ECONOPHYSICS Section

THE NEOCLASSICAL THEORY OF A FIRM; CORRECTIONS FOR ITS ERRORS

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Abstract. The static neoclassical theory of a firm is logically inconsistent, and it has not got any support in empirical tests. In spite of this, the theory is dominating in mainstream textbooks. The main logical inconsistencies in the neoclassical theory are: 1) Assuming that a firm produces at a constant positive profit maximizing flow of production does not allow its growth or the possible bankruptcy of the firm, which are common events in firms' behavior. 2) The theory does not explain how firms find their equilibrium; only optimal behavior is modeled. 3) The dynamic neoclassical theory of a firm obtained by dynamic optimization is inconsistent with the static one. The main empirical shortcomings of the theory are: 1) most data of production flows obeys a unit root which implies that the time series has a linear or a more complicated time trend; business cycles are also common in production data. These observations question the assumption that firms produce at a constant flow of production. 2) In the neoclassical theory, price explains the flow of production but price and the flow of production do not always correlate positively. 3) The existence of a profit maximizing flow of production requires decreasing returns to scale in production, but increasing returns have been observed in various studies. 4) The assumed duality in the neoclassical theory has been rejected in empirical tests. As a solution to these problems, we present a dynamic theory of a firm that corrects the shortcomings in the neoclassical theory. We define the "economic force" that acts upon the production of a firm and show that firms' profit-seeking adjustment of production may be stable or unstable. Economic growth, business cycles, and bankruptcies of firms are modeled by using a single framework, and the static neoclassical theory is obtained as a special case in this framework: the zero-force situation. (JEL D21, O12)

Keywords: Errors in the neoclassical theory, Dynamics of production, Economic force, Causes for firms' bankruptcies.

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1. Introduction

Static neoclassical theory of a firm is the dominating one in economics textbooks. However, by static analysis we cannot explain the observed evolutionary behavior of economies. MasColell et al. [1 p. 620] state the problems of the neo-classical framework as follows: "A characteristic feature that distinguishes economics from other scientific fields is that, for us, the equations of equilibrium constitute the center of our discipline. Other sciences, such as physics or even ecology, put comparatively more emphasis on determination of dynamic laws of change. ... The reason, informally speaking, is that economists are good (or so we hope) at recognizing a state of equilibrium but are poor at predicting precisely how an economy in disequilibrium will evolve. Certainly there are intuitive dynamic principles: if demand is larger than supply then price will increase, if price is larger than marginal cost then production will expand. ... The difficulty is in transforming these informal principles into precise dynamic laws."

The definition of "precise dynamic economic laws" requires a dynamic framework for modeling economic behavior, and we introduce here such. We believe that the willingness of economic agents (entrepreneurs, consumers, workers etc.) to better their situation in a competitive environment is the fundamental cause of economic dynamics. The assumption that economic units behave in an optimal way prohibits understanding economic dynamics because no economic unit likes to change its optimal behavior. The error in the neoclassical framework is the same as if in physics it were assumed that the initial position of a particle is in its point of minimum potential energy where it has "no willingness" to move anywhere. Thus to understand economic dynamics we need to assume that economic units are not in their optimum, for example, after a price change that has shifted the equilibrium point. Economic units like to improve their welfare and optimal behavior results when no improvement is possible, see [2].

We start by presenting the neoclassical theory of a firm to point out its theoretical weaknesses. Next we give empirical evidence that is in contrast with the assumptions made in the neoclassical framework. Then we introduce a theory that corrects the weaknesses in the neoclassical framework and gives the static neoclassical theory as a special case: the zero-force situation. Finally, we give examples of dynamic behavior that can be explained by using the proposed framework.

2. The neoclassical theory of a firm and its weaknesses

Let the flow of production of a firm be q(unit/y), where y is a time unit e.g. a week or a month. The unit price of the product of the firm is denoted by $p(\in/unit), p(q)$ is the sales or inverse demand function, $C(q)(\notin/y)$ the cost function, and $\Pi(p,q)(\notin/y)$ the profit of the firm,

$$\Pi(p,q) = p(q)q - C(q), \ C'(q) > 0, \ C''(q) < 0.$$

In the neoclassical framework it is assumed that the firm produces at its profit maximizing flow:

$$\frac{\partial \Pi}{\partial q} = 0 \Leftrightarrow p(q) + p'(q)q = C'(q) \Leftrightarrow q^* = f(p),$$

where p(q) + p'(q)q is marginal revenue, C'(q) marginal cost, and q^* the optimal flow of production of the firm. The essential weaknesses of the theory are:

- 1) In the theory time is abstracted away and thus q^* depends on a fixed price p. This is in contrast with the observed growth of firms, business cycles, and firms' bankruptcies.
- 2) The theory assumes that price determines the flow of production, but in the theory it is not explained mathematically how the firm reaches its new optimum after a price change. Only equilibrium situations are modeled.
- 3) The static and the dynamic neoclassical theory obtained by dynamic optimization are inconsistent with each other, see [3].
- 4) The interactions between firms' production decisions are not taken account properly in modeling the behavior of a single firm, see [4].
- 5) The theory does not explain the motivation for firms to develop their production technology or the quality of their products that are essential elements in firms' competition.

In Figure 7 in Appendix are graphed annual industrial flows of production and price levels of Finnish manufacturing industries: DA: Food products, beverages and tobacco, DB+DC: Textiles, textile products,

leather and leather products, DD: Wood and wood products, DE: Pulp, paper and paper products, publishing and printing, DF: Refined petroleum products, coke and nuclear fuel, DG: Chemicals and chemical products, DH: Rubber and plastic products, DI: Other non-metallic mineral products, DJ: Basic metals and fabricated metal products, DK: Machinery and equipment, DL: Electrical and optical equipment, DM: Transport equipment, DN: Other manufacturing and recycling. The industrial prices are computed as $p_tq_t/(p_0q_t) = p_t/p_0$, i.e. current value time series p_tq_t are divided by fixed price series p_0q_t .

The following observations of the data are in contrast with the neoclassical framework:

- 1) All industrial flows of production and prices obey a unit root (Table 1 in Appendix; the power values of all Augmented Dickey-Fuller tests are greater than 0.01). Thus a linear or a more complicated time trend exists in the series, and fluctuating behavior is also visible. These observations question the omitting of time in the neoclassical framework.
- According to the *T*-tests of the correlations (Table 1), two of the industrial prices do not correlate significantly with the flows of production, and two of the correlations are significantly negative. Thus only in 9 cases of 13 a significant positive correlation is observed between the flow of production and the price in an industry.
- 3) The theory assumes decreasing returns to scale(C''(q) < 0), but increasing returns have been observed in various studies, see e.g. [5, 6].
- 4) In [7] the duality theory of a firm fails in empirical testing because the estimated parameters of production and cost functions turned out to differ significantly. The reason for this may be that some of the following assumptions in the neoclassical framework are erroneous: cost minimizing behavior, perfect competition, static framework, and no uncertainties in modeling.

Due to these theoretical and empirical weaknesses of the neoclassical theory, we introduce a dynamic theory of a firm that corrects them. This theory is similar to the Newtonian one in physics, and it is based on a force that is acting upon the production of a profit-seeking firm. This framework has been presented earlier in refs [8, 9], but here we give some new results and data that support the theory. In [8] is described how uncertainties can be handled in the theory, and in [9] it is shown that the Newtonian theory to be presented here outperforms the neoclassical one in every studied industry. Thus we have evidence of the superiority of the theory to be introduced next.

3. Kinematics of production

The accumulated production of a firm till time moment t (the accumulated kilometers a car has driven) denoted by Q(t)(unit) (a marginal change in time ds is measured in time units y) is

$$Q(t) = Q(t_0) + \int_{t_0}^t q(s)ds, \quad Q'(t) = q(t), \quad Q''(t) = q'(t),$$

where $Q(t_0)$ is the accumulated production of the firm from its foundation till moment $t_0, Q'(t)$ with unit *unit*/y the momentous velocity of accumulated production, and Q''(t) = q'(t) with unit *unit*/y² the momentous acceleration of accumulated production. This kinematics of production is a necessary prelude for production dynamics analogous to Newtonian mechanics.

4. A dynamic theory of production

A common way to transform the neoclassical theory into dynamic form is to use dynamic optimization. The applications of this technique e.g. [10] use time dependent profit functions in contrast with the static theory, however, and thus the two theories are inconsistent. The dynamic optimization problem of a firm gives the same result as the static problem if identical profit functions are applied, see [3]. Dynamic optimization as a mathematical technique then does not solve the problem, and we omit here the dynamic optimization technique.

The decision-making concerning the dynamics of production of the firm – the cause of this dynamics – can be studied by assuming that the decision-makers are planning whether to increase the accumulated production of the firm by a certain amount or not, or whether to change the velocity of accumulated production of the firm measured from the last week, month, or year by a certain quantity. We study here only the latter

alternative; for the former see [8]. The function, that expresses the maximum unit price $p(t) (\notin / unit)$ by which the firm can sell the quantity q(t) (unit/y) of products during time unit y, the inverse demand or the sales function of the firm, is

$$p(t) = f(q(t), t), \frac{\partial f}{\partial q} \le 0.$$

Our modeling covers a monopoly firm and a firm in monopolistically and also in "roughly perfectly" competing market. We assume here that the product is heterogeneous with those of other firms to avoid taking account other firms' production decisions in the production decision of the studied firm, see [4]. Including time in function $f(\cdot)$ allows that the sales of the firm may change with time due to changes in customers' habits or due to marketing the good.

The costs of firms can be divided in four categories: 1) costs from starting production (buying or renting the necessary buildings, machines and tools), 2) fixed costs in a given time unit independent of the level of activity (monthly rents, salaries, heating costs etc.), 3) costs in a given time unit that depend on the level of activity (costs from working hours, raw materials etc.), and 4) costs from maintaining and expanding the production capacity (gross and net investments). Here we do not treat starting costs and investment decisions, and so these costs are omitted in the following.

Without losing generality we can write the cost function as

$$C(q(t),t) = h(t) + g(q(t),t)q(t).$$

We denote by $C(\cdot)(\in/y)$ the costs of the firm during time unit y at moment t realized at the velocity of production q(t), by $h(t), (\in/y)$ fixed costs during time unit y independent of the velocity of production, and unit costs $g(\cdot)(\in/unit)$ may depend on the velocity of production and time. Technological development, for example, may decrease unit costs with time.

The profit Π with unit \notin / y is then

$$\Pi(q(t),t) = f(q(t),t)q(t) - h(t) - g(q(t),t)q(t), \frac{\partial f}{\partial q} \le 0.$$

Taking the time derivative of the profit function gives

$$\Pi'(t) = \frac{\partial \Pi}{\partial q} q'(t) + \frac{\partial \Pi}{\partial t}$$
$$= \left(\left(\frac{\partial f}{\partial q} - \frac{\partial g}{\partial q} \right) q(t) + f(q(t)t) - g(q(t)t) \right) q'(t) + \left(\frac{\partial f}{\partial t} - \frac{\partial g}{\partial t} \right) q(t)$$
(1)
$$-h'(t).$$

The second additive term in the first form of Eq. (1) shows that time affects profit via functions f(.), g(.), and h(.) independently on the firm's production decisions, which motivates the firm to do marketing and develop its technology. The firm's leaders like to increase the profit of the firm with time, and then they change q(t) as follows:

$$q'(t) > 0$$
 if $\frac{\partial \Pi}{\partial q} > 0$, $q'(t) < 0$ if $\frac{\partial \Pi}{\partial q} < 0$, and $q'(t) = 0$ if $\frac{\partial \Pi}{\partial q} = 0$. (2)

These rules make the first additive term in the first form of Eq. (1) non-negative, i.e. the rules increase the profit with time. The last rule implies that there is no reason to change the velocity of production if it does not affect profit. These rules are in accordance with the intuitive dynamic rules stated in [1] referred earlier, and [11 p. 387] writes about the behavior of a firm in a perfectly competed market as: "... *if* $p - \Delta C / \Delta q > 0$, *then the firm can increase its profits by producing more*". Thus also in mainstream textbooks adjustment rules (2) have been accepted.

Imitating Newtonian mechanics we identify $\partial \Pi / \partial q$ – the reason for the acceleration of production – as the "force acting upon the production of the firm". A relation that fulfills rules (2) is

$$q'(t) = F\left(\frac{\partial\Pi}{\partial q}\right), \quad F'\left(\frac{\partial\Pi}{\partial q}\right) > 0, \quad F(0) = 0,$$
 (3)

where $F : \mathbb{R} \to \mathbb{R}$. In Eq. (3), the firm adjusts its velocity of production according to the deviation between marginal revenue and cost (this assumption is made in [12] too). Taking the Taylor series approximation of

function *F* in the neighborhood of point $\partial \Pi / \partial q = 0$, and assuming the error term to be zero, we can approximate function *F* by a linear one,

$$mq'(t) = \frac{\partial \Pi}{\partial q}.$$
(4)

Constant m > 0 with unit $(\mathbb{E} \times y^2)/unit^2$ is the ratio of force and acceleration; its magnitude measures the inertia in the firm's adjustment of production: rigid technology, bottlenecks in the production process, fear of decreasing product price etc. Following Newton we call *m* the "*inertial mass of the velocity of production*", and Eq. (4) the Newtonian theory of a firm adjusting its velocity of production. Neoclassical theory corresponds to zero-force in Eq. (4): $\partial \Pi / \partial q = 0 \Leftrightarrow q'(t) = 0$.

We can also introduce the concept of static friction into economics. Static friction is a force component resisting all changes. By reflecting on this concept we can explain that many times firms (and people) do not change their behavior unless the reasons become compelling enough, i.e. the acting force component exceeds a limit. This can be formulated as

$$mq'(t) = \frac{\partial \Pi}{\partial q} + F_f, \qquad (5)$$

where the static friction force with unit $\notin/unit$ is denoted by F_f ; it is zero when $\partial \Pi/\partial q = 0$, its direction is opposite to that of $\partial \Pi/\partial q$, and $|F_f| \leq |\partial \Pi/\partial q|$. The absolute value of F_f increases with that of $\partial \Pi/\partial q$ and it keeps the total force zero until $\partial \Pi/\partial q$ exceeds a limit. Static friction could be added in all examples we study later but is omitted for simplicity.

Case 1: A dynamic extension to the neoclassical theory

Let the sales and unit cost functions be

$$f(q(t)) = a - \frac{b}{2}q(t), \quad g(q(t)) = A,$$

where q(t) is as before and a,b,A positive constants with units: $\frac{e}{unit}, \frac{e}{y}{unit}^2$, and $\frac{e}{unit}$, respectively. Parameter *a* measures the maximum unit price at which the first produced unit during time unit *y* can be sold, and *b* the relation between unit price and the sales of the firm. Taking account the problems in the definition of perfect competition (see [4]), the more competition there exists the closer to zero *b* is; monopolistic competition and a monopoly firm correspond to b > 0. Parameter A > 0 measures unit costs from the first produced unit during time unit *y*. Assuming the fixed costs during time unit *y* to be constant h_0 , the profit function $\Pi(\notin y)$ becomes

$$\Pi(t) = f(q(t))q(t) - h_0 - g(q(t))q(t) = aq(t) - \frac{b}{2}q^2(t) - h_0 - Aq(t).$$

Assuming no static friction in production, the Newtonian equation is:

$$\frac{\partial \Pi}{\partial q} mq'(t) \Leftrightarrow a - A - bq(t) = mq'(t).$$
(6)

Eq. (6) shows that the firm increases its flow of production (g'(t) > 0)if $g(t) < (a - A) / b = q^*$, and decreases its flow of production if $g(t) > q^*$, where g^* is the profit maximizing flow of production. The solution of differential equation (6) is

$$q(t) = \frac{a-A}{b} + C_0 e^{-\frac{b}{m}t},$$
(7)

where $C_0(unit/y)$ is the constant of integration and (b/m)t is a dimensionless quantity because time t has unit y. According to Eq. (7), $q(t) \rightarrow q^* = (a - A)/b$ with time which situation corresponds to zero force. Setting t = 0 in Eq. (7) gives $q(0) = (a - A)/b + C_0 \Rightarrow C_0 = q(0) - (a - A)/b$. Thus q(t) increases (decreases) with time if C_0 is negative (positive), that is, whether q(0) is smaller or greater than g^* . The neoclassical theory corresponds to $q(0) = (a - A)/b = q^*$, and then the firm produces at the constant optimal flow of production. This example adds dynamics in the neoclassical theory and shows how the firm reaches its optimum with time if it does not be in it; see Figures 1, 2 where $g^* = 10$.



Case 2: Growth in production, business cycles, and the bankruptcy of the firm

Let the sales function be

$$f(q(t),t) = a - \frac{b}{2}q(t) + ct + k_1 Sin(h_1 + w_1 t) + k_2 Sin(h_2 + w_2 t),$$

where parameter c with unit $\notin/(unit \times y)$ measures changes in sales with time due to changes in consumers' preferences or wealth (c may be positive or negative), the amplitude parameters k_1 of the sine functions have unit $\notin/unit$, the frequency parameters w_i have unit 1/y, and the phase parameters h_i are pure numbers, i=1,2; the other terms are as before. Parameters k_i are needed for the dimensional consistency of the function, and the unit of the frequency parameters w_i makes the arguments of the sine functions dimensionless as they should be (a system of measurement units for economics is given in [13] and also in [14]. The sine functions represent business cycles and seasonal fluctuations that affect the sales of the firm with different phase and frequency.

Let the unit costs be:

$$g(q(t),t) = A + \frac{B}{2}q(t) - Ct,$$
(8)

where A, B, C are dimensional constants with units $\notin /unit$, $(\notin \times y)/unit^2$, and $(\notin \times y)/unit$, respectively. Parameter *B* measures either decreasing (B > 0) or increasing (B < 0) returns to scale, and parameter *C* measures the development in technology that decreases unit costs with time if (C > 0). Assuming fixed costs to be constant h_0 the profit Π during time unit y becomes:

$$\Pi(t) = \left(a - \frac{b}{2}q(t) + ct + k_1 Sin(h_1 + w_1 t) + k_2 Sin(h_2 + w_2 t)\right)q(t) - \left(A + \frac{B}{2}q(t) - Ct\right)q(t) - h_0.$$
(9)

The Newtonian equation is then:

$$mq'(t) = z_0 - z_1q(t) + z_2t + k_1Sin(h_1 + w_1t) + k_2Sin(h_2 + w_2t),$$
(10)

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where $z_0 = a - A$, $z_1 = b + B$, and $z_2 = c + C$. If the unit price by which the firm can sell its first produced unit during time unit y exceeds the unit costs of the first unit, then $z_0 > 0$. The second term in the force consists of possible linear relations unit price and unit costs may have with q(t). It always holds $b \ge 0$, and decreasing returns to scale corresponds to B > 0, and increasing returns to scale to B < 0. We obtain different behavior depending on the sign of z_1 , that is, whether the increasing returns to scale effect (B < 0) dominates the negative demand effect (b > 0). Constant z_2 shows that changes in consumers' preferences or wealth and technological development create an identical linear time trend in the force; Eq. (10) thus does not distinguish between demand and supply based growth. Notice that z_2 may be negative, too, which corresponds to situations where consumers substitute this product by others or the unit costs of the firm increase with time.

The solution of Eq. (10) is:

$$q(t) = a_0 + a_1 t + a_2 Sin(w_2 t) + a_3 Cos(w_2 t) + a_4 Sin(h_1 + w_1 t) + a_5 Cos(h_1 + w_2 t) + a_6 e^{-\left(\frac{z_1}{m}\right)t},$$
(11)

where:

$$a_{0} = \frac{(z_{0}z_{1} - mz_{2})(z_{1}^{2} + m^{2}w_{1}^{2})}{z_{1}^{4} + m^{2}z_{1}^{2}w_{1}^{2}}, a_{1} = \frac{(z_{1}z_{2}(z_{1}^{2} + m^{2}w_{1}^{2})}{z_{1}^{4} + m^{2}z_{1}^{2}w_{1}^{2}},$$

$$a_{2} = \frac{k_{2}(z_{1}Cos(h_{2}) + mw_{2}Sin(h_{2}))}{z_{1}^{2} + m^{2}w_{2}^{2}}, a_{3} = \frac{k_{2}(z_{1}Sin(h_{2}) - mw_{2}Cos(h_{2}))}{z_{1}^{2} + m^{2}w_{2}^{2}},$$

$$a_{4} = \frac{k_{1}z_{1}^{3}}{z_{1}^{4} + m^{2}w_{1}^{2}z_{1}^{2}}, a_{5} = -\frac{mk_{1}w_{1}z_{1}^{2}}{z_{1}^{2} + m^{2}w_{2}^{2}},$$

and $a_6(unit/y)$ is the constant of integration. In Eq. (11), q(t) has a linear and an exponential time trend together with cyclical behavior. The time trends are caused by returns to scale and time dependent sales and cost functions. A profit maximizing flow of production exists only if $z_0, z_1 > 0$ and $z_2 = k_1 = k_2 = 0$. Then Eq. (7) results and the firm will end into the neoclassical optimum. Setting different values for the constants we get varying realizations of Eq. (11), see Figures 3-6.





Figure 3 shows a growing flow of production with business cycles and seasonal fluctuations due to increasing sales or decreasing unit costs with time. Figure 4 shows exponential growth with fluctuations due to increasing returns to scale. In Figure 5, the firm ends up into bankruptcy due to decreasing sales or increasing unit costs with time, and Figure 6 shows that decreasing returns to scale and no time dependencies in sales and unit costs lead to stationary fluctuations.

We can remark here that our assumption of linear time relations in demand and unit costs are unrealistic. They were made only to get a simple Newtonian equation for production and more complicated functions would give more realistic results. However, the numerical value of z_2 can be chosen small enough that the growth rate obtained is e.g. 2%/year, which is a common growth rate of real GDP in industrialized countries. In Figure 7, the time paths of flows of production have a linear or an exponential time trend together with cyclical behavior. From this we have evidence at firm level too. For example, the average growth rate of the turnover of Nokia Corporation was 25.1%/year, during 1979-88, -2.5%/year, during 1989-92, and 28.5%/year, during 1993-8. This demonstrates the non-steady-state nature of the production process.

4. Conclusions

We introduced a dynamic framework for modeling firms' behavior that can explain firms' growth, business cycles, and bankruptcies. The "economic force" acting upon the production of a profit-seeking firm was defined, and the model was shown to give the neo-classical theory as a special case: the zero-force situation. The studied force fields were "time and velocity dependent". Physics too operates with velocity dependent forces, and engineering operates with time-dependent forces e.g. when the motion of a body is controlled by a force. The defined forces allow us to analyze the production of a firm as a controllable system. In this study the controller was the decision-maker of the firm, but the analysis can be extended to economic policy-making too; e.g. tax policy can be studied by assuming that government controls by tax parameters the productions of firms.

REFERENCES

- A. MasColell, M. D. Whinston, J. R. Green, *Microeconomic Theory*, Oxford University Press, New York, 1995.
- [2] Fisher, F. M., *Disequilibrium Foundations of Equilibrium Economics*, Cambridge University Press, 1983.
- [3] Estola, M., Consistent and Inconsistent ways to Dynamize the Neo-classical Theory, Hyperion International Journal of Econophysics & New Economy, Vol. 6, Issue 1, 2013.
- [4] Keen, S. and R. Standish. Debunking the theory of a firm a chronology. *Real-world economics review*, Issue no. **53**, 2010.
- [5] Lee, F. Post Keynesian Price Theory. Cambridge University Press, Cambridge, 1998.
- [6] Blinder, A. SA., Canetti, E. Lebow, D., & J. Rudd. Asking about Prices: A New Approach to Understanding Price Stickiness. Russel Sage Foundation, New York, 1998.
- [7] Appelbaum, E. Testing neoclassical production theory. Journal of Econometrics, 7, 1978, pp. 87-102.
- [8] Estola, M. A., *Dynamic Theory of a Firm: An Application of 'Economic Forces'*. Advances in Complex Systems, Vol. **4**, No. **1**, 2001.
- [9] Estola, M. & A. Dannenberg, *Testing the neo-classical and the Newtonian theory* of production, Physica A 391, 2012, pp. 6519-6527.
- [10] Evans, G. C., *The Dynamics of Monopoly*, American Mathematical Monthly, February, 1924, pp. 77-83.
- [11] Varian, H., *Intermediate Microeconomics*, A Modern Approach 5th Ed. W.W. Norton & Company, 1999.
- [12] Dixit, A. *Comparative statics for oligopoly*, International Economic Review **27** (1), 1986.
- [13] de Jong, F. J., *Dimensional Analysis for Economists*, North-Holland Publishing Company, Amsterdam, 1967.
- [14] Estola, M. & V-M. Hokkanen, Consumer, Firm, and Price Dynamics: An Econophysics Approach. Modeling by Economic Forces. VDM Verlag Dr. Muller, Saarbrücken, Germany, 2008.

APPENDIX

Table 1.

Industry	ADF, volume (Prob.)	ADF, price (Prob.)	Correlations, (T-test)
DA	0.9 (0.99)	-2.6 (0.10)	r = 0.34(2.03)
DB+DC	-0.6 (0.86)	-3.6 (0.01)	r = -0.87(-9.85)
DD	-0.6 (0.85)	-2.7 (0.09)	r = 0.45(2.86)
DE	-1.2 (0.65)	-2.0 (0.30)	r = 0.88(10.51)
DF	2.3 (0.99)	-1.9 (0.32)	r = -0.11(-0.64)
DG	-0.5 (0.89)	-1.0 (0.76)	r = 0.88(10.42)
DH	-0.4 (0.89)	-1.4 (0.59)	r = 0.94(14.94)
DI	-0.7 (0.83)	-2.2 (0.21)	r = 0.79(7.29)
DJ	1.9 (0.99)	-2.9 (0.06)	r = 0.86(9.58)
DK	1.2 (0.99)	-2.1 (0.23)	r = 0.72(5.93)
DL	1.5 (0.99)	-0.1 (0.94)	r=-0.75(-6.48)
DM	-3.2 (0.03)	-1.5 (0.51)	r = 0.13(0.75)
DN	-1.0(0.73)	-2.5(0.12)	r = 0.77(6.82)

Unit root and correlation tests of industrial flows of production and prices





Figure 7. Industrial flows of production and prices in Finnish manufacturing.