of a physical meaning of both parts of the Cole and Cole diagram, the number n in the expression of a dynamics (abstract or real), gives equivalently the meaning of both hyperbolic lengths linking the three limits, $[0, \infty_+, \infty_-]$ of the dynamic diagrams. Hence, the functional relations between the transfer functions must have their equivalences in the functional relationships upon zeta; (*ii*) the alpha phase angle is an element among others of the problem raised by the Riemann hypothesis, but this element is fundamental because, it determines the relevance of the Yoneda lemma for this problematic. It is indeed the only factor that sets the real part of the power law, therefore the phase required for the definition of the rotation given by the factor $\pm it_{\theta_\alpha}$ and hence a détermination without any ambiguity of the morphisms $Z_{\theta_\alpha} \to \zeta_{\theta_\alpha}(s+it)$; (*iii*) the functional perspectives established on the transfer functions and therefore upon zeta functions allows us to understand the depthness of Riemann's hypothesis – in term of commutative singularity in the framework of non-commutative geometry – that imposes $\alpha = 1/2$ and hence the value for the phase angle: $\theta_\alpha = \pi/4$, some kind of Zeeman condensation. Highlighting the role of primes, this strong constraint involves the disappearance of the entropic factor and a return to a reversible temporality that is to say, to the main condition of existence of 'objects of science' (invariant or stationary state) with respect to standard experimentation.

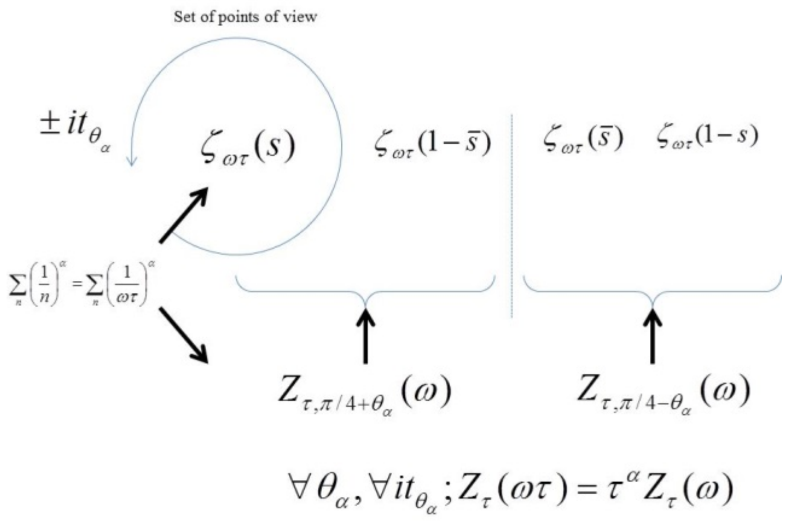

Figure 6. Schematic representation of the relationships betwwen fractional Cole and Cole transfer function and zeta Riemann fonction emphasing the role of the phase angle with respect to the sign of the rotation.

This singular behaviour is justified because it must exist the link between the zeros of the zeta function, and the universal properties of steady states such as those pointed in quantum mechanics [12]. However, the issue concerning the properties of 'complex systems' is more complicated than the one given by the zeros-spectrum. It is more precisely the following: "*what can we tell about the physic of the pseudo-states, located aside the stable states, that is to say, aside of the zeros?*"

At this stage the concepts of monads, of Kan extension and the ones of Kleisli categories play a fundamental role in terms of understanding the Riemann hypothesis in the framework of a more general level of analysis of the complexity. For instance, the Kleisli categories highlight the reason of the transition between the reversible time and the irreversible time; the deepness of the meaning of the decoherence process when changing the coarse graining for zooming upon a complex object. This is rather well known in quantum mechanics, but many other classes of macro-systems, named 'zeta complex systems' by the authors [28], feature the same type of properties, and among them, the econometric systems.

