THE OKUN'S LAW AND THE CHAOTIC UNEMPLOYMENT RATE GROWTH MODEL: G7

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Abstract. *The basic aim of this analysis is to set up a relatively simple chaotic unemployment rate growth model that is capable of generating stable equilibria, cycles, or chaos. This paper analyzes the local growth stability of unemployment rate in the G7 countries in the period 1990-2013. (www.imf.org) and confirms stable but monotonically increasing movement of unemployment rate in the G7 countries in the observed period.*

Keywords: *Single equation model, Unemployment, Investment, Budget Deficit.*

JEL classification numbers: C2, J64, E22, H62.

1. Introduction

Almost 202 million people were unemployed in 2013 around the world. There is an increase of almost 5 million compared with the year before. Employment is not growing sufficiently fast to keep up with the expanding labor force. The euro area emerged from recession during the second quarter of 2013. In the United States, growth reached more than 4 per cent in the last quarter of 2013. Similarly, in Japan, growth remained steady in the first half of 2013. However, improvements in both productivity and competitiveness have not yet been strong enough to make a significant difference to the still large employment gap. It remains, so far, a recovery in economic activity, not in jobs. In the developed economies and European Union region, the unemployment rate remained in 2013 at 8.6 per cent, or 45.2 million people. It is expected to gradually decline to below 8 per cent around 2018. In 2013, 18.3 per cent of young people in this region were out of a job (ILO, 2014).

The analysis focuses on the G7 countries: Canada, France, Germany, Italy, Japan, the United Kingdom (UK), and the United States (USA). The unemployment rate measures the number of people actively looking for a job as a percentage of the labor force. Unemployment rate in the G7 countries in the period 1990-2013 is presented in Figure 1.

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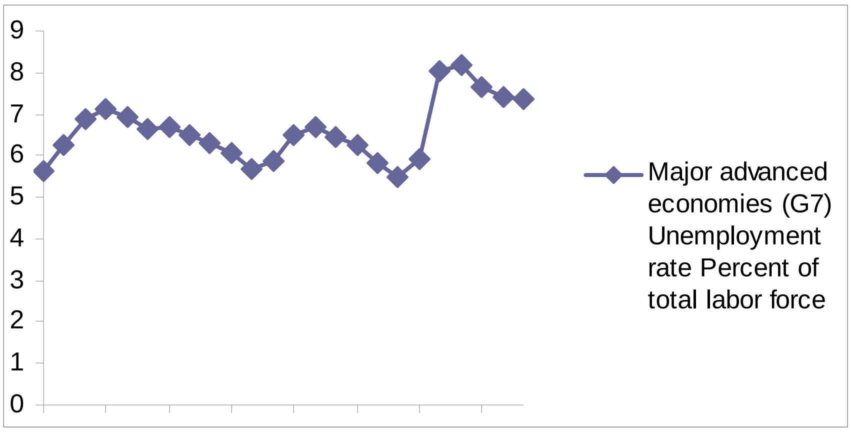

Figure 1. Unemployment rate in the period 1990-2013: G7 (www.imf.org).

Namely, in an open economy, government budget deficit, as a negative public saving, raises real interest rates, crowds out domestic investment, decreases net capital outflow, decreases the level of asset prices, causes the domestic currency to appreciate. A decrease in aggregate demand causes output, unemployment and prices to fall. The recession may further increase budget deficit, unemployment, and public debt. Large government spending can lower private investment expenditures (crowding out). If the government consumes a larger share of the total output of goods and services via deficit finance, an equally smaller amount of output is available for private use.

When the government runs a budget deficit, it reduces the national saving. The interest rate rises. The asset prices decrease. Further, the higher interest rate reduces domestic and net foreign investment (net capital outflow). Reduced net foreign investment (net capital outflow), in turn, reduces the supply of domestic currency in the market for foreign-currency exchange, which causes the real exchange rate of domestic currency to appreciate.

