INTEGRATING FLEXIBLE BEHAVIORS INTO THE ECONOMIC TRANSMISSION MECHANISMS[1]

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Abstract. *The paper suggests a method for the analysis of the economic transmission mechanisms based on directional graphs.*
The relations between the economic variables are defined based on a vector of conditional probabilities which reflect a mix of specific economic behaviours. At the same time, the conditional probabilities are defined for complex events which reflect economic behaviours characterized by: a) different triggering factors; b) different frequencies and patterns.
This approach facilitate the integration of a variety of economic behaviours in one economic transmission mechanism make it a more flexible and realistic instrument for the economic analysis.

Keywords: *economic transmission mechanisms, conditional probabilities, flexible behaviours, functionalities, complex, systems*.

Jel classification: C15, C63.

1. Introduction

The economy is a dynamic system with specific trajectories for the variables of interest. This rises a challenge to define the probability for elementary events associated with a given states of one or more variables. Taking the classical example of coin tossing as a metaphor for the economic system, each tossing of a coin, in the case of the economic system each economic event doesn't have the same initial conditions. This is equivalent with saying that we cannot define a set of elementary events having the same set of conditions. This suggests that the sequence of relative frequencies may not converge, thus the limit of the relative frequency may not exist.

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Having this is mind the paper has three objectives.

The first objective is to analyze the probability of complex events associated with the phase space of the economic dynamic system.

The second one is the analysis of the stability of the relative frequencies of elementary events.

The third one is the characterization of the economic transmission mechanisms using conditional probability.

The paper is structures in six chapters.

The second chapter focuses on the literature review of the main applications of the dynamic systems in economics as well as on describing the place of the paper in relation to these contributions.

In the third chapter the paper presents the impact of structural factors on the economic variables of interest. The focus is on inflation due to the characteristics of the analyzed structural factors, namely the downward rigidity of wages which is a nominal variable strongly connected with inflation.

The fourth chapter presents the model focusing on the blocks of the model and on the specific system of differential equations.

The fifth chapter presents the results of the model's simulation focusing on the three objectives enumerated above.

The sixth chapter is a synthesis of the main findings of the paper.

2. Systems of differential equations – applications in economics

The analysis of the economic processes using systems of differential equations underlines the complex behaviour of the economic system: bifurcations, critical points, possible chaotic behaviours. The main economic problems analyzed using the dynamic framework are diverse indicating the versatility of the approach: economic cycles (see Goodwin 1967, Desai 1973, Wolfstetter 1982, Sportelli 1995, Purica and Caraiani 2009), the dynamic theory of oligopoly (see Yoshida 2011), demographic problems and its application in economics (see Lucas 2002, Galor and Moav 2002, Hansen and Prescott 2002, Jones 2001), economic growth (see Kaldor 1956, Pasinetti 1962, Samuelson and Modigliani 1966 as well as unified economic growth theory developed by Galor 2011), the problem of stability of the economic equilibrium (see Woodford 1990, Bewley 1998, Bohm and Kaas 2000).

From the perspective of the variables used in the model the differences in comparison with the cited approaches are not significant.

These approaches are based on the fundamental relations in economics between demand, offer, inflation, wages etc. The current paper introduces no innovative new relations. The main differences are related with the hypothesis of a unique market. The large majority of the models follow the tradition set by the dynamic stochastic general equilibrium models (DSGE) working with a single market, thus not taking into account the relation between economic sectors and the impact of these relations. Due to the nature of the problem studied in this paper, namely the impact of structural factors, the model builds a market with two sectors with all the implications that derive from this approach. The main implications are the capacity to capture the relation between sectors and the effect of the modifications of one of the sectors on the others.

From the perspective of the behaviours of the solutions there are some differences in „the logic" of the approaches which should be underlined. We are going to focus on those approaches which are relevant to the current analysis.

The approach which is steadily gaining ground is based on the use of non-linear systems of differential equations in describing and analyzing economic processes (see Rand 1978, Day 1982, Puu 2000, Nishimura and Yano 1995, Vosvrda 2001). Admittedly there is a richness of behaviours of these systems but also artificiality in defining the non-linear behaviours of the economic variables. One approach is to build trigonometric functions to reflect the relations between variables (see Hommes 1997) or to use equations known for their non-linear behaviours as in the case of the Faigenbaum equation (see Jensen and Urban 1984) or Van der Pol equation (see Bouli 1999). The use of these equations is made without clearly presenting their capacity to reflect the specific economic mechanisms. In this context it is not clear if the economic variables exhibit a chaotic behaviour or if there is a selection of those equations which allow the chaotic behaviour to manifest itself. In these cases the analysis of the chaotic behaviours is made in the logic of Thom 1989 and Zeeman 1977 (interestingly these papers are not analyzing economic processes), namely analyzing the behaviour of the systems for different intervals of the coefficients. In this context, the perturbations on the system are described by the modifications in these coefficients. These modifications affect the stability and the behaviour of the system's solutions only if they generate a change of the above mentioned interval of variation (see Goodwin 1990 for a presentation of the problem). The current paper investigate the processes which are behind these approaches, namely it focuses on understanding the connection between the trajectories of the system's solutions and the characteristics of the economic transmission mechanisms specific to the analyzed system. The problem is presented in chapter 5.

3. The impact of structural factors on the economic indicators

The different ways to quantify and theoretically define structural changes led to different approaches to analyze the structural component of the economic variable of interest. Dobrescu (2009) analyzes structural inflation from the perspective of the relation between the modifications of the weights of different sectors in the total production and the changes in the prices calculated as a ratio between the sectorial price index and the total aggregated price index. Balke and Wynne (1996) showed that the sectorial technological changes are reflected in the transversal distribution of price changes. Sheedy (2005) analyzed the impact of a shock (changes in the oil price) on the firm's costs which led to price adjustments with different lags. In this approach the shocks that affect the economy are structural because they reflect the structure of the firms in the economy and the differences in their behaviours.

The paper follows the logic of Sheedy (2005) in the sense that the changes in the key variables due to structural shocks are a result of the structure of the economy and its characteristics. The idea that we want to underline in this paper is that the economic system through its structure (the relation between the components of the system, the feed-back structure, behaviour characteristics) facilitates the understanding of the relations between the trajectories of the prices and the characteristic of the economic transmission mechanisms and their relations.

4. The model's presentation

The chapter will focus on presenting the system of differential equations and the algorithm for running the model. The mathematical relations will be presented for a model with n components (consumers, firms). This version of the model uses an accommodative monetary policy. In the context of a constant money velocity, the monetary mass varies to equate the product of the quantity of goods in the economy and the prices of these goods.

4.1. The system of differential equations

The system of differential equations models the variation of prices, wages, demand and offer in the context of the interaction between producers and consumers on two markets. The symbols used stand for p^{ri} – the price of the economic good, for the sector *i*, c^{ri} – demand for the good produced by the sector *i*, o^{ri} – offer for the good produced by the sector *i*, w^{ri} – nominal wage for the sector *i*, e^p – expected inflation, i^s – the wage index, a^s – wage adjustment index, r^s – downward rigidity of wages index, a_{prod} – production adjustment index, $profit^{ri}$ – the profit of the firm in sector *i* calculated as the difference between income and costs. The relations that model the evolution of prices, wages, demand and offer are described below:

$$p_t^{r_i} = p_{t-1}^{r_i} * \frac{C_{t-1}^{r_i}}{O_{t-1}^{r_i}} * e^p; \quad (1)$$

$$w_t^{r_i} = w_{t-1}^{r_i} * \frac{p_t^{r_i}}{p_{t-1}^{r_i}} = w_{t-1}^{r_i} * \frac{C_{t-2}^{r_i}}{O_{t-2}^{r_i}} * e^p * i^s; \quad (2)$$

$$i^s = \begin{cases} a^s & \frac{C_{t-1}^{r_i}}{O_{t-1}^{r_i}} > 1 \\ r^s & \frac{C_{t-1}^{r_i}}{O_{t-1}^{r_i}} < 1 \end{cases}; \quad (3)$$

$$O_t^{r_i} = O^{r_i}{}_{t-1} * a_{prod} \quad ; \quad (5)$$

$$a_{prod} = \begin{cases} a_{prod}^1 & \text{Profit}^{r_i} > 0 \\ a_{prod}^2 & \text{Profit}^{r_i} < 0 \end{cases}; \quad (6)$$

$$C_t^{r_i} = \frac{w_t^{r_i}}{p_t^{r_i}} = \frac{w_1^{r_i}}{p_1^{r_i}} * \frac{C_0^{r_i}}{O_0^{r_i}} * \frac{O_{t-1}^{r_i}}{C_{t-1}^{r_i}} . \quad (7)$$

The system of differential equations reflects the following economic behaviour mechanisms.

The variation in prices is proportional with the ratio between demand and offer and depends on the inflation expectations. The prices are increasing if the demand is higher than the offer, remain unchanged in the case of equality and decrease otherwise (see relation 1). The variation in wages depends on the variation of prices. An increase in prices generates pressures in the direction of an increase in wages. The wage adjustment is captured by the wage index which can be an index higher than unity (the wages increase more than the prices) equal to unity (the wages increase is equal to the price increase) and lower then unity (the wage increase is lower than the price increase). By algebraic manipulation of the relation (9), the wages can be written as a function of demand and offer with lag two (t–2), the inflation expectations and the wage index. The output variation depends on the differences between output and demand and on the production adjustment capacity (which can be seen as the elasticity of output to demand changes). This capacity is endogenously defined and it depends on the specificities of the production process and on the characteristics of the market (see relation 4). There are two possible values of the adjustment indicator, which corresponds to the case of positive profits or negative ones (see relation 5). The demand at the moment t depends on the relation between income and prices. By algebraic manipulation of relation 13, the demand can be written as a function of the demand and offer with one lag ($t-1$), of the demand and offer at time t_0 and of the ratio of income and price and time t_1 (see relation 6).

4.2. The algorithm of the model

The model has the following theoretical framework. The market is described by representative producers from two sectors. The producers use row materials (intermediate consumption) and labour force to produce the output. The relations between the sectors are reflected by the technological coefficients. The consumers income depend on the number of hours worked and the hourly cost of the labour force in the sector of work. The demand for the goods in the economy reflects the consumers' choices taken into account their disposable income and preferences. The algorithm is based on the following steps:

Step 1. At the beginning of the simulation there are defined: 1) a set of prices for the goods produced; 2) a set of prices for the labour force; 3) the technological coefficients matrix; 4) the consumer preferences. The above mentioned variables are randomly defined but they respect a set of constraint that confers economic consistency. In this respect the technological matrix reflects the ratio between the weight of the intermediate consumption and the labour force in the production process. The set of prices is chosen as not to be smaller than the marginal costs.

Step 2. The output at the sectoral level is the solution to the problem of profit maximization. The demand is the solution to the problem of utility maximization.

Step 3. Prices increase if $C > O$ and decrease if $C < O$. The changes are proportional with the difference between demand (C) and offer (O) (see relation 8). The process illustrates the price adjustment mechanism of the output to the demand.

The wages vary proportionally to the changes in prices taken into account the difference between the demand and offer (see relation 2). A supplementary constraint is added, namely the wages cannot decrease below the minimum wage in the economy.

If the profit is negative after the price adjustment then the firms adjust their output to match demand. The adjustment is made with a lag reflecting the production characteristics of the firm, market strategies, the characteristics of the labour market etc.

Step 4. Steps 2 and 3 are repeated for a number of periods (45 in the case of the simulations in the paper).

5. Running the model

The simulation investigates the limits of describing the evolution of a dynamic economic system using probabilities. To make the arguments more transparent some clarifications are in order.

There are different types of events, specific to the economic system, for which we want to know their probability. Three types of events are of a particular interest.

First of all, we want to know what is the probability that at time t_i we have event A = {an increase in demand is accompanied by an increase in offer}. If the system is deterministic, as in the case of the model, the answer is 1. If the dynamic system can have more than one initial conditions then $P(A) = \lim_{n \to \infty} \frac{n(A)}{n}$ and the values is a direct result of the probability of occurrence of the initial conditions and of the trajectories of the system specific to those initial conditions.

Second of all given a deterministic system and the trajectories of the key variables we want know what is the probability that event A will occur at a random chosen time period given that no relevant random events will take place. The probability will be then defined by the relative frequency of the event A given the deterministic evolution of the system.

Third of all we want to know how the probability of the event *A* changes if one of the variables of the system changes it value in an interval ε.

Having this in mind, we propose the following procedure.

Step 1. We run the model described by equations 1 to 6 for 45 periods obtaining the economic variables of interest for the two sectors.

Step 2. We apply the Granger causality test between the series demand and offer, prices and offer, prices and demand for the two sectors.

Step 3. Based on the results of the Granger causality we define the events: $A_1 = \{d(O_t) < 0$ given that $d(C_{t-i}) < 0\}$, $A_2 = \{d(P_t) > 0$ given that $d(O_{t-i}) < 0\}$ and $A_3 = \{d(P_t) < 0$ given that $d(C_{t-i}) < 0\}$. We calculate the probability that event A_i will occur at a random chosen time period given that no relevant random events will take place. As mentioned before, the probability will be defined by the relative frequency of the events A_i given the deterministic evolution of the system.

Step 4. We run the model 100 times. At each run the structural variables downward rigidity of wages r^s is perturbed by a random term, where $r^s \in [1.01\ \ 1.01+\varepsilon]$ and ε is the random term $\varepsilon \in [0\ \ 0.01]$.

Step 5. The output of the model give us 4 key series of length 100, namely r^s and $P(A_i)$ where I = 1 to 3.

Step 6. We defined the functions $f_i : S_{r^s} \to S_{P(A_i)}$ where S_{r^s} is the set of all values obtained for the 100 runs of the model of r^S and $S_{P(A_i)}$ is the set of all values obtained for the 100 runs of the model for $P(A_i)$. The functions are then analyzed to see if they were monotonic.

Step 7. An interval of variation is defined for $P(A_i)$, namely $P(A_i) \in [\min(f(r^s))\ \ \max(f(r^s))]$.

The application of the procedure generated the following results (see Table 1). The relevant statistics can be found in the appendix of the paper.

The probability $P(A_i)$ describes the behaviour and stability of the economic transmission mechanism regarding demand and offer, offer and prices, demand and prices. The most stable mechanism to the fluctuations in the downward rigidity of wages is the one described by event A_1 between demand and offer and the least stable the one described by A_3 between demand and prices.

Table 1

Synthesis of the simulation's results

Relevant indicators	Interval of variation for the indicators – sector 1	Interval of variation for the indicators – sector 2
r^s	(1.010 1.019)	(1.010 1.019)
$P(A_1)$	(0.962 0.969)	(0.942 0.88)
$P(A_2)$	(0.52 0.83)	(0.5 0.77)
$P(A_3)$	(0.32 0.54)	(0.5 0.77)

Source: Own computations

The function f_1 is weakly decreasing for both sectors, thus it preserve the order. The functions f_2 and f_3 are not monotonically. This indicates that we are in a case of demand driven economy. The decrease in prices is not accompanied by a decrease in wages of the same magnitude. This suggests that there are situations in which an increase in demand is not accompanied by an increase in prices and an increase in offer is not accompanied by a decrease in prices. Both mechanisms are possible due to the different lags between them.

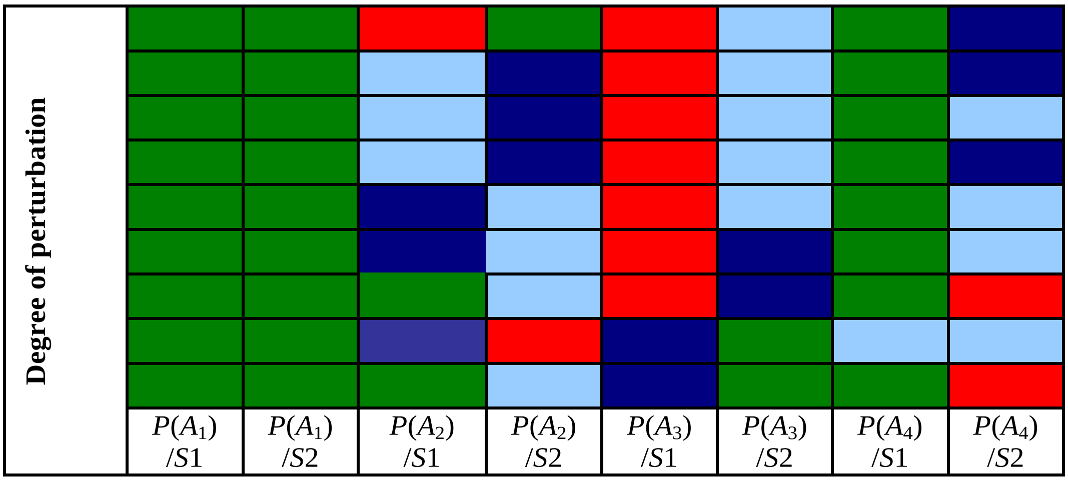
A way of looking at the results in Table 1 is to look at the economic transmission mechanism as functionalities of the economic system and to ask in what conditions (perturbation of r^s) are the functionalities preserved. To make the argument more clearly we are going to analyze a more complex transmission mechanism, namely a decrease in demand generates a decrease in offer which generates an increase in price. This mechanism can be seen as a directional graph with the nodes demand, offer and prices. The event specific to this mechanism is A_4 = {a decrease in demand generates an increase in price given that a decrease in demand generates a decrease in offer}.

The simulation of the model for 100 runs indicates the following results for the mechanism presented above $P(A_4)$ is constant for sector 1, its value being 0.5 and $P(A_4)$ varies in the interval (0.2 0.33) for the second sector.

The robustness of functionalities to perturbation in r^s is analyzed by looking at the correspondence between the increase in the value of r^s in comparison with the starting value of 1.01 and the change in the probability of the events A_i. The approach is similar with the one proposed by Kitano (2007). As a starting point we defined intervals for the probabilities $P(A_i)$. The intervals reflect if and in what degree the functions of the system are kept. The functions correspond to the events A_i. Red colour indicates that the system fails to have/maintain its function, different blue colours show the level of degradation of the function and the green colour indicates the system maintains the functionality, white colour indicates that the indicator of robustness could not be calculated. A system is considered more robust if it better preserves it functionalities.

Table 2

The robustness of the functionalities to perturbations in r^s



Features that are perturbed

Legend: $f_i(r^s) \geq 0.75$, $f_i(r^s) \geq 0.65$, $f_i(r^s) \geq 0.5$, $f_i(r^s) < 0.5$

Source: Own computations

The analysis of Table 2 suggests an interesting question: "Are the complex functionalities more robust then simple ones?". In the case of our analysis the complexity of functionality is easy to be defined, and it corresponds to events which are the combination of other events. In this respect, $A_4 = A_2 \backslash A_1$ where "\" should be read as "given". In the case of the analysis the robustness of A_4 is a direct consequence of the robustness of A_2 and A_1 having the lowest performance of those two. In this respect the increase in complexity leads to a decrease in robustness.

6. Conclusions

The paper theoretically analyzed the probability of complex events associated with the phase space of an economic dynamic system. It proposed a procedure for the analysis of the stability of the relative frequencies of elementary events and the characterization of the economic transmission mechanisms using conditional probability. The procedure allows for the definition of the interval of variation for the probability of different economic transmission mechanisms.

In the context of the analysis of the probability associated with complex transmission mechanisms the paper discussed the robustness of different economic transmission mechanisms to perturbations in r^s and finds that complex functionalities are less stable in comparison with simpler functionalities.

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APPENDIX 1

Statistical tests

Pairwise Granger Causality Tests
Date: 04/30/12 Time: 15:22
Sample: 1 45
Lags: 1

Null Hypothesis:	Obs	F-Statistic	Probability
OFERTA1 does not Granger Cause CERERE1	44	14.0765	0.00054
CERERE1 does not Granger Cause OFERTA1		71.8107	1.5E-10

Pairwise Granger Causality Tests
Date: 04/30/12 Time: 15:22
Sample: 1 45
Lags: 1

Null Hypothesis:	Obs	F-Statistic	Probability
OFERTA2 does not Granger Cause CERERE2	44	11.3302	0.00167
CERERE2 does not Granger Cause OFERTA2		9.63126	0.00346

Pairwise Granger Causality Tests
Date: 04/30/12 Time: 15:18
Sample: 1 45
Lags: 1

Null Hypothesis:	Obs	F-Statistic	Probability
OFERTA1 does not Granger Cause IPC1	43	3.58946	0.06539
IPC1 does not Granger Cause OFERTA1		0.99193	0.32526

Pairwise Granger Causality Tests
Date: 04/30/12 Time: 16:50
Sample: 1 45
Lags: 3

Null Hypothesis:	Obs	F-Statistic	Probability
IPC2 does not Granger Cause OFERTA2	41	4.78036	0.00694
OFERTA2 does not Granger Cause IPC2		3.12014	0.03867

Pairwise Granger Causality Tests
Date: 04/30/12 Time: 15:52
Sample: 1 45
Lags: 2

Null Hypothesis:	Obs	F-Statistic	Probability
IPC1 does not Granger Cause CERERE1	42	9.74905	0.00040
CERERE1 does not Granger Cause IPC1		6.75465	0.00316

Granger Causality Tests
Date: 04/30/12 Time: 15:24
Sample: 1 45
Lags: 1

Null Hypothesis:	Obs	F-Statistic	Probability
IPC2 does not Granger Cause CERERE2	43	0.69833	0.40831
CERERE2 does not Granger Cause IPC2		2.16304	0.14919