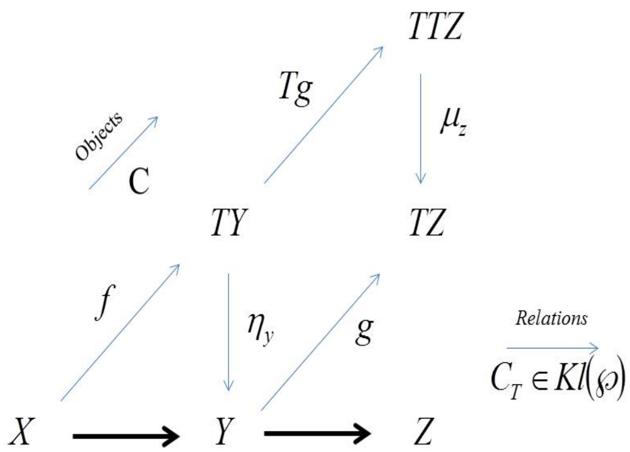

Figure 7. Relationships between Kleisli categories and natural monad $C(T, \eta, \mu)$. The monad is characterized by basic internal relations between internal states that establish both identity and local closure. The relationship with other monads gives rise to a geometry of relations.

As has been seen above, the validation of the Voronin theorem requires the compactification of the multiplicative monoid $(N\times)$ in order to have at disposal a limit, before any folding down this limit upon the natural co-limit of the additive monoid $(N+)$. The conceptual difficulty is that the algebra associated with the multiplicative monoid has no final algebra that means that there is not any limit without an additional mathematical transformation, precisely called compactification. But this operation is far from trivial. However, the authors argue that the compactification can be obtained using a monadic 'projection' upon Kleisli categories $Kl(\wp)$, that is to say, upon a category only concerned by relations, as shown in figure 7 and 8. Recall that all category of relations C_T is a Kleisli category and that it is almost always linkable to a monad $C(T, \eta, \mu)$. The monad plays the role of categorical limit. If we add that we may describe the compactification in terms of adjunction (Figure 8), the aim of our approach is then reached, and a limit of multiplicative monoid is found.

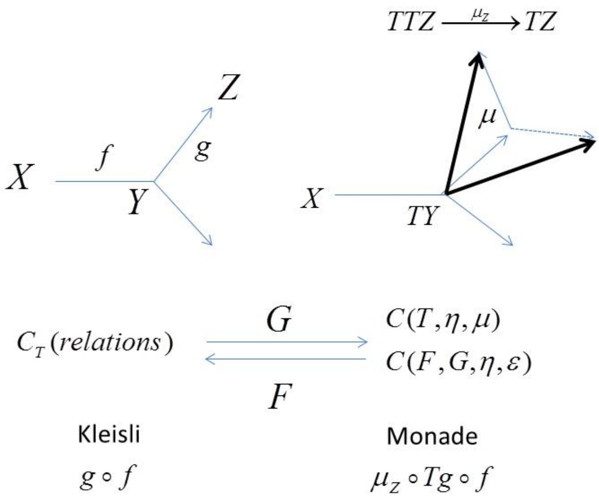

Figure 8. Diagram that highlights how the relationship between Kleisli categories and natural monad and bases the emergence of the renormalization group through the loop of self coherence.

More precisely, Given the monad is defined by $C(F, G, \eta, \varepsilon)$, with $F: C \to C_T$; $G: C_T \to C$; $\varepsilon_Y = 1_{TY}$; $\eta_{TX} = 1_X$ then the Kliesli categories are defined from an entanglement of the morphisms through the fonctorial relation, between C the category of monad *and* C_T the categories of relations. In the frame of group theory, the approach appears perfectly focused upon the limit because as shown in the diagram above (Figure 8), the monadic adjunction requires the existence of a basic pattern, the folding down of the multiplicative pattern on the monadic pattern causing a natural emergence of the generator of a renormalisation group. Let us also observe that the monadic adjunction applies to the transfer functions (Figure 9) where it expresses a morphism concerning the functional relationship. Indeed, we notice herein that the categorical adjunction is at the foundation of the relationship between $Z_{\theta_\alpha}(\omega\tau)$ and $Z_{\theta_{1-\alpha}}(\omega\tau)$, the disjunct sum '*Z*' being viewed as a virtual transfer function based upon a Kan extension $Z_{\theta_\alpha}(\omega\tau)$ along $Z_{\theta_{1-\alpha}}(\omega\tau)$. According to our above arguing; this extension is the basis of the functional relationship well known for the zeta functions. It is in fact possible to create the extension to ensure that Ran *Z* is a functor likely to switch the parametrization orientation. The transfer functions $Z_{\theta_\alpha}(\omega\tau)$, $Z_{\theta_{1-\alpha}}(\omega\tau)$ are indeed the result of a combination of injection points of $Z_{\theta_\alpha}(\omega\tau)$ upon $Z_{\theta_{1-\alpha}}(\omega\tau)$ the inverse injection insuring not only the commutation (*RanZ*=*LanZ*), but also, according to the Yoneda Lemma, a functorial link leading the symmetry between s and 1-s in the relationship: $\zeta(s) = 2^s \pi^{1-s} \sin\left(\frac{\pi s}{2}\right)\Gamma((1-s)\zeta(1-s)$ with $s \in C \setminus \{1\}$ Since there is a structural connection between Kan extension and Fourier transformation, it is confirmed that the functional $\zeta(s)$ equation finds its origin in the existence of a set of eigenfunctions of the Fourier transform of zeta into the certain class of $L^2(R)$ functions. Practically, the entanglement -untied via the Kleisli categories and an unfolded by using the coincidence of the additive and multiplicative monoïds-, involves not only the emergence of a renormalisation group, but also the emergence of a functional relation between $\zeta(s)$ *et* $\zeta(1-s)$. This physical meaning of this relation is remind via the Figure 9.

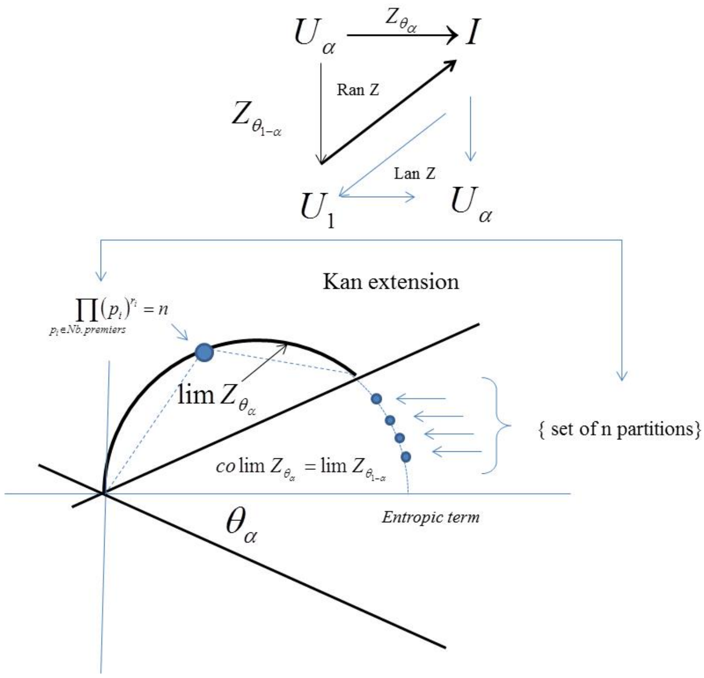

Figure 9. How the commutative Kan extension highlights the irreversible time component when analysed through the Cole et Cole transfer functions. This specific source of irreversibility was first experimentally disclosed 35 years ago [24].

All this issues will be considered in detail in future mathematical publications [37].

5. Consequences in econo-physics and monetary theory

As we noted above, the categorical approach of the Riemann hypothesis allows to understand the following major idea: *out of the field of existence of the non-trivial zeros (Primary steady states), – that is to say out of the field of a deterministic symmetric imposed by* $\alpha = 1/2$ *and* $\theta_{1/2} = \pi/4$ *with* $\zeta_{-\pi/4}(s) = \zeta_{\pi/4}(\bar{s})$ *– the Riemann zeta function leads one to think the fractional dynamic systems and more generally the complex systems -and/or the auto-organized systems-, like couples of dual and asymmetrical components*. The emergence of the duality is due to a non commutativity of the geometry of the dynamics that is to say, the presence of a different phase angle for both sign of rotation (+,–) in other words time singularities at boundaries. In the complex environment, any irreversible dynamics must be divided into two parts (disjoint but able to be adjonct), into two reciprocal categorical 'extensions' leading both projection and surjection. Specifically, one of the components of our representation concerns a geodesic of action and/or relations, parameterized by a frequency, that is to say the coarse graining of the dynamic groups, while the second component relates the distribution of the Fourier transform (the above parameters), onto discret normalized degree of freedom (time constant). This splitting imposes an almost general irreversibility of the processing (to create a relation is obviously not the inverse of cutting it). At this step, the nature of the irreversibility has still to be defined, but sole the couple allows retrieving symmetry through the ability of switching inductive and projective diagrams. In spite of the entanglement of both components, the duality allows to find a combinatorial form of causality and steadyness. We train the assumption that the coupling of both components is a general principle to illuminate complex behaviours. As asserted Clement Rosset [38], for being relevant, we must always consider the real and its shade. From this point of view, the thinking of complexity is moving away of a functional vision (like physical or pure numerical model) for coming close a functorial vision (research of homeomorphic folded upon boundary structures). It is in this context that should probably be revisited what we call 'econo-physics'.

But, in economy, the critical posture is quite difficult [13]. The econo-physics is based on the following idea: the objects of economical exchanges are considered like 'scientific substance', because they have a tangible reality. In this framework, and out of any social considerations, the scientist paradigms want their behaviour must follow some 'natural laws'. They must be searched and found. But the majority of neoclassical concepts appear epistemologically too simplistic with respect to the complexity of the global economy. Maurice Allais at the end of his life like Steve Keen [18] or Gael Girault, doubt of the truth of this point of view even though it represents the main stream of econometrics and monetary studies [10]. The economic crisis and their resolution show that natural laws are in fact social rules. Like the aghast economists, we also doubt of the trueness of the axioms of the standard econometry (too close to physics concepts), and obviously, of its conclusions (too close to equilibrium thermodynamics point of view). This is particularly the case of the assumed separation between physical capital and intangible capital. What makes emergente the 'beings for the exchange' is none other than its social value or the 'price', in the broad meaning of both intangible and macroscopic, that is to say: social and symbolic [18]. There is clearly an irreducible duality between the social value of an economic entity and its existence as part of exchange. The relationships between entities are not even nonlinear, but combinatorial and entangled because they are part of a symbolic whole. Moreover, even if the exchanging laws are simple to express, ("it is bit" or "bit is it"), they no less create the emergence of a great multiplicity of complex structures, markets, whose erratic behaviours certainly does depend neither of the action of an invisible hand nor of a perfect competition, - finding their sources into quasi-thermodynamic point of view. Indeed, the presence of States, monopolies or insiders exclude the efficiency of the performative ideal-scheme even if it is certainly convenient for computing abstract models, or for publishing academics notes.

To sum up, the authors think that we are still very far from an 'econo-physics' that would be able to be regarded as a science, that is to say, as a field of treatment of 'objects of science'; outside of any social symbolic representation. However, that skepticism must not suppress the certitude of a real need for a rational relation to the complex systems (living systems, social systems catastrophic systems, economic world, etc) based upon physical analysis. The 'econo-physics' is a part of a broader problem, that of rationalizing the complexity, seen like an entangled system of relationships and exchanges. Both require an understanding effort based upon very similar formal will. The above analysis suggests that a complex object, as exchange structure, is the component of a class of 'uncompleted scientific objects'. No simple causality, may explain their behaviours. Nevertheless, the field of existence of such ‘tempo-beings’ can be understood using some general rules of category theory. The rules concerning the morphisms are able to highlight some internal degree of freedom, always balanced by some internal symmetry characterizing the duality. By reaching the limits, these relationships are able to lead compactification achievements, Kan extensions, and forcing [4]. The duality that leads to an extended causality and forced steadiness is based upon the closure between what is accounted (flow, cash and stock) depending of a projective computation $(N+)$ and what is measured (price, savings rates,...), that is dependent of an inductive computation $(N\times)$. In the classical theory, this interaction is hidden into both Exp and Log functions giving rise to temperature and time constant concepts. The matching between both monoids for computing does not lead to reproducible universal laws like in physics because it does not give birth to a 'scientific object' and because the resulting algebras are neither incremental nor linear in the space (projective) and in the time (surjective). The matching gives birth to distributions of fractal set of singularities, partly defined from environmental variables. They veil the internal variables into a fog of uncertainties when course grained. Naturally open, these systems do not possess any imanente categorical limits. They have to be constructed. However, as it has been seen concerning the resolution of the Riemann hypothesis, a duality can be built in suitable fields of interactions to rebuild an over-structural causality, involving the environment as a categorical limit of the complex object. Among the specific spaces that are able to help to formalization exists a new class of causality based upon duality that we can consider in the space Υ of primes suggested above [36].

To illustrate the proposal one can analyse for instance, the axioms of monetarism that consider the currency as the alpha and the omega of the economy [10]. The monetarism aims an adjusted monetary injection, to reach a 'steady state' of the economic system. It artificially distinguishes two components: currency and goods , both being combined stochastically for giving to the economy a stationary state of growing, for instance 5%, defined from outside institutions. The behaviour of economy should then match a deterministic behaviour, Z. In this frame, the anticipation of the economic growth defines the rate of interest of the currency supply, while the economic balance is achieved through the fluctuation of the interest rates around a fantasized fair value. This fair value only exists if the system matches the field of the zeros of the Riemann function that is to say in the neoclassical theory if the full employement is reached at each level of the economy. Within the formal framework outlined by our analysis, the omniscience of the market and its infinite speed of adjustment between coupled variables lead the pattern given of the figure 10 where money is a mirror of the transactions.

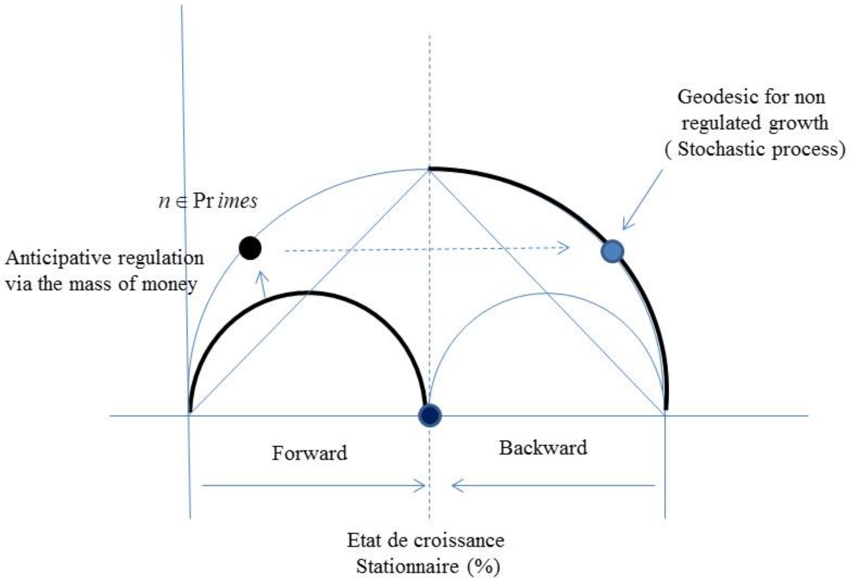

Figure 10. Riemann hypothesis situation analysed from the adjunction of a couple of random 2D Cole and Cole diagrams into a first order *Z* spectral representation. This mirror situation is implicitly asummed by the neoclassical theory.

