THE SYSTEