COMPARATIVE ANALYSIS OF ELECTROMAGNETIC FIELD STUDY USING OPERATOR DIAGONALIZATION METHODS

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Abstract. *Two types of operator diagonalization procedures are analyzed for the differential Maxwell system in its classical form and in the specific case of electromagnetic field expofunctional excitation. The first diagonalization approach concerns operator analogy of the algebraic Gauss method. The second one deals with the inverse matrix operator construction. In both cases, the general wave PDE (partial differential equation) is got with respect to all unknown scalar components of electromagnetic field intensities. Detailed comparison of the suggested methods is done basing on their virtues and drawbacks. The common features and essential differences of the aforesaid Maxwell systems of PDEs are found owing to the obtained general wave equations.*

Keywords and phrases: *classical Maxwell system, symmetrical Maxwell system, operator analogy of Gauss method, general wave equation.*

1. Introduction

Though, so many computer numerical programs are used nowadays solving various industrial and engineering problems including technical electrodynamics and so on, analytical explicit methods basing on the respective mathematical models are encouraged as well [1, 2]. Really, it is better putting the given experimental data into the known exactly obtained expression, than dealing with the approximate numerical algorithm taking into account its all computing inaccuracies. Moreover, all of them should be in conformity with the original physical or engineering problem statement.

In this case, analysis of the different constructive analytical methods becomes rather important, as from the theoretical, as from the applied viewpoints. Since systems of PDEs (partial differential equations)

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represent the main mathematical models in electromagnetic field theory, their research remains rather urgent just in this scientific direction.

Basing on the fundamental differential Maxwell system [3], so many other versions of it were considered in the current engineering phenomena [1, 2]. In particular, it concerns the so called expofunctional excitations in the framework of the classical electromagnetic field theory [4].

Further, even analytically almost precise solving procedure has its own virtues and drawbacks. That is the main reason why in the present paper, two new pure mathematical methods are compared in terms of their influence on exact study of electrodynamic problems.

The first one is the operator analogy [5] of the algebraic Gauss method [6]. The second approach deals with the inverse matrix operator construction [7], again basing on the well known numerical algebraic technique [6]. Both methods act only in the class of ordinary, non generalized functions, simplifying and not infringing the original industrial problem statement. Later, this functional class can be modified taking into account the particular boundary problem conditions.

2. Preliminaries

At first, let the symmetrical differential Maxwell system [4, 5] be given for homogeneous isotropic linear media in the presence of "expo functional" influences when their kernels are studied

$$\begin{cases} \mathbf{rot}\vec{H} = (\sigma \pm \lambda\varepsilon_a)\vec{E} + \varepsilon_a \partial_0 \vec{E} + \vec{j}^{\,OS} \\ -\mathbf{rot}\vec{E} = (r \pm \lambda\mu_a)\vec{H} + \mu_a \partial_0 \vec{H} + \vec{e}^{\,OS}. \end{cases} \tag{2.1}$$

In (2.1): the unknown vector functions $\vec{E}, \vec{H} = \vec{E}, \vec{H}(x, y, z, t)$ with scalar components $E_k, H_k = E_k, H_k(x, y, z, t), (k = \overline{1, 3})$ represent electric and magnetic field intensities; positive constants $\sigma, \mu_a, \varepsilon_a$ are the specific conductivity, absolute and dielectric permeability of the medium respectively; vector functions $\vec{j}^{\,OS}, \vec{e}^{\,OS} = \vec{j}^{\,OS}, \vec{e}^{\,OS}(x, y, z, t)$ are known and describe the outside current sources and intensities. Their scalar components look like $j_k^{OS}, e_k^{OS} = j_k^{OS}, e_k^{OS}(x, y, z, t), (k = \overline{1, 3})$. Parameter of the signal exciting medium is $\lambda = const > 0$. Sign reversal in front of λ corresponds to the absorption of signal for "+" and activity of the medium for the "–". Theoretical constant $r > 0$ is assumed at the current research stage only. It is responsible for the system symmetry and simplification of the further mathematical computation. At the end, it can be omitted not influencing the original problem statement and final numerical calculation. Partial differential temporal operator and classical rotor definition look like

$$\partial_0 = \frac{\partial}{\partial t}, \qquad \mathbf{rot} = \det\begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial_1 & \partial_2 & \partial_3 \\ F_{i1} & F_{i2} & F_{i3} \end{bmatrix}, \quad \text{where} \quad \partial_1 = \frac{\partial}{\partial x}, \partial_2 = \frac{\partial}{\partial y}, \partial_3 = \frac{\partial}{\partial z}.$$

Functions $F_{ik} = F_{ik}(x, y, z, t)$, $(i = 1,\ 2;\ k = \overline{1,\ 3})$, designate scalar components of the appropriate vector electromagnetic field intensities $\vec{F}_1, \vec{F}_2 = \vec{F}_1, \vec{F}_2(x, y, z, t)$; $\vec{F}_1, \vec{F}_2 = \vec{E}_1, \vec{H}_2$.

Solution of (2. 1) was got using two operator diagonalization methods [5, 7] in the meaning of this system reduction to the general wave equation with respect to all scalar components of the original unknown electromagnetic field vector intensities. The first approach [5] dealt with operator analogy of the classical algebraic Gauss method and simply fulfilled diagonalization procedure constructing equivalent matrix problem whose all equations depended on the only one scalar component of the initially unknown vector field function. The second procedure [7], though pursued the same goal of getting general wave equation, was more diverse in its direct applications. Thus, it allowed obtaining explicit solvability conditions of (2.1), and corresponding criterion of the solution existence was proved.

Nevertheless, elegant technique of the second algorithm based on the matrix operator kernel study [8], could not be used for the diagonalization of homogeneous systems. It was completely explained by the main tendency of the given procedure of inverse matrix operator construction generated by the analogous algebraic numerical matrix theory [6].

Before coming to the straight analytic realization of the aim of the present paper, the “diagonalization” term should be reminded. In general, it means the reduction of the original matrix problem to the equivalent system of scalar ones, where each of them depends on the only one component of the initially unknown vector field function.

3. Two diagonalization methods in short for the symmetrical Maxwell system

At first, the outline of operator analogy [5] of the classical Gauss method [6] is proposed as the corresponding diagonalization procedure for the symmetrical Maxwell system (2.1).

Using additional operator notations

$$A = \mathbf{rot};\ C = \sigma + \varepsilon_a \partial_0^*;\ \ P = r + \mu_a \partial_0^*;\ \partial_0^* = \partial_0 \pm \lambda \tag{3.1}$$

system (2.1) can be rewritten like that

$$\begin{cases} A\vec{H} - C\vec{E} = \vec{j}^{\,OS}, \\ -P\vec{H} - A\vec{E} = \vec{e}^{\,OS}. \end{cases} \tag{3.2}$$

Then, diagonalization of (3.2) "by blocks" begins. It means operator $P,\ A$ application to the first and the second equations of (3.2) respectively and term-by-term addition of both transformed equations. Further, to the first and the second equations of the new equivalent system

$$\begin{cases} -(PC + A^2)\vec{E} = P\vec{j}^{\,OS} + A\vec{e}^{\,OS}, \\ \qquad -P\vec{H} - A\vec{E} = \vec{e}^{\,OS} \end{cases} \tag{3.3}$$

two operators $(-A),\ (PC + A^2)$ are applied, and both transformed equations are added again term-by-term. It is easy to notice that the first equation in (3.3) depends only on $\vec{E}$ now.

The last equivalent system at this diagonalization stage is the following

$$\begin{cases} \quad -(PC + A^2)\vec{E} = P\vec{j}^{\,OS} + A\vec{e}^{\,OS}, \\ -(PC + A^2)P\vec{H} = PC\vec{e}^{\,OS} - PA\vec{j}^{\,OS}, \end{cases} \tag{3.4}$$

and its second equation depends only on $\vec{H}$. So, diagonalization procedure of (2.1) on its "block stage" is closed, and after application to the second equation of (3.4) the inverse operator P^{-1}, the general wave vector equation regarding two electromagnetic field vector intensities is got

$$-(A^2 + PC)\vec{F}_i = \vec{\varphi}_i, \quad (i = 1, 2). \tag{3.5}$$

In (3.5):

$$\vec{F}_1 = \vec{E},\ \vec{F}_2 = \vec{H};\ \vec{\varphi}_1 = A\vec{e}^{\,OS} + P\vec{j}^{\,OS},\ \vec{\varphi}_2 = C\vec{e}^{\,OS} - A\vec{j}^{\,OS};$$

$$A^2 = \mathbf{grad}\ \mathrm{div} - \Delta;\ \Delta = \sum_{k=1}^{3} \partial_k^2, \tag{3.6}$$

and last two formulas are classical operators from the electromagnetic field theory [3].

Taking into account symbols (3.6) and introducing operator polynomial

$$\tilde{\partial}_0^2 = PC = (\sigma + \varepsilon_a \partial_0^*)(r + \mu_a \partial_0^*) = \varepsilon_a \mu_a (\partial_0^*)^2 + (\sigma\mu_a + r\varepsilon_a)\partial_0^* + \sigma r, \quad (3.7)$$

partial differential operator from the left side of (3. 5) can be expressed as follows

$$A^2 + PC = \mathbf{grad}(\partial_1 + \partial_2 + \partial_3) - \Delta + \tilde{\partial}_0^2. \quad (3.8)$$

Using (3.7), (3.8), the former general vector wave equation (3.5) can be described in the coordinate form

$$\begin{cases} A_{23}F_{i1} - B_{12}F_{i2} - B_{13}F_{i3} = \varphi_{i1}, \\ -B_{12}F_{i1} + A_{13}F_{i2} - B_{23}F_{i3} = \varphi_{i2}, \\ -B_{13}F_{i1} - B_{23}F_{i2} + A_{12}F_{i3} = \varphi_{i3}, \ (i = 1, 2). \end{cases} \quad (3.9)$$

In (3.9): operators

$$A_{jk} = \partial_j^2 + \partial_k^2 + \tilde{\partial}_0^2, \ (j \neq k); \ B_{jk} = \partial_j \partial_k, \ (j \neq k), \ (j, k = \overline{1, 3}) \quad (3.10)$$

and scalar functions

$$F_{ik}, \varphi_{ik} = F_{ik}, \varphi_{ik}(x, y, z, t), \ (k = \overline{1, 3}; \ i = 1, 2) \quad (3.11)$$

are given in terms of (3. 6), where

$$\vec{F}_i = \{F_{ik}\}_{k=1}^3; \ \vec{\varphi}_i = \{\varphi_{ik}\}_{k=1}^3; \ (i = 1, 2). \quad (3.12)$$

Applying directly diagonalization procedure from [5] to (3.9) – (3.12) one gets the required diagonal matrix problem equivalent to both systems, – (3.9), (2.1). Moreover, the latter can be expressed as the general wave scalar equation regarding all unknown components of the electromagnetic field vector intensities

$$\tilde{\partial}_0^2(\tilde{\partial}_0^2 - \Delta)F_{ik} = (\partial_k^2 - \tilde{\partial}_0^2)\varphi_{ik} + \partial_k(\partial_\nu \varphi_{i\nu} + \partial_l \varphi_{il}),$$

$$(\nu \neq l, k \neq \nu, k \neq l; k, \nu, l = \overline{1, 3}; \ i = 1, 2). \quad (3.13)$$

In other words, (3.13) represents the attainment of (2.1) solution in terms of diagonalization procedure based on the operator analogy of the algebraic Gauss method.

Now, as it was mentioned above, the main features of the inverse matrix operator construction should be proposed here in comparison with the preceding analytic approach. Turning to [7], it is useful to show how both diagonalization methods can act together successfully. Thus, after the previous algorithm action “by blocks”, the inverse matrix operator construction for (3.9) appears. Writing (3.9) in a matrix form:

$$KF_i = \varphi_i, \ (i = 1, 2);$$

$$K = \begin{bmatrix} A_{23} & -B_{12} & -B_{13} \\ -B_{12} & A_{13} & -B_{23} \\ -B_{13} & -B_{23} & A_{12} \end{bmatrix}, \quad F_i = \begin{bmatrix} F_{i1} \\ F_{i2} \\ F_{i3} \end{bmatrix}, \quad \varphi_i = \begin{bmatrix} \varphi_{i1} \\ \varphi_{i2} \\ \varphi_{i3} \end{bmatrix} \ (i = 1, 2) \quad (3.14)$$

the inverse matrix operator regarding K can be obtained

$$K^{-1} = (\det K)^{-1} \begin{bmatrix} K_{11} & K_{21} & K_{31} \\ K_{12} & K_{22} & K_{32} \\ K_{13} & K_{23} & K_{33} \end{bmatrix}, \quad (3.15)$$

where

$$\det K = -\tilde{\partial}_0^2 (\tilde{\partial}_0^2 - \Delta)^2; \ K_{mm} = (\Delta - \tilde{\partial}_0^2)(\partial_m^2 - \tilde{\partial}_0^2);$$

$$K_{mn} = K_{nm} = \partial_m \partial_n (\Delta - \tilde{\partial}_0^2), \ (m, n = \overrightarrow{1, 3}; \ m \neq n). \quad (3.16)$$

In (3.16), the first expression is the determinant of (3.14), and all others are operator analogies of algebraic adjuncts from (3.15) with respect to the original matrix K from (3.14). Substitution of (3.16) for (3.15) gives

$$K^{-1} = (\tilde{\partial}_0^2)^{-1} (\tilde{\partial}_0^2 - \Delta)^{-1} \begin{bmatrix} \partial_1 - \tilde{\partial}_0^2 & \partial_1 \partial_2 & \partial_1 \partial_3 \\ \partial_1 \partial_2 & \partial_2 - \tilde{\partial}_0^2 & \partial_2 \partial_3 \\ \partial_1 \partial_3 & \partial_2 \partial_3 & \partial_3 - \tilde{\partial}_0^2 \end{bmatrix}. \quad (3.17)$$

Simple verification confirms $K^{-1}K = KK^{-1} = I = \mathrm{diag}\{1, 1, 1\}$ and agrees with algebraic matrix analogy [6].

Application of (3.17) to (3.14) leads to the following equivalent diagonal system

$$F_i = \begin{bmatrix} F_{i1} \\ F_{i2} \\ F_{i3} \end{bmatrix} = K^{-1} \varphi_i = K^{-1} \begin{bmatrix} \varphi_{i1} \\ \varphi_{i2} \\ \varphi_{i3} \end{bmatrix} \Leftrightarrow$$

$$\Leftrightarrow \begin{bmatrix} F_{i1} \\ F_{i2} \\ F_{i3} \end{bmatrix} = (\tilde{\partial}_0^2 (\tilde{\partial}_0^2 - \Delta))^{-1} \begin{bmatrix} \partial_1^2 - \tilde{\partial}_0^2 & \partial_1 \partial_2 & \partial_1 \partial_3 \\ \partial_1 \partial_2 & \partial_2^2 - \tilde{\partial}_0^2 & \partial_2 \partial_3 \\ \partial_1 \partial_3 & \partial_2 \partial_3 & \partial_3^2 - \tilde{\partial}_0^2 \end{bmatrix} \begin{bmatrix} \varphi_{i1} \\ \varphi_{i2} \\ \varphi_{i3} \end{bmatrix}, (i = 1, 2). \quad (3.18)$$

It is easy to find that in reality (3.18) is identical to (3.13) and looks like

$$\tilde{\partial}_0^2(\tilde{\partial}_0^2-\Delta)\begin{bmatrix}F_{i1}\\F_{i2}\\F_{i3}\end{bmatrix}=\begin{bmatrix}(\partial_1^2-\tilde{\partial}_0^2)\varphi_{i1}+\partial_1(\partial_2\varphi_{i2}+\partial_3\varphi_{i3})\\(\partial_2^2-\tilde{\partial}_0^2)\varphi_{i2}+\partial_2(\partial_1\varphi_{i1}+\partial_3\varphi_{i3})\\(\partial_3^2-\tilde{\partial}_0^2)\varphi_{i3}+\partial_3(\partial_1\varphi_{i1}+\partial_2\varphi_{i2})\end{bmatrix},\ (i=1,2). \quad (3.19)$$

Further detailed study of the inverse operator (3.17) existence brings the following

Solvability criterion. System (2.1) is solved in the meaning of its equivalence to the general scalar wave PDE (3.13) = (3.19) if such conditions are true

$$\left[\begin{array}{l}\Delta<-\left(\frac{1}{2}\left(\sigma\sqrt{\frac{\mu_a}{\varepsilon_a}}-r\sqrt{\frac{\varepsilon_a}{\mu_a}}\right)\right)^2\\ \left\{\begin{array}{l}\Delta\geq-\left(\frac{1}{2}\left(\sigma\sqrt{\frac{\mu_a}{\varepsilon_a}}-r\sqrt{\frac{\varepsilon_a}{\mu_a}}\right)\right)^2\\ \partial_0=\frac{\partial}{\partial t}\neq\begin{bmatrix}-\lambda\\+\lambda\end{bmatrix}-\frac{1}{2}\left(\left(\frac{\sigma}{\varepsilon_a}+\frac{r}{\mu_a}\right)\mp\sqrt{\left(\frac{\sigma}{\varepsilon_a}-\frac{r}{\mu_a}\right)^2+\frac{4\Delta}{\mu_a\varepsilon_a}}\right)\end{array}\right.\end{array}\right., \quad (3.20)$$

and only non generalized functions are taken into account.

In (3.20), sign reversal in front of λ is independent of the sign value near the square root, and inequalities are understood in the meaning of the corresponding operator influence on the functions from certain classes.

4. Diagonalization procedure in the case of the classical differential Maxwell system

Let the classical differential Maxwell system with constitutive equations be given in macroscopic theory of electromagnetic field [3, p. 10]

$$\begin{cases}\mathbf{rot}\,\vec{E}+\frac{1}{c}\frac{\partial\vec{B}}{\partial t}=0,\\ \mathbf{rot}\,\vec{H}-\frac{1}{c}\frac{\partial\vec{D}}{\partial t}=\frac{4\pi}{c}\vec{j},\\ \vec{D}=\varepsilon_0\vec{E};\quad\vec{B}=\mu_0\vec{H};\quad\vec{j}=\sigma\vec{E}.\end{cases} \quad (4.1)$$

In (4.1), the unknown vector functions $\vec{E}, \vec{D} = \vec{E}, \vec{D}(x, y, z, t)$ and $\vec{H}, \vec{B} = \vec{H}, \vec{B}(x, y, z, t)$ characterize both the electric and magnetic fields correspondingly; $\vec{j} = \vec{j}(x, y, z, t)$ is known and describes current density; $\sigma, \varepsilon_0, \mu_0$ are parameters of the medium and represent its specific conductivity, electric and magnetic permeability. Those mentioned attributes are positive constants in the case of homogeneous media and become functions regarding (x, y, z) for heterogeneous ones. Light speed in vacuum $c \approx 3 \cdot 10^{10}\,\text{cm}/\text{sec}$.

The aim of the present section consists of explicit solution of (4.1) basing on the diagonalization procedures of the preceding section 3. It should be reminded once more that diagonalization implies reduction of the original matrix problem regarding the unknown vector field function $\vec{F}$ to the equivalent system of such equations where each of them depends on the only one required scalar component of $\vec{F}$.

At first, substitution of the last three equations in (4.1) for the first and the second ones reduces (4.1) to the equivalent homogeneous system of PDEs:

$$\begin{cases} \mathbf{rot}\,\vec{E} + \dfrac{\mu_0}{c}\partial_0 \vec{H} = 0, \\ -\dfrac{1}{c}(\varepsilon_0 \partial_0 + 4\pi\sigma)\vec{E} + \mathbf{rot}\,\vec{H} = 0, \ \ \partial_0 = \partial / \partial t. \end{cases} \tag{4.2}$$

It is easy to notice that diagonalization of (4.2) using method of inverse matrix operator construction (look previous section 3) is impossible because of the last system homogeneity. Really, basing on [6] (4.2) has non zero solutions when the following condition is true:

$$\det \mathbf{M} = 0, \ \ \mathbf{M} = \begin{bmatrix} \mathbf{rot} & \dfrac{\mu_0 \partial_0}{c} \\ -\dfrac{1}{c}(\varepsilon_0 \partial_0 + 4\pi\sigma) & \mathbf{rot} \end{bmatrix}, \tag{4.3}$$

where $\mathbf{M}$ is the matrix operator generated by (4.2), and $\det \mathbf{M}$ is its determinant.

In its turn, the inverse matrix operator $\mathbf{M}^{-1}$ with respect to $\mathbf{M}$ from (4.3) exists then and only then when $\det \mathbf{M} \neq 0$. This fact eliminates presence of non trivial (non zero) solutions. Thus, only operator analogy of the classical Gauss method from section 3 can be applied here.

