I- st Section:

ECONOPHYSICS

CONSISTENT AND INCONSISTENT WAYS TO DYNAMIZE THE NEO-CLASSICAL THEORY OF ECONOMICS

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Abstract. *With equal profit functions, the static neo-classical theory of a firm and its dynamization by dynamic optimization give equal results. Thus without additional assumptions, dynamic optimization does not dynamize the theory. In the existing models, the solution paths obtained by dynamic optimization converge to the static optimum only under very restricting initial conditions. On the other hand, the static neoclassical theory of a consumer and its dynamization by dynamic optimization explain different quantities: the real consumption of various goods and the money allocated for consumption over time. Thus the dynamic neo-classical micro theory obtained by dynamic optimization is either inconsistent with the static one, or models a different quantity. The neo-classical framework is then lacking consistent static and dynamic theories of a firm and a consumer, and here we introduce such. We define the "economic forces" acting upon the production of a firm and the consumption of a consumer, and show that these dynamic theories analogous to Newtonian mechanics are consistent with the static neo-classical ones; the latter correspond to zero-force situations in the former. The proposed theories are expressed with measurable quantities and they explain economic growth too.*

Keywords: *Economic dynamics, Neo-classical theory, Economic force, Newtonian economics.*

JEL: D11, D21, D91.

1. Introduction

According to Mirowski (1989a), the progenitors of neo-classical economics imitated classical mechanics. For example, the concept of equilibrium was introduced in economics from physics by Canard at 1801 (Mirowski 1989b). Even though equilibrium is a "balance of forces" situation, in economics the balancing forces have not been defined. In spite of this, the term "force" is common in economics, see Lucas (1988). The

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"invisible hand" by Adam Smith is an example of how the concept of "force field" has been applied in economics.

Fisher (1983 pp. 9-12) writes: "... I now briefly consider the feature that a proper theory of disequilibrium adjustment should have ... under what conditions the rational behaviour of individual agents drives an economy to equilibrium. ... Such a theory must involve dynamics with adjustment to disequilibrium over time modelled. ...the most satisfactory situation would be one in which the equations of motion of the system permitted an explicit solution ... as known functions of time. ...the theory of the household and the firm must be reformulated ... to allow agents to perceive that the economy is not in equilibrium and to act on that perception. ... a satisfactory underpinning for equilibrium analysis must be a theory in which the adjustments to disequilibrium made by agents are made optimally."

We follow Fisher in proposing a new framework for modelling in economics. We believe that the *"willingness of economic agents to better their situation"* is the fundamental cause of economic dynamics, and we demonstrate the applicability of this framework in modelling the behaviour of a firm and a consumer. Our approach offers several advantages as compared with the existing way to dynamize the static neo-classical theory by dynamic optimization: 1) The dynamic theories obtained by dynamic optimization are inconsistent with the corresponding static ones. This means that both of them cannot be accepted simultaneously. On the other hand, static neo-classical theory is a special case in our modelling – the zero-force situation – and our framework covers both static and dynamic situations. 2) Our framework does not require dynamic optimization, and 3) it covers also cases where an optimum does not exist. Thus, for example, permanent growth can be modelled in our framework.

2. The Static and the Dynamic Neo-classical Theory of a Firm

We study the dynamization of the neo-classical theory of a firm by dynamic optimization according to Evans (1924), because other such models share the same problems, see e.g. Jorgenson (1963). Evans assumes the cost and demand functions for a monopoly firm as:

$$C(q_s) = Aq_s^2 + Bq_s + C_0,\ q_d = ap + b + hp'(t), \tag{1}$$

where q_s is the amount of production of the firm in a time unit, q_d the demand of the product of the firm at the time unit, p the price of the product, A, B, C_0, b are positive constants while a is negative. The sign of constant h is left open. Assuming $q_s = q_d = q,$ that is the whole production gets sold, the profit function becomes the following:

$$\pi(p, p'(t)) = pq - C(q) =$$

$$= p(ap + b + hp'(t)) - A(ap + b + hp'(t))^2 - B(ap + b + hp'(t)) - C_0. \quad (2)$$

Assuming $h = 0,$ we get the corresponding static optimization problem with the solution:

$$\frac{\partial\pi}{\partial\pi} = 0 \Leftrightarrow p^* = \frac{b - 2aAb - aB}{2a(aA-1)} \Leftrightarrow q^* = ap^* + b = \frac{b + aB}{2(1-aA)}. \quad (3)$$

The corresponding dynamic optimization problem is following:

$$\max_{p(t), p'(t)} \int_0^{t_1} \pi(p(t), p'(t))dt, \ \ p(0) = p_1, \ \ p(t_1) = p_2. \quad (4)$$

The Euler equation of this problem gives the following differential equation:

$$\frac{\partial \pi}{\partial p} - \frac{\partial}{\partial t}\left(\frac{\partial \pi}{\partial p'(t)}\right) = 0 \Leftrightarrow 2Ah^2 p''(t) + 2a(1-aA)p(t) = aB + 2aAb - b, \quad (5)$$

that has solution:

$$p(t) = p_0 + C_1 e^{kt} + C_2 e^{-kt}, \ \ p_0 = p^* = \frac{b - 2aAb - aB}{2a(aA-1)}, \ \ k = \frac{\sqrt{a(aA-1)}}{h\sqrt{A}},$$

$$C_1 = \frac{(p_2 - p_0) - (p_1 - p_0)e^{-kt_1}}{e^{kt_1} - e^{-kt_1}}, \ \ C_2 = \frac{(p_1 - p_0)e^{kt_1} - (p_2 - p_0)}{e^{kt_1} - e^{-kt_1}}. \quad (6)$$

Now, in thinking whether the static and the dynamic optimization problem are consistent with each other, we see firstly that if $h = 0$ in (5), the Euler equation reduces to that in (3). Thus if we apply the same profit function in the dynamic problem as in the static one, we get the same result: $p(t) = p^*, 0 < t < t_1$. Dynamic optimization then does not give a differential equation to be solved unless a different profit function is assumed as in the static case, namely, $h \neq 0.$

On the other hand, the solution $p^* = p_0$ of the static problem in (3) is obtained as a special case of Eq. (6) by assuming $C_1 = C_2 = 0,$ i.e. $p_1 = p_2 = p_0$. Another correspondence between the two solutions is obtained by assuming $C_1 = 0$ in (6) if $h > 0$ and $C_2 = 0$ if $h < 0.$ In this case $\lim_{t\to\infty} p(t) = p_0$, i.e. the solution path of the dynamic problem converges to the static optimum, even though during time $(0, t_1)$ this convergence does not occur.

Thus the solution of the static problem is obtained as a special case from the dynamic problem only under restricting initial conditions that either degenerate the solution path to a fixed point $p(t) = p_0$, or restrict the solution path to be stable. In cases $C_1, k \neq 0, \lim_{t\to\infty} p(t) = \pm\infty$ depending on the sign of C_1 if $h > 0$, and on the sign of C_2 if $h < 0$. Thus in most cases the solution path of the dynamic problem is inconsistent with that of the static one.

2.1. Modelling the Present Value of Profit

The dynamic optimization problem of the current value of profit of a firm from time unit $(0, t_1)$ is:

$$\max_{\mathrm{p(t)}} \int_0^{t_1} F(p(t), t)dt = \max_{\mathrm{p(t)}} \int_0^{t_1} e^{-rt} \pi(p(t))dt, \tag{7}$$

where e^{-rt} the discount factor with constant interest rate r. The necessary condition for (7) – together with possible boundary conditions – is the following Euler equation:

$$\frac{\partial F}{\partial p(t)} - \frac{d}{dt}\left(\frac{\partial F}{\partial p'(t)}\right) = 0 \Leftrightarrow e^{-rt}\frac{\partial \pi}{\partial p(t)} = 0 \Leftrightarrow \frac{\partial \pi}{\partial p(t)} = 0.$$

The necessary condition of problem (7) equals with that in the static theory. Thus assuming present value of profit as the target function in the dynamic problem does not affect the result.

3. The Static and the Dynamic Neo-classical Theories of a Consumer

3.1. The Static Neo-classical Theory of a Consumer

We assume a consumer's decision-making as simple as possible like in all textbooks of elementary economics. The length of the time horizon is assumed to be one week, and the consumer is assumed to choose his/her weekly consumption of only two goods the consumer consumes every week. For clarity, let good 1 be "food" and good 2 "playing video games" according to the traditional choice between food and fun. The consumer is assumed to have budgeted a fixed amount of money $I(\$/week)$ for his/her weekly consumption, and the unit prices of food and playing video games are $p_1(\$/kg)$ and $p_2(\$/h)$, respectively[1]. The weekly budget of the consumer is $I = p_1q_1 + p_2q_2$ where $q_1(kg/week)$ and $q_2(h/week)$ are the consumption flows of the two goods.

The consumer has a continuous real valued weekly utility function $u = f(q_1, q_2)$. To be able to write well-defined expressions with this function, measurement unit ut is given for utility (or satisfaction). However, because the consumer spends all the money he/she has budgeted for the week during the week, the satisfaction he/she gains from consumption takes place at the week. Thus function u with unit $ut/week$ measures the consumer's weekly flow of satisfaction.

The consumer's marginal utilities of the two goods,

$$\frac{\partial f(q_1,q_2)}{\partial q_1} > 0, \ \frac{\partial f(q_1,q_2)}{\partial q_2} > 0,$$

have units $(ut/week)/(kg/week) = ut/kg$ and $(ut/week)/(h/week) = ut/h$, respectively. Substituting the budget in the utility function gives the following unrestricted problem:

$$\max_{q_1 q_2} u, u = f(q_1,q_2) = f(q_{1,}(I - p_1q_1)/p_2) \equiv F(q_1, p_1, p_2, I). \tag{8}$$

The optimal weekly consumption of food q_1^* can be solved from the following equation:

$$\frac{du}{dq_1} = 0 \Leftrightarrow \frac{\partial f}{\partial q_1} - \frac{p_1}{p_2}\frac{\partial f}{\partial q_2} = 0 \Leftrightarrow \frac{1}{p_1}\frac{\partial f}{\partial q_1} = \frac{1}{p_2}\frac{\partial f}{\partial q_2} \Rightarrow q_1^* = g(p_1, p_2, I),$$

that can also be presented according to Eq. (8) as:

$$\frac{\partial F}{\partial q_1} = \frac{\partial f}{\partial q_1} - \frac{p_1}{p_2}\frac{\partial f}{\partial q_2} = 0. \tag{9}$$

The sufficient condition for maximum is:

$$\frac{d^2u}{dq_1^2} = \frac{\partial^2 f}{\partial q_1^2} - \frac{p_1}{p_2}\frac{\partial^2 f}{\partial q_2 \partial q_1} - \frac{p_1}{p_2}\frac{\partial^2 f}{\partial q_1 \partial q_1} + \frac{\partial^2 f}{\partial q_2^2}\left(\frac{p_1}{p_2}\right)^2 < 0. \tag{10}$$

[1] Measurement units are in parenthesis after the quantities, see De Jong (1967).

Non-increasing marginal utility makes $\partial^2 f / \partial q_1^2$ and $\partial^2 f / \partial q_2^2$ non-positive, and if the partial functions are continuous, then $\partial^2 f / \partial q_1 \partial q_2 = \partial^2 f / \partial q_2 \partial q_1$ (Apostol 1979 p. 360). Assuming the partial functions continuous, the sufficient condition for maximum is: $\partial^2 f / \partial q_1 \partial q_2 > 0$. Thus, the greater q_1 is, the higher the marginal utility of good 2 and vice versa.

3.2. Dynamic Consumer Behaviour by Dynamic Optimization

Here we dynamize the theory given in section 3.1 by dynamic optimization. The consumption flows of the two goods are assumed to depend on time t with unit *week*, while the other quantities are assumed fixed. Substituting the budget equation in the utility function, gives:

$$u(t) = f\left(q_1(t), \frac{I - p_1 q_1(t)}{p_2} \right) \equiv F(q_1(t), p_1, p_2, I). \tag{11}$$

Assuming that the consumer lives an infinite time with ρ as his/her time preference, the unrestricted dynamic optimization problem for the consumer becomes the following:

$$\max_{q_1(t)} \int_0^\infty e^{-\rho t} u(t) dt = \max_{q_1(t)} \int_0^\infty e^{-\rho t} F(q_1(t), I, p_1, p_2) dt$$

$$= \max_{q_1(t)} \int_0^\infty G(q_1(t), I, p_1, p_2, \rho, t) dt.$$

The Euler equation of this problem is:

$$\frac{\partial G}{\partial q_1} - \frac{d}{dt}\left(\frac{\partial G}{\partial q_1^{'}(t)} \right) = 0 \Leftrightarrow e^{-\rho t} \frac{\partial F}{\partial q_1} = 0 \Leftrightarrow \frac{\partial F}{\partial q_1} = 0 \text{ since } e^{-\rho t} > 0.$$

The necessary condition for this dynamic problem then equals with that in the static case in (9). Thus for dynamic optimization to give an equation of motion for the consumption of a consumer, either the target function or the budget equation must be changed from that of static analysis. However, in that case the two frameworks would not be consistent with each other as we observed in the case of a firm.

3.3. The Ramsey-Cass-Koopmans Model of Consumption

The inability of dynamic optimization to yield an equation of motion for the consumption of a consumer described in Section 3.2 has led to the situation that a dynamic model for the real consumption of a consumer does not exist. On the other hand, the allocation of money for consumption over time has been modelled by Ramsey (1928), Cass (1965), and Koopmans (1965). The RCK-model is presented here in the form of Romer (1996, pp. 39-44). In the model there are H identical households with the size of each household growing at rate n. The household's lifetime utility function of infinite horizon with discount rate ρ is the following:

$$U = \int_0^\infty e^{-\rho t} u(C(t)) \frac{L(t)}{H} dt, \tag{12}$$

where $C(t)$ is the consumption of each member of the household at time t. $u(C(t))$ is the utility function that gives each member's utility at a given date, $L(t)$ the total population in the economy, and $L(t)/H$ average number of members in a household. Thus $u(C(t))L(t)/H$ is the household's utility at time t. The budget constraint of this optimization problem is:

$$\int_{t=0}^\infty e^{-R(t)} C(t) \frac{L(t)}{H} dt \le \frac{K(0)}{H} + \int_{t=0}^\infty e^{-R(t)} A(t)\, w(t) \frac{L(t)}{H} dt,$$

where $R(t) = \int_{\tau=0}^t r(\tau) d\tau, r$ is the interest rate, $K(0)/H$ the initial capital holdings of the household, $A(t)$ the efficiency factor of a worker in work, and $A(t)w(t)$ a worker's labour income. The budget constraint takes care that the present value of lifetime consumption does not increase the present value of lifetime income of the household. To get clear results, the following utility function is assumed for the household:

$$u(C(t)) = \frac{C(t)^{1-\theta}}{1-\theta}, \ \theta > 0, \ \rho - n - (1-\theta)g > 0, \tag{13}$$

where θ is the coefficient of risk aversion, and $g = A'(t)/A(t)$ the growth rate of efficiency of labour. The latter condition guarantees that the lifetime utility does not diverge. This constrained dynamic optimization problem yields the following optimum condition (Romer 1996 p. 44):

$$\frac{C'(t)}{C(t)} = \frac{r(t) - \rho}{\theta}.$$

The optimal growth rate of consumption of a worker $C'(t)/C(t)$ is thus increasing, if interest rate exceeds the time preference of the household and vice versa.

Now, the RCK-model explains the time path of money used in consumption of a household, but it does not dynamize the static neo-classical theory of real consumption of a consumer given in section 3.1. The two theories explain different quantities and we have found no article where the link between these theories has been studied. The difference in the theories is seen in the arguments of the utility functions. The static theory assumes that utility originates from the real consumption of various goods, and the dynamic theory assumes that utility originates from the total money used in consumption without noticing the structure of consumption.

4. A Dynamic Theory of Production Consistent with the Static Neo-classical One

4.1. Kinematics of Production and Consumption

Let $q(t)(unit/time)$ be the flow of production or consumption of a good at time moment t. The accumulated production (consumption) of the good till time moment $t, Q(t)(unit)$, is then:

$$Q(t) = Q(t_0) + \int_{t_0}^{t} q(s)ds, \;\; Q'(t) = q(t), \;\; Q''(t) = q'(t),$$

where $Q(t_0)(unit)$ is the accumulated amount of production (consumption) of the good till moment t_0, $Q'(t) = q(t)(unit/time)$ the momentous flow, and $Q''(t) = q'(t)(unit/time^2)$ the momentous acceleration of accumulated production (consumption) at time moment t. This kinematics serves as a prelude for Newtonian theories of production and consumption.

4.2. Newtonian Theory of Production

The decision-making for the dynamics of production of a firm can be studied by assuming the decision-makers to plan whether to increase the accumulated production of the firm by a certain amount or not, or by assuming that they are planning to change the flow of production of the firm from the last week, month, or year by a certain quantity. We study here only the latter case by the profit function applied in Evans (1924) with $h = 0$. Assuming the units for the quantities as $q(unit/week), p(\$/unit)$, the Newtonian theory of a firm in Estola (2001) gives the following equation of motion for production of the profit-seeking firm:

$$mq'(t) = \frac{\partial \pi}{\partial q} \Leftrightarrow mq'(t) = \left(\frac{2}{a} - 2A\right) q(t) - \frac{b}{a} - B. \tag{14}$$

The units of the constants are:

$A: \$ \times week/unit^2, B: \$/unit, a: unit^2/(\$ \times week), b: unit/week$, and positive constant m with unit $\$ \times week^2/unit^2$ is the inertial factor of production. It takes care that changing the flow of production does not occur instantaneously but takes some time. These units make the equation dimensionally correct, see De Jong (1967). Marginal profitability $\partial\pi/\partial q$ with unit $\$/unit$ is the "force" acting upon the production of the firm. According to Eq. (14), there is positive acceleration in production if $\partial\pi/\partial q > 0$ and vice versa. The solution of the differential equation in (14) is:

$$q(t) = \frac{b + aB}{2(1 - aA)} + C_1 e^{\frac{2(1-aA)}{am}t}, \ q(\infty) = \frac{b + aB}{2(1 - aA)}, \ \frac{2(1 - aA)}{am} < 0,$$

$$p(\infty) = \frac{b - 2aAb - aB}{2a(aA - 1)}, \ C_1 \text{ constant.}$$

This dynamic model gives the static neo-classical theory as asymptotic case with $t \to \infty$. The static neo-classical equilibrium is obtained in (14) also as the "zero-force" situation. Notice that instead of production dynamics we could have modelled price dynamics and get the same solution because the demand function defines a one-to-one relation between p and q_1. Non-stable solution paths, that correspond to permanent growth, are obtained from (14) by assuming time dependencies in the demand or the cost function, or assuming increasing returns to scale, see Estola (2001).

5. A Dynamic Theory of Real Consumption Consistent with the Static One

Here we model dynamic consumer behaviour so that the static neo-classical framework corresponds to a "zero-force" situation in this. We continue analyzing the two-good situation in section 3.1 and assume a Cobb-Douglas – form for utility, $u = A(bq_1)^{\alpha}(cq_2)^{1-\alpha}$, where the units of the positive constants are: $A: ut/week, b: week/kg, c: week/h,$ and α is a pure number. Utility is thus measured in units $ut/week$ and the two power functions are dimensionless. Substituting the budget in the utility function, we get:

$$u(t) = A(bq_1(t))^{\alpha}\left(\frac{c(I - p_1q_1(t))}{p_2}\right)^{1-\alpha}. \quad (15)$$

Omitting the time dependency in q_1 we get the corresponding static neo-classical solution for optimal q_1 from Eq. (15) as follows:

$$\frac{\partial u}{\partial q_1} = \frac{Ac(bq_1)^{\alpha}\left(\frac{c(I - p_1q_1)}{p_2}\right)^{-\alpha}(\alpha I - p_1q_1)}{p_2q_1} = 0 \Leftrightarrow q_1^* = \frac{\alpha I}{p_1}.$$

According to Estola & Hokkanen (2008), we can write the following Newtonian equation of motion for q_1 for a utility-seeking consumer with time:

$$m_1q_1^{'}(t) = \frac{\partial u}{\partial q_1}$$

$$\Leftrightarrow m_1q_1^{'}(t) = \frac{Ac(bq_1(t))^{\alpha}\left(\frac{c(I - p_1q_1(t))}{p_2}\right)^{-\alpha}(\alpha I - p_1q_1)(t))}{p_2q_1(t)} \quad (16)$$

where positive constant m_1 with unit $ut \times week^2/kg^2$ has the same role as inertial mass has in physics; it takes care that the adjustment in consumption does not occur immediately but takes some time. Marginal utility $\partial u/\partial q_1$ is the "force" acting upon the consumption of good 1. Notice that the budget equation is included in the utility function before calculating the marginal utility. Because of the non-linearity of Eq. (16), we solve it numerically by assuming the values for the constants as: $\alpha = 0.5, I = 100, p_1 = 5, p_2 = 10, m_1 = 2, A = b = c = 1.$ With $t \to \infty,$ the solution path of Eq. (16) in Figure 1 converges to the static neo-classical optimum: $q_1^* = \alpha I / p_1 = 10.$

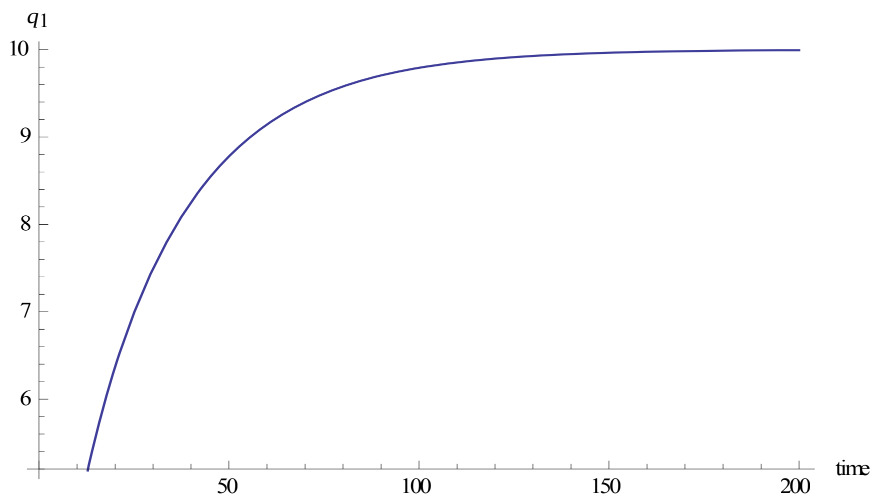

Figure 1. The solution path for consumption of good 1.

In order to test the theory in (16), the marginal utility of the consumer should be measurable. However, the consumer's marginal utility of food consumption with unit ut/kg is not measurable because we have no way to measure a consumer's satisfaction. Thus, for testing the theory we need to transform the force in a measurable form. We can factor $\partial u/\partial q_1$ as:

$$\frac{\partial u}{\partial q_1} = \frac{(1-\alpha)Ac(bq_1)^{\alpha}\left(\frac{c(I-p_1q_1)}{p_2}\right)^{-\alpha}}{p_2}\left[\left(\frac{\alpha}{(1-\alpha)}\right)\left(\frac{I-p_1q_1}{q_1}\right)-p_1\right], \quad (17)$$

where positive term Z:

$$Z \equiv \frac{(1-\alpha)Ac(bq_1)^{\alpha}\left(\frac{c(I-p_1q_1)}{p_2}\right)^{-\alpha}}{p_2},$$

does not affect the sign of the force. The active part of the force – the term in brackets in (17) – can be understood so that the positive term:

$$\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{I-p_1q_1}{q_1}\right)=\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{I}{q_1}-p_1\right)$$

is the "marginal willingness-to-pay" of the consumer for food, and p_1 is its price, see Estola & Hokkanen (2008). Thus the term in brackets in (17) can be understood so that the force is positive if the marginal willingness-to-pay of the consumer exceeds the price of food, and vice versa. This term has the same zero point as $\partial u / \partial q_1$ has, namely $q_1^* = \alpha I / p_1$. Thus the acceleration of food consumption of this consumer is positive if the consumer's marginal willingness-to-pay exceeds the price of food, and vice versa.

To simplify the equation, we leave the positive factor Z out from the force and denote this new measurable force with unit $\$/kg$ as F_1,

$$F_1 = \frac{\alpha}{1-\alpha}\left(\frac{I}{q_1} - p_1\right) - p_1.$$

The corresponding equation of motion for food consumption is:

$$\hat{m}_1 q_1'(t) = \frac{\alpha}{(1-\alpha)}\left(\frac{I}{q_1(t)} - p_1\right) - p_1, \qquad (18)$$

where positive inertial constant $\hat{m}_1$ ("mass") with unit $\$ \times week^2 / kg^2$ makes the equation well defined, see De Jong (1967). Due to the non-linearity of the equation in (18), we solve it numerically. Assuming $\alpha = 0.5,\ p_1 = 5,\ I = 100$ as before, the solution path is shown in Figure 2. The consumer's food consumption converges to the static neo-classical optimum: $q_1^* = \alpha I / p_1 = 10$. Thus analogous results are obtained by the two forces. The advantage of the former is that it is derived from the assumption that the consumer's aim is to increase his/her utility with time, and that of the latter is that it is measurable in units $\$/kg$.

Our framework implies that the explicit measuring of utility is not needed in modelling consumer behaviour. A consumer's decision-making can always be expressed in the form "*marginal willingness-to-pay minus price*". Utility is only an auxiliary quantity required in defining the marginal willingness-to-pay of a consumer for various things. All utility functions with the same preference order give equal marginal willingness-to-pay values for goods near the consumer's optimum, see Estola & Hokkanen (2008).

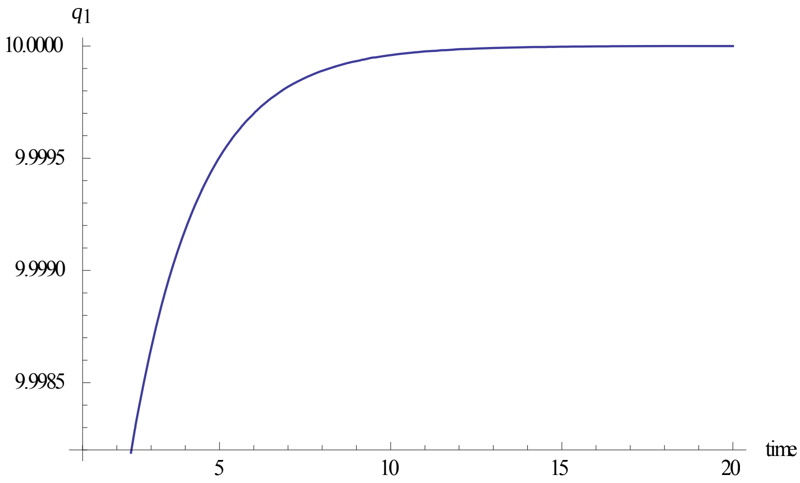

Figure 2. The path of food consumption of the consumer.

The dynamic consumer behaviour introduced in this section has still one advantage as compared with the static neo-classical framework. Because time is omitted in the static neo-classical analysis, in that framework we cannot model how changes in consumers' wealth or in prices with time affect their consumption. In our framework this can be done. Suppose a consumer gains wealth so that he/she can steadily increase funds for his/her consumption. The budgeted funds for his/her weekly consumption become then a function of time, and in the real world the prices of goods are functions of time. Assuming the Newtonian equation for food consumption as in (18), we can set there functions $I(t)$, $p_1(t)$ and solve the equation in this case. Thus our framework can be extended for modelling changes in consumption due to changes in income and prices that cannot be done in the static neo-classical one. Time dependent consumer preferences can be included in the model analogously.

6. Conclusions

We showed that the static neo-classical theory of a firm and a consumer are either inconsistent with the corresponding theories obtained by dynamic optimization, or they model a different quantity. As a solution to this, we extended the static neo-classical theory of a firm and a consumer to a dynamic form consistent with the static analysis. In this we defined the "economic forces" acting upon the production of a profit-seeking firm and a utility-seeking consumer. An isomorphism between economic dynamics and classical mechanics was thus proposed that gives equilibrium and non-equilibrium analysis by using a single framework.

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