THE CHAOTIC PRODUCTION GROWTH MODEL OF THE MONOPOLY FIRM AND INCENTIVES TO INNOVATE

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Abstract. *Monopoly is a market with only one seller. Characteristics of the model of a monopolistic market structure are: a) a firm is a price-maker; b) a firm do not behave strategically; c) entry into the industry is completely blocked; d) buyers are price taker; e) no close substitutes for the firm's product; f) well informed buyers.*

Monopoly has the ability to influence the market price. Except a monopolist's choice of price and output, there are other decisions a monopolist must make. One of the most important is how much to invest in research and development activities. Process innovation explains an idea the lowers the cost of producing existing products. Monopoly will invest in a new technology whenever doing so lowers its costs.

The basic aim of this paper is to construct a relatively simple chaotic growth model of the monopoly quantity that is capable of generating stable equilibria, cycles, or chaos. Incentives to innovate are included in model.

A key hypothesis of this work is based on the idea that the coefficient

$$\pi = \frac{b\,(d-1)}{m\,(1-\alpha)\left[1+\left(\frac{1}{e}\right)\right]},$$

plays a crucial role in explaining local stability of the monopoly's production, where, b represents the coefficient of the marginal cost function of the monopoly firm, m – the coefficient of the inverse demand function, e – the coefficient of the price elasticity of the monopoly's demand, α *– the coefficient which explain effect of the investment in research and development activities on the monopolist's marginal cost curve.*

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1. Introduction

Monopoly is the form of market structure in which a single firm sells a commodity for which there are no close substitutes. Under monopoly, there is a single producer and entry into the market by additional sellers is

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blocked. Monopoly is a price maker because it can influence the price through this choice of quantity. A monopolist recognizes that the amount it sells influences the price it receives for its output. Except a monopolist's choice of price and output, there are other decisions a monopolist must make. One of the most important is how much to invest in research and development activities. For example, process innovation is an idea that lowers the cost of producing existing products. The innovation leads to lower production costs. Monopoly will invest in a new technology whenever doing so lowers its costs.

Chaos theory can explain effectively unpredictable economic long time behavior arising in a deterministic dynamical system because of sensitivity to initial conditions. It must be emphasized that a deterministic dynamical system is perfectly predictable given perfect knowledge of the initial condition, and is in practice always predictable in the short term. The key to long-term unpredictability is a property known as sensitivity to (or sensitive dependence on) initial conditions.

Chaos theory started with Lorenz's (1963) discovery of complex dynamics arising from three nonlinear differential equations leading to turbulence in the weather system. Li and Yorke (1975) discovered that the simple logistic curve can exhibit very complex behaviour. Further, May (1976) described chaos in population biology. Chaos theory has been applied in economics by Benhabib and Day (1981, 1982), Day (1982, 1983, 1997), Grandmont (1985), Goodwin (1990), Medio (1993, 1996), Lorenz (1993), among many others.

The basic aim of this paper is to provide a relatively simple chaotic growth model of the monopoly's production that is capable of generating stable equilibria, cycles, or chaos. Incentives to innovate are included in the model.

A Simple Chaotic Model of a Profit-Maximizing Monopoly

In the model of a profit-maximizing monopoly, take the inverse demand function:

$$P_t = n - m\, Q_t \tag{1}$$

where P – monopoly price; Q – monopoly output; n, m – coefficients of the inverse demand function.

Further, suppose the quadratic marginal-cost function for a monopoly is:

$$MC_t = a + b\, Q_t + c\, Q_t^{\,2} \tag{2}$$

where MC represents marginal cost; Q – monopoly output; a, b, c – coefficients of the quadratic marginal-cost function.

Monopoly will invest in a new technology whenever doing so lowers its costs. With innovation, the firm's marginal cost curve falls to:

$$MC_t = (a + bQ_t + cQ_t^2) - d\,(a + bQ_t + cQ_t^2) \qquad (3)$$

or:

$$MC_t = a(1-d) + b(1-d)\,Q_t + c\,(1-d)\,Q_t^2, \qquad (4)$$

where MC represents marginal cost; Q – monopoly output; a, b, c – coefficients of the quadratic marginal-cost function, d – the coefficient which explain effect of the investment in research and development activities on the monopolist's marginal cost curve.

Marginal revenue is:

$$MR_t = P_t\left[1+\left(\frac{1}{e}\right)\right] \qquad (5)$$

where MR represents marginal revenue; P – monopoly price; e – the coefficient of the price elasticity of demand.

A monopoly firm maximizes profit by producing the quantity at which marginal revenue equals marginal cost. Thus the profit-maximizing condition is that:

$$MRt = MC. \qquad (6)$$

Further,

$$Pt + 1 = Pt + \Delta P \qquad (7)$$

or:

$$Pt + 1 = Pt + \alpha\, Pt + 1, \qquad (8)$$

i.e.

$$(1-\alpha)\, P_{t+1} = P_t. \qquad (9)$$

Thus, the chaotic model of the profit-maximizing monopoly is presented by the following equations:

$$(1-\alpha)\, P_{t+1} = P_t \qquad (10)$$

$$MR_t = MC_t \qquad (11)$$

$$MR_t = P_t\left[1+\left(\frac{1}{e}\right)\right] \qquad (12)$$

$$MC_t = a\,(1–d) + b\,(1–d)\,Q_t + c\,(1–d)\,Q_t^{\,2} \tag{13}$$

$$P_t = n - m\,Q_t \tag{14}$$

where: Q represents output of the monopoly firm; MC – marginal cost; MR – marginal revenue; P – monopoly price; e – the coefficient of the price elasticity of demand; n, m – coefficients of the inverse demand function; a, b, c – coefficients of the quadratic marginal-cost function, d – the coefficient which explain effect of the investment in research and development activities on the monopolist's marginal cost curve.

Firstly, it is supposed that $a = 0$ and $n = 0$.

By substitution one derives:

$$Q_{t+1} = \frac{b\,(d-1)}{m\,(1-\alpha)\left[1+\left(\frac{1}{e}\right)\right]}Q_t - \frac{c\,(1-d)}{m\,(1-\alpha)\left[1+\left(\frac{1}{e}\right)\right]}Q_t^{\,2}. \tag{15}$$

Further, it is assumed that the monopoly quantity is restricted by its maximal value in its time series. This premise requires a modification of the growth law. Now, the monopoly output growth rate depends on the current size of the monopoly output, Q, relative to its maximal size in its time series Q^m. We introduce q as $q = Q/Q^m$. Thus q range is between 0 and 1. Again we index q by t, i.e., write q_t to refer to the size at time steps $t = 0, 1, 2, 3,...$ Now, growth rate of the monopoly output is measured as:

$$Q_{t+1} = \frac{b\,(d-1)}{m\,(1-\alpha)\left[1+\left(\frac{1}{e}\right)\right]}Q_t - \frac{c\,(1-d)}{m\,(1-\alpha)\left[1+\left(\frac{1}{e}\right)\right]}Q_t^{\,2}. \tag{16}$$

This model given by equation (11) is called the logistic model. For most choices of b, c, d, m, and e there is no explicit solution for (11). Namely, knowing b, c, d, m, and e and measuring q_0 would not suffice to predict q_t for any point in time, as was previously possible. This is at the heart of the presence of chaos in deterministic feedback processes. Lorenz (1963) discovered this effect – the lack of predictability in deterministic systems. Sensitive dependence on initial conditions is one of the central ingredients of what is called deterministic chaos.

This kind of difference equation (11) can lead to very interesting dynamic behaviour, such as cycles that repeat themselves every two or more periods, and even chaos, in which there is no apparent regularity in the behaviour of q_t. This difference equation (11) will possess a chaotic region. Two properties of the chaotic solution are important: firstly, given a starting point q_0 the solution is highly sensitive to variations of the parameters *b, c, d, m*, and *e*; secondly, given the parameters *b, c, d, m*, and *e* the solution is highly sensitive to variations of the initial point q_0. In both cases the two solutions are for the first few periods rather close to each other, but later on they behave in a chaotic manner.

2. Logistic Equation

The logistic map is often cited as an example of how complex, chaotic behaviour can arise from very simple non-linear dynamical equations. The logistic model was originally introduced as a demographic model by Pierre François Verhulst. It is possible to show that iteration process for the logistic equation:

$$z_{t+1} = \pi z_t (1 - z_t), \quad \pi \in [0, 4], \quad z_t \in [0, 1] \tag{17}$$

is equivalent to the iteration of growth model (9) when we use the following identification:

$$z_t = \frac{c(1-d)}{b(d-1)} q_t \tag{18}$$

and:

$$\pi = \frac{b(d-1)}{m(1-\alpha)\left[1+\left(\frac{1}{e}\right)\right]} \; . \tag{19}$$

Using (11) and (13) we obtain:

$$z_{t+1} = \left[\frac{c(1-d)}{b(d-1)}\right] q_{t+1} =$$

$$= \left[\frac{c(1-d)}{b(d-1)}\right] \left\{ \frac{b(d-1)}{m(1-\alpha)\left[1+\left(\frac{1}{e}\right)\right]} q_t - \frac{c(1-d)}{m(1-\alpha)\left[1+\left(\frac{1}{e}\right)\right]} q_t^{\,2} \right\} =$$

$$= \frac{c(1-d)}{m(1-\alpha)\left[1+\left(\frac{1}{e}\right)\right]} q_t - \frac{c^2 (1-d)^2}{mb(d-1)(1-\alpha)\left[1+\left(\frac{1}{e}\right)\right]} q_t^{\,2} \, .$$

On the other hand, using (12), (13), and (14) we obtain:

$$z_{t+1} = \pi\, z_t\, (1 - z_t) = \left\{ \frac{b\,(d-1)}{m\,(1-\alpha)\left[1+\left(\frac{1}{e}\right)\right]} \right\} \left[\frac{c\,(1-d)}{b\,(d-1)}\right] q_t \left\{ 1 - \left[\frac{c\,(1-d)}{b\,(d-1)}\right] \right\} =$$

$$= \frac{c\,(1-d)}{m\,(1-\alpha)\left[1+\left(\frac{1}{e}\right)\right]} q_t - \frac{c^2\,(1-d)^2}{mb\,(d-1)\,(1-\alpha)\left[1+\left(\frac{1}{e}\right)\right]} {q_t}^2 .$$

Thus we have that $q_{t+1} = \frac{b\,(d-1)}{m\,(1-\alpha)\left[1+\left(\frac{1}{e}\right)\right]} q_t - \frac{c\,(1-d)}{m\,(1-\alpha)\left[1+\left(\frac{1}{e}\right)\right]} {q_t}^2$ iterating is really the same as iterating $z_{t+1} = \pi\, z_t(1 - z_t)$ using (13) and (14).

It is important because the dynamic properties of the logistic equation (12) have been widely analyzed (Li and Yorke (1975), May (1976)).

It is obtained that:

(i) For parameter values $0 < \pi < 1$ all solutions will converge to $z = 0$;

(ii) For $1 < \pi < 3{,}57$ there exist fixed points the number of which depends on π;

(iii) For $1 < \pi < 2$ all solutions monotonically increase to $z = (\pi - 1)/\pi$;

(iv) For $2 < \pi < 3$ fluctuations will converge to $z = (\pi - 1) / \pi$;

(v) For $3 < \pi < 4$ all solutions will continuously fluctuate;

(vi) For $3{,}57 < \pi < 4$ the solution become "chaotic" which means that there exist totally aperiodic solution or periodic solutions with a very large, complicated period. This means that the path of z_t fluctuates in an apparently random fashion over time, not settling down into any regular pattern whatsoever.

3. Conclusions

This paper suggests conclusion for the use of the simple chaotic model of a profit – maximizing monopoly in predicting the fluctuations of the monopoly output. The model (11) has to rely on specified parameters *b*, *c, m, d,* and *e*, and initial value of the monopoly output, q_0. But even slight deviations from the values of parameters *b, c, m, d,* and *e* and initial value of the monopoly output, show the difficulty of predicting a long-term behaviour of the monopoly output, q_0.

A key hypothesis of this work is based on the idea that the coefficient:

$$\pi = \frac{b\,(d-1)}{m\,(1-\alpha)\left[1+\left(\frac{1}{e}\right)\right]}$$

plays a crucial role in explaining local stability of the monopoly output where, *b* represents the coefficient of the marginal cost function of the monopoly firm, *m* – the coefficient of the inverse demand function, *e* – the coefficient of the price elasticity of monopoly's demand, α – the coefficient of the price growth, *d* – the coefficient which explain effect of the investment in research and development activities on the monopolist's marginal cost curve.

The quadratic form of the marginal cost function of the monopoly firm is important ingredient of the presented chaotic monopoly output growth model (11).

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