Influences of **rot** and $-\frac{\mu_0}{c}\partial_0$ upon the first and second equations from (4.2) respectively, after term-by-term addition of those transformed equations give the equivalent system where the first equation depends on the only one vector field function

$$\begin{cases} \left(\mathbf{rot}^2 + \frac{1}{c^2}\mu_0\partial_0(\varepsilon_0\partial_0 + 4\pi\sigma)\right)\vec{E} = 0, \\ -\frac{1}{c}(\varepsilon_0\partial_0 + 4\pi\sigma)\vec{E} + \mathbf{rot}\,\vec{H} = 0, \ \ \partial_0 = \partial/\partial t. \end{cases} \tag{4.4}$$

Further application to the first and the second equations from (4.4) operators $\frac{1}{c}(\varepsilon_0\partial_0 + 4\pi\sigma)$ and $\left(\mathbf{rot}^2 + \frac{1}{c^2}\mu_0\partial_0(\varepsilon_0\partial_0 + 4\pi\sigma)\right)$ respectively and again term-by-term addition of both transformed equations lead to the final diagonal system regarding each of the required vector field intensities

$$\begin{cases} \left(\mathbf{rot}^2 + \frac{1}{c^2}\mu_0\partial_0(\varepsilon_0\partial_0 + 4\pi\sigma)\right)\vec{E} = 0, \\ \mathbf{rot}\left(\mathbf{rot}^2 + \frac{1}{c^2}\mu_0\partial_0(\varepsilon_0\partial_0 + 4\pi\sigma)\right)\vec{H} = 0, \ \ \partial_0 = \partial/\partial t. \end{cases} \tag{4.5}$$

Introduction of new operator notation:

$$\bar{\partial}_0^2 = \frac{1}{c^2}\mu_0\partial_0(\varepsilon_0\partial_0 + 4\pi\sigma) \tag{4.6}$$

allows writing determinant of the initial system (4.2), and it is equal to $\det\mathbf{M}$ from (4.3):

$$\det\mathbf{M} = \det(4.2) = \mathbf{rot}^2 + \bar{\partial}_0^2 = \mathbf{grad}\,\mathrm{div} - \Delta + \bar{\partial}_0^2. \tag{4.7}$$

The right part of (4.7) is obtained owing to the well known formulas of classical electrodynamics [3] and expression (4.6).

In its turn, (4.5) in the symbols of (4.6), (4.7) looks like

$$\begin{cases} (\mathbf{rot}^2 + \bar{\partial}_0^2)\vec{E} = 0, \\ \mathbf{rot}\,(\mathbf{rot}^2 + \bar{\partial}_0^2)\vec{H} = 0, \ \ \partial_0 = \partial/\partial t, \end{cases} \tag{4.8}$$

and

$$(\mathbf{rot}^2 + \bar{\partial}_0^2)\vec{E} = (\mathbf{grad}\,\mathrm{div} - \Delta + \bar{\partial}_0^2)\vec{E} = (\det(4.2))\ \vec{E}. \tag{4.9}$$

Taking into account (4.8), (4.9) and [6], we can assert that non trivial (non zero) solution of homogeneous system, (4.2) in particular, exists only of its determinant zero value ($\det(4.2) = 0$ here). But in the case of $\det(4.2) = 0$ and (4.9), the first equation from (4.8) is zero identically while $(4.7) = 0$ in vector sense. It means that even at this first diagonalization stage (4.8) and (4.9) need very accurate detailed analytic further investigation.

5. Conclusions

Comparing virtues and drawbacks of both diagonalization methods it is easy to find that operator analogy of the Gauss procedure effectively acts as for homogeneous, as for inhomogeneous systems. Also, it can be applied to arbitrary block matrix structure beginning from the external and coming to the required inner block, moving through all levels of vector function and stopping at any desired step. Moreover, this algorithm does not depend either on the coordinate system, or specific boundary problem statement. The latter is suggested only at the end, when the equivalent system of scalar equations appears. It is obvious that formulation of the appropriate scalar boundary problem and creation of its solution is incomparably more evident than dealing with the initial matrix form.

The main drawback of the same method consists of the natural requirement of all matrix operator elements' invertibility. It implies here investigation of all operator kernels and their intersection. Such computation and analysis are rather complicated and even vast, especially for matrices (systems of operator equations) whose order is greater than two.

Turning to the method of inverse matrix operator construction, it is not difficult to confirm that its main shortcoming is impossibility to work with homogeneous systems of operator equations. Nevertheless, the proposed algorithm allows constructing solvability conditions that are very useful in correct theoretical and physical scalar boundary problem statement instead of the vector one. Moreover, both methods can be applied in the framework of one and the same problem simultaneously.

At last, precise analysis of final results from section 4 is under consideration yet and is prepared for nearest publications.

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