According to this scheme, the unclosed economy is pinned on a 2D hyperbolic geodesic, which has for basis only the chance and not any environment. The stochastic market, in fact the banks, as actors replace the state to produce symbolic value. The variables of anticipation and / or projection of the economy into the future, based upon the currency supply, are folded upon the monetary geodesic. The economy is no more than a casino-economy. Monetary policy and economy are coupled via the set of primes without any freedom. As shown in figure 10 above the symmetry is total. The dynamics of the economy is then clearly determined by the sole monetary growth rate [10]. The hyperbolic morphism between both components merges the dynamics onto a geodesic of a global economy (trading in goods, added with the trading in currencies) matching a simple first order system *Z* (Newton type equation), indeed implicitly a very simple causal system herein based upon a splitting of *Z* into two random symmetrical geodesic. The dictatorship of the market is assured. The monetarists claim that the global dynamics would then be optimal. But what kind of optimality is concerned? If we use the Nash embedding theorem – that establishes the existence of an isomorphism between any hyperbolic manifold and a Euclidean space – to project the process in adapted spaces, we can show (Figure 10) that the distributions do not interferes together. In fact the monetarism is not concerned by the real economy. It is a platonic ideology in terms of values and and ideal type of performative device, in terms of governance ! Without any consideration of the being it is a performative ideal template used to impose a self realizing prediction written within a fight for individual utility. However, this ideology possesses a unanswerable quality, we wish to emphasize: in the purely 2D random case (in fig 10, the phase angle is strictly equal to $\pm\pi/4$), the pure market is said efficient; it emerges from a symmetrical construction that reduces the general problem which involves a symmetry in the set $Z_{\theta_\alpha}(\omega\tau)$ vs $Z_{\theta_{1-\alpha}}(\omega\tau)$ / $\zeta_{\theta_\alpha}(s)$ vs $\zeta_{\theta_{1-\alpha}}(s)$ that is to say, it asserts the existence of steady states (or stationary) obviously singular and isolated in an abstract space. The monetary model hence implicitly assumes the existence of a reversible time variable. In other words, while pointing the relevance of the couple currency / economy as the basic element of the economy, monetarism gives a simplistic vision clearly not suited to understand what a complex system is, but that can be extended. In spite of its simplicity, the monetarist perspective, has a virtue, it allows considering an unlarged case to the modern monetary theory based upon money seen as a Kan extension of the real economic exchanges.

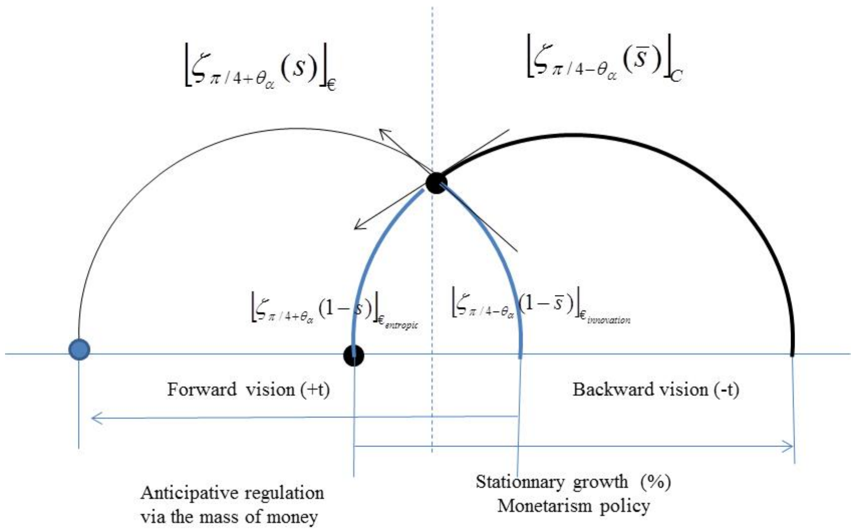

Figure 11. Diagram that highlights the duality between active backward economic and forward monetary visions. The Figure 10 is simply a degenerate form of this diagram.

If this hypothesis is relevant, the theoretical analysis that led to a new approach to the resolution of the Riemann hypothesis can be of a great help, especially for understanding why in economy, the financial field plays a very similar role to the entropy/negentropy factor in physics. By considering some properties related to the general case – not solely focussed upon stationary states –, the monetary point of view may be enlarged to point in the direction of Modern Monetary Theory, without any exclusion of the existence of an optimal prospective geodesic (Figure 11). In this case, money may express the symbolic correlations established in the dynamic social structure of exchanges which behavior is partially condensed through the concept of phase angle, that is to say, through the factor of non-commutativity of the dynamics, in other words, through a noncommutative manifold

6. Discussion

According to the above analysis, the steering control of the economy using currency is always possible, but in the above enlarged framework, this control has to take into account the correlations that characterize the dual structure of the economy. In addition, the field of interactions between prospective point of view, and inductive point of view has to be perfectly mastered. In an asymmetrical fluid exchange state – between goods and currency –, mainly unstable, the currency, – seen like an entropic extension-, aims to combine a dynamic state of the real economy represented here by $Z_{\theta_\alpha}(\omega\tau)$ or $\zeta_{\theta_{\theta_\alpha}}(s)$ with a 'virtual stabiliser' always prospective. This prospective component is herein expressed through $Z_{\theta_{1-\alpha}}(\omega\tau)$ or $\zeta_{\theta_{1-\alpha}}(s)$. Therefore:

- The currency expresses a confidence in a representable future new stable state. The geodesics to this state must be seen as a catenation of two fractional stochastic branches characterized by internal correlations serving as dynamical reference. The future, designed from these geodesics, is seen as an opportunity and a calling for investments. In the language developed above, despite the underlying complexity and the stochastic character of the projection to the future, the coherence of the dynamics is perceived at all scales of the processing. While staying in the instability, it is possible to stage the future by the mean of a Kan extension of the existing, upon dreaming states. A collective investment associated with the creation of assets is herein used to change the time constant of action while strengthening the stability of the overall dynamics. The social trust can jointly be increase. This stabilization is provided by money creation assigned to precise investments.

This creation is seen like the change of a stock of currency upon a fractional probabilistic set and not through the credit. The extension is seen as a modification of the economic action itself. In this frame, the value of the currency and thus the rate of interest reflects the overall expansion of the dynamic, that is to say, the confidence that the society places within the future.

- This projection is essentially composite, because, as shown in the model, it gives the distribution of all the time constants of acts that can be considered inside the real economy. Practically, there are many ways to distribute the currency upon a sole optimal geodesic. The key for coherence is here the conservation of the correlations that allows the maintenance of the social trust, registered into the internal correlation field (herein the phase angle).
- The phase angle defines precisely the internal correlations of the complex system. Therefore it defines the coherence of the economic structure and the meaning of the collective dynamics toward the future. As we seen, if exchanges are left to the free market fluctuations, only constrained by the amount of currencies, even mathematically optimal, the system is no more socially ruled because the monetary structure, constraint by the random distribution of primes, can no longer be composite. Human being becomes particle. The chance is hence the sole engine for a development only based upon the strong hierarchy of a society without any exteriority. Due to the symmetry, the market is finally left to the financial monopoles and the prices are defined by them (that means lower prices and salaries and higher incomes for the gentry). That is the well known social limit of the monetarist choices! At the opposite, if the market is fully regulated in a deterministic way, currency is no longer required and exchange becomes barter. Any pure dictatorial model has to face this limit.

This analysis leads to a certainty: our ability of re-enchantment of the world is located in an in-between always unstable but where the money plays the role of symbolic social stabilizator. This positioning is based upon an incompleteness of our 'numerical' representation of the real, and upon a duality of the reality always designed like an uncompleted motion. The incompleteness must be partially filled with tools that we put in place to answer to the questions addressed by our imagination. As shown above, these tools have both symbolic and operational status; symbolic because they reflect the internal consistency of collective dynamics; operational because it is mediated by investments for a common future. These investissements are called 'assets' in economy. In this context the time coupled to the action is clearly irreversible because it is associated with some kind of confusion between the real able to be compute and the virtual. This aspect disappears in the monetarist framework. The difference between these two temporalities reversible and irreversible is not only related to the functionality of tools but always linked to a measure of distance, but it is a hyperbolic distance (here given using the phase angle) involving singular behavior at boundaries. It is in this frame and taking a position opposite to the modern newtonians asserting that 'time does not exist', that Lee Smolin asserts the time as one of the key variables of future physics [40]. Conversely, if the world is plunged into the stream of irreversible time – as assumed by Smolin and Connes –, this irreversible time flow necessarily induces correlations at all levels of complexity and thus justifies the duality of uncertainties. Therefore the phylogenetic evolution becomes natural. We then understand what the cognitivists call with Francisco Varela 'embodied cognition' translated in french by the word 'énaction', where the language unveil the idea of action as a pivot of prospective toward future and the idea of free correlations between both action and dream. We find again herein the idea of the Spanish writer Antonio Machado that the trail is always draw by the walker (*Caminente no hay camino, se camino al andar*).

7. Conclusion

As noted in the book published under the provocative title: 'The trouble with physics', Lee Smolin criticizes the Newtonian and Cartesian attitude, which, far from the complex systems analysis, bases the physical models upon the concept of computation upon 'scientific objects'. Smolin expresses a critic of the paradigm that separates body and mind and organization and its reasons. This splitting is less and less adapted to the complex reality to which we have to face. The modern crisis of the Euro is a perfect example of what happens if the symbolic social role of the money is forgotten. The duality of economy and finance is only a particular mathematical category of complexity. Facing this complexity, it is very difficult to build new models because our education is patterned by paradigms of single and simple causality without any feedback. Nevertheless, henceforth, the categorical concepts allow us to think the limits and the duality of objects that cannot be reduced to points or numbers. In this frame, the monetary policy may be considered as being the initial algebra with respect to a co algebraic economy, seen as final co algebra of endofunctors into a social set. Thinking an econo-physics on the basis of current physics is to ignore, at least partially, the current conceptual limitations of this science. The epistemological assumptions which built and justified the concept of 'objects of science' (eternal, reproducible, able to be falsified, etc) cannot be neglected at this stage of our reflexion. There is no doubt than, with the sole reference to physical paradigms, our mind traps itself in an unspoken jail from which it will be not so simple to escape. Faced with this difficulty we propose to follow an aside trail, by observing that the economy is just a peculiar field of issues in the frame of more general complex systems. Far from classical physic, the main issues are related, (*i*) to the rôle of singularities, (*ii*) to absence of falsifiability, (*iii*) to the requirements of simulations, (*iv*) to the rôle of environmental variables, (*v*) to the treatment of 'big data', etc, all questions being outside the field of traditional sciences. Hence, the emergence of the idea of extending the concept of rationality and call 'Sciance', with 'a' [42] the design of adapted dual rationality. Also emerges the idea of calling 'Zeit Objekt' [Husserl] the object of study of complex systems. At this step, it is only a moot intention nevertheless the practical scope of the categorical first approach of the resolution of Riemann hypothesis herein delivered, is a fundamental reason for going in this direction. Indeed this approach takes the status of a Swiss Army Knife for the hiker in the wild landscape of complex systems. What is telling through this approach is mainly that pure physical state is an extremely singular situation and that our representation of the world must be dual. The traditional 'object of science' contains an artificial temporal symmetry, the projection into the future, being identical to the projection into the past. In the general case our representation of the world can not be so simple, that why, except in very simple cases, the 'Object of Science' must be revisited for leaving the way to a 'Zeit Objekt'. The internal self-similarity of this new 'objekt' of Sciance' must be of algebraic essence; the irreversibility must be the foundation of its open internal dynamics, and the ultrametric its fundamental metric. That is why we propose to leave aside the concept of 'econo-physics'. The ontological crisis of the post modern physics should no longer be transfered in vivo econo-physics issues. We suggest calling 'econology' the Sciance of the economic systems, henceforth considered as a part of the Sciance of Complex Systems.

Knowledgement: Bloomon SARL for financial support and Materials Design Inc for scientific support and exchanges.

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