Further, a rise in the equilibrium real interest rate induces greater saving and lower consumption. The increase in the equilibrium real interest rate also causes investment to decline. The total decline in private consumption and investment is equal to the rise in the government budget deficit. An expansionary fiscal policy action that increases the government budget deficit causes an increase in the real rate of interest that induces reductions in private consumption and investment. The total decline in private spending, the sum of the amount by which private saving rises and private investment falls, exactly equals the amount by which the deficit rose. This reduction in private spending and aggregate demand caused by an increase in the government deficit is known as the crowding-out effect.

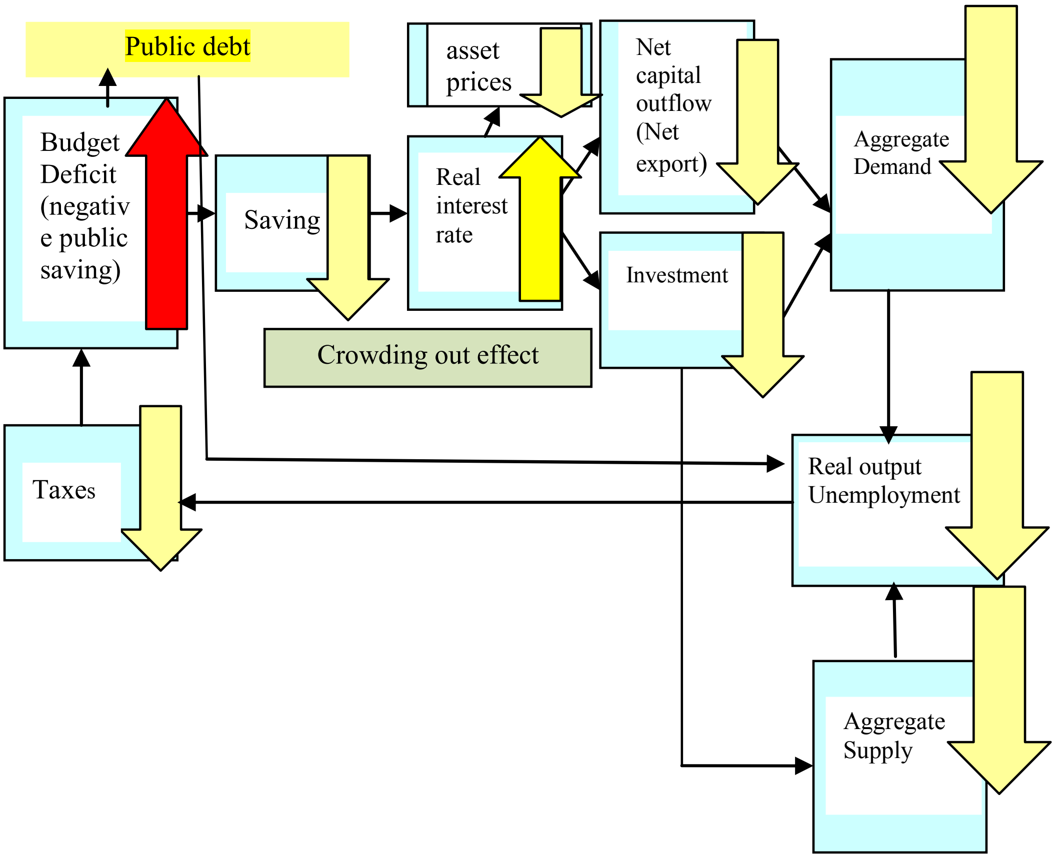

Figure 2. Causes of the financial crisis or the effects of a government budget deficit and public debt.

In Figure 2 we can examine the effects of the crowding out effect on unemployment. Figure 3 shows investment and general government net debt in the G7 countries in the period 1991-2013.

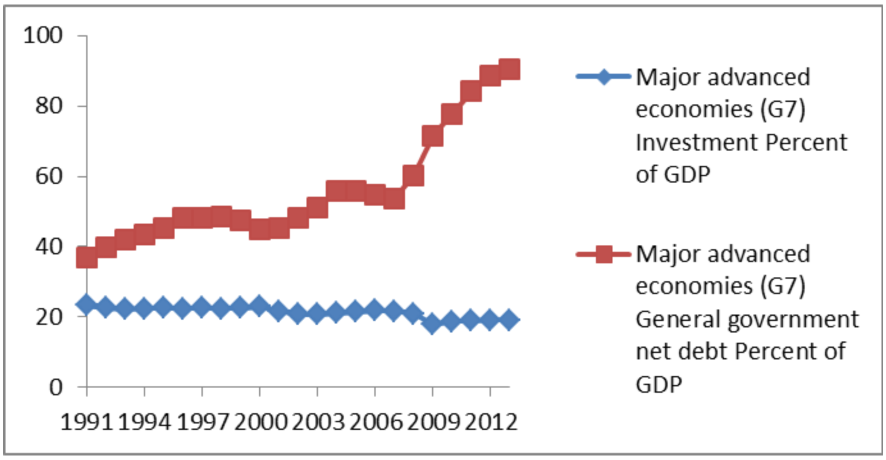

Figure 3. Investment (% of GDP) and general government net debt (% of GDP) in the period 1991-2013: G7 (www.imf.org).

2. The Model

Chaos theory started with Lorenz's (1963) discovery of complex dynamics arising from three nonlinear differential equations leading to turbulence in the weather system. Li and Yorke (1975) discovered that the simple logistic curve can exhibit very complex behavior. Further, May (1976) described chaos in population biology. Chaos theory has been applied in economics by Benhabib and Day (1981, 1982), Day (1982, 1983), Grandmont (1985), Goodwin (1990), Medio (1993), Lorenz (1993), Jablanovic (2011, 2012, 2013), Puu, T. (2003), Zhang W. B. (2006) etc.

Chaos theory started with Lorenz's (1963) discovery of complex dynamics. Deterministic chaos refers to irregular or chaotic motion that is generated by nonlinear systems evolving according to dynamical laws that uniquely determine the state of the system at all times from a knowledge of the system's previous history. Chaos embodies three important principles: (i) extreme sensitivity to initial conditions; (ii) cause and effect are not proportional; and (iii) nonlinearity.

The basic aim of this analysis is to set up a relatively simple chaotic unemployment rate growth model that is capable of generating stable equilibria, cycles, or chaos. It is important to analyze the local stability of unemployment rate growth in the G7 countries in the period 1990-2013 (www.imf.org).

Okun's (1962) paper regarding the unemployment – output relationship considers the measurement of potential output. Okun believed that the potential output should not be defined as the maximum output the economy could produce. Instead, he argued that the potential should be measured at full employment, which he characterized as the level of employment absent inflationary pressures.

We can write the Okun's Law in this form:

$$(Y_t - Y_p) = -\rho\,(U_t - U_n) \tag{1}$$

where $\rho > 0$. Namely, deviations of the real output from its natural level are inversely related with deviations of unemployment rate, U, from its natural level, U_n. When unemployment rate increases above its natural level ($U_t > U_n$) real output tends to decrease below its natural level and vice versa.

Further, it is supposed:

$$U_n = \lambda\,U_t \tag{2}$$

$$Y_p = \mu\,Y_t \tag{3}$$

where: U_t – unemployment rate; U_n – natural rate of unemployment; Y_t – real output; Yp – potential output; ρ – the coefficient which explains relation between deviations of real output from its natural level and deviations of unemployment rate from its natural level; β – the coefficient which relates unemployment rate and natural rate of unemployment; μ – the coefficient which explains relation between real output and potential output:

$$\frac{I_{t+1} - I_t}{I_t} = \alpha - \beta Bd_t \tag{4}$$

$$I_t = \gamma\,Y_t\; 0 < \gamma < 1 \tag{5}$$

$$Bd_t = \delta Y_t \tag{6}$$

where: α – an autonomous investment growth rate; γ – an investment rate; δ – the budget deficit rate.

By substitution one derives:

$$U_{t+1} = (1+\alpha)\,U_t - \left[\frac{\beta\,\delta\,\rho\;(\lambda-1)}{(1-\mu)}\right] U_t^2\,. \tag{7}$$

Further, it is assumed that the current value of unemployment rate is restricted by its maximal value in its time series. This premise requires a modification of the growth law. Now, the unemployment rate growth rate depends on the actual value of the unemployment rate, U, relative to its maximal size in its time series Um. We introduce u as $u = U/U^m$. Thus y ranges between 0 and 1. Again we index u by t, i.e., write u_t to refer to the size at time steps $t = 0,1,2,3,...$ Now the unemployment rate growth rate is measured as

$$u_{t+1} = (1 + \alpha)\, u_t - \left[\frac{\beta\,\delta\,\rho\ (\lambda-1)}{(1-\mu)}\right] u_t^2\ . \tag{8}$$

This model given by equation (8) is called the logistic model. For most choices of α, β, δ, ρ, λ, and μ there is no explicit solution for (8). Namely, knowing of α, β, δ, ρ, λ, μ, and measuring u_0 would not suffice to predict u_t for any point in time, as was previously possible. This is at the heart of the presence of chaos in deterministic feedback processes. Lorenz (1963) discovered this effect – the lack of predictability in deterministic systems. Sensitive dependence on initial conditions is one of the central ingredients of what is called deterministic chaos.

This kind of difference equation (8) can lead to very interesting dynamic behavior, such as cycles that repeat themselves every two or more periods, and even chaos, in which there is no apparent regularity in the behavior of u_t. This difference equation (8) will possess a chaotic region. Two properties of the chaotic solution are important: firstly, given a starting point u_0 the solution is highly sensitive to variations of the parameters of α, β, δ, ρ, λ, and μ; secondly, given the parameters of α, β, δ, ρ, λ , and μ the solution is highly sensitive to variations of the initial point u_0. In both cases the two solutions are for the first few periods rather close to each other, but later on they behave in a chaotic manner.

3. The Logistic Equation

The logistic map is often cited as an example of how complex, chaotic behavior can arise from very simple non-linear dynamical equations. The map was popularized in a seminal 1976 paper by the biologist Robert May. The logistic model was originally introduced as a demographic model by Pierre François Verhulst. It is possible to show that iteration process for the logistic equation:

$$z_{t+1} = \pi\, z_t\, (1 - z_t),\ \pi \in [0,4],\ z_t \in [0,1] \tag{9}$$

is equivalent to the iteration of growth model (8) when we use the identification (see Fig. 4):

$$z_t = \frac{\beta\,\delta\,\rho\,(\lambda-1)}{(1+\alpha)\,(1-\mu)}\ u_t \text{ and } \pi = 1 + \alpha. \tag{10}$$

Using (8) and (10) we obtain:

$$z_{t+1} = \frac{\beta\,\delta\,\rho\,(\lambda-1)}{(1+\alpha)\,(1-\mu)}\; u_{t+1} =$$

$$= \frac{\beta\,\delta\,\rho\,(\lambda-1)}{(1+\alpha)\,(1-\mu)} \left\{(1+\alpha)\,u_t - \left[\frac{\beta\,\delta\,\rho\,(\lambda-1)}{(1-\mu)}\right] u_t^2\right\} =$$

$$= \left[\frac{\beta\,\delta\,\rho\,(\lambda-1)}{(1-\mu)}\right] u_t - \left[\frac{\beta^2\,\delta^2\,\rho^2\,(\lambda-1)^2}{(1+\alpha)\,(1-\mu)^2}\right] u_t^2.$$

Using (8) and (9) we obtain:

$$z_{t+1} = \pi z_t\,(1 - z_t) =$$

$$= (1+\alpha) \left[\frac{\beta\,\delta\,\rho\,(\lambda-1)}{(1+\alpha)\,(1-\mu)}\right] u_t \left\{1 - \left[\frac{\beta\,\delta\,\rho\,(\lambda-1)}{(1+\alpha)\,(1-\mu)}\right] u_t\right\} =$$

$$= \left[\frac{\beta\,\delta\,\rho\,(\lambda-1)}{(1-\mu)}\right] u_t - \left[\frac{\beta^2\,\delta^2\,\rho^2\,(\lambda-1)^2}{(1+\alpha)\,(1-\mu)^2}\right] u_t^2.$$

Thus we have that iterating (8) is really the same as iterating (9) using (10). It is important because the dynamic properties of the logistic equation (9) have been widely analyzed (Li and Yorke (1975), May (1976)).

It is obtained that:

(i) For parameter values $0 < \pi < 1$ all solutions will converge to $z = 0$;

(ii) For $1 < \pi < 3{,}57$ there exist fixed points the number of which depends on π;

(iii) For $1 < \pi < 2$ all solutions monotnically increase to $z = (\pi - 1)/\pi$;

(iv) For $2 < \pi < 3$ fluctuations will converge to $z = (\pi - 1) / \pi$;

(v) For $3 < \pi < 4$ all solutions will continuously fluctuate;

(vi) For $3{,}57 < \pi < 4$ the solution become "chaotic" which means that there exist totally aperiodic solution or periodic solutions with a very large, complicated period. This means that the path of z_t fluctuates in an apparently random fashion over time, not settling down into any regular pattern whatsoever.

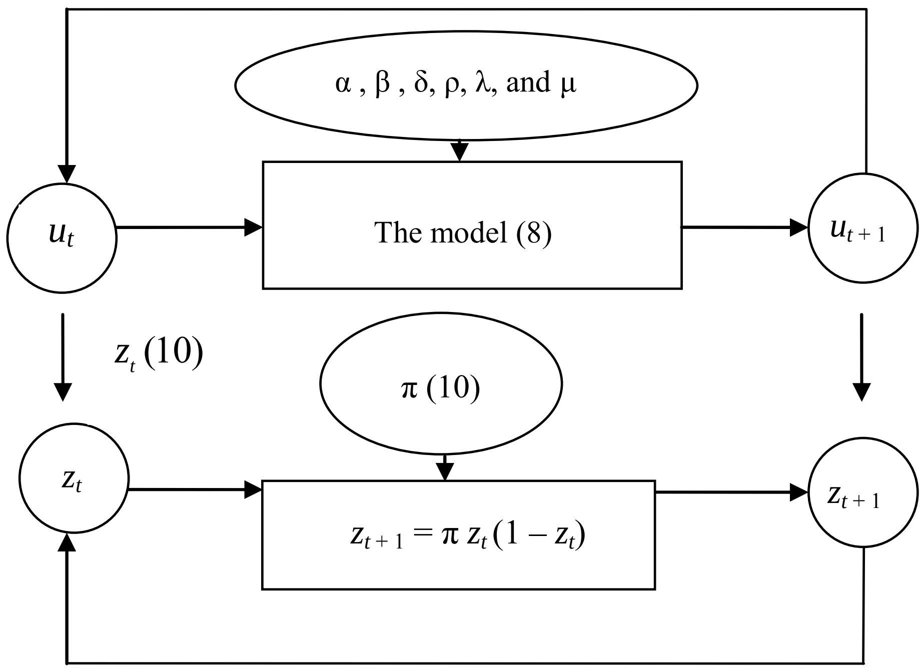

Figure 4. Two quadratic iterations running in phase are tightly coupled by the transformations indicated.

4. Empirical Evidence

The main aim of this paper is to analyze the unemployment rate growth stability in the G7 countries in the period 1990-2013, by using the presented non-linear, logistic unemployment rate growth model (11):

$$u_{t+1} = \pi u_t - \vartheta u_t^2 \qquad (11)$$

where: u – unemployment rate, $\pi = 1 + \alpha$, $\vartheta = \left[\frac{\beta \delta \rho (\lambda - 1)}{(1 - \mu)} \right]$.

Firstly, data on unemployment rate are transformed (www.imf.org) from 0 to 1, according to our supposition that actual value of unemployment rate, U, is restricted by its highest value in the time-series, U^m. Further, we obtain time-series of $u = U / U^m$. The estimated results are presented in Table 1.

Table 1.

The estimated model (11): G7, 1990-2013.
(R = 0.69119 Variance explained: 47.774%) (www.imf.org)

	π	ɵ
Estimate	1.296460	0.351578
Std.Err.	0.150934	0.182414
t(21)	8.589606	1.927359
p-level	0.00000	0.067571

5. Conclusion

The presented chaotic unemployment rate growth model (8) has to rely on specified parameters α, β, δ, ρ, λ, and μ, and initial value of unemployment rate, u_0. But even slight deviations from the values of parameters: α, β, δ, ρ, λ, μ, and/or initial value of unemployment rate, u_0, show the difficulty of predicting a long-term unemployment rate.

A key hypothesis of this work is based on the idea that the coefficient $\pi = 1 + \alpha$ plays a crucial role in explaining local growth stability of unemployment rate, where, α – an autonomous investment growth rate.

The estimated value of the coefficient π is 1.296460. This result confirms stable but monotonically increasing movement of unemployment rate in the G7 countries in the period 1990-2013. Decreasing budget deficits and increasing investment would generate decreasing unemployment rate in the observed countries.

REFERENCES

[1] Benhabib, J., & Day, R. H. (1981), Rational choice and erratic behavior. *Review of Economic Studies*, 48, 459-471.

[2] Benhabib, J., & Day, R. H. (1982), *Characterization of erratic dynamics in the overlapping generation model* Journal of Economic Dynamics and Control, **4,** 37-55.

[3] Day, R. H. (1982), *Irregular growth cycles,* American Economic Review, **72,** 406-414.

[4] Day, R. H. (1983), *The emergence of chaos from classical economic growth*. Quarterly Journal of Economics, **98,** 200-213

[5] ILO (2014), *Global Employment Trends 2014 – Risk of a jobless recovery?* International Labour Office. Geneva: ILO, 2014,

[6] Goodwin, R. M. (1990), *Chaotic economic dynamics,* Oxford: Clarendon Press .

[7] Grandmont, J. M. (1985), *On endogenous competitive business cycles.* Econometrica, **53,** 994-1045

[8] Jablanović V. (2011), *A Chaotic Unemployment Rate Growth Model*, Hyperion International Journal of Econophysics &New Economy**,** Volume **4,** Issue 1, 45-51.
[9] Jablanović, V. (2012), *Budget Deficit and Chaotic Economic Growth Models,* Aracne editrice S.R.L, Roma.
[10] Jablanović V. (2013), *The Chaotic Economic Growth Model: China,* Advances in Management and Applied Economics, Volume **3,** Issue 4, 89-94
[11] Li, T., & Yorke, J. (1975), *Period three implies chaos,* American Mathematical Monthly, **8,** 985-992.
[12] Lorenz, E. N. (1963), *Deterministic nonperiodic flow,* Journal of Atmospheric Sciences, **20,** 30-141.
[13] Lorenz, H. W. (1993), *Nonlinear dynamical economics and chaotic motion* (2nd ed.), Heidelberg: Springer-Verlag.
[14] May, R. M. (1976), *Mathematical models with very complicated dynamics*, Nature, 261, 459-467.
[15] Medio, A. (1993), *Chaotic dynamics: Theory and applications to economics*, Cambridge: Cambridge University Press.
[16] Okun, Arthur (1962), *Potential GNP, its measurement and significance*, Cowles Foundation, Yale University, http://cowles.econ.yale.edu/P/cp/p01b/p0190.pdf
[17] Puu, T. (2003), *Attractors, Bifurcations, and Chaos - Nonlinear Phenomena in Economics*, Springer.
[18] Zhang W.B. (2012), *Discrete Dynamical Systems, Bifurcations and Chaos in Economics*, Elsevier B.V
[19] www.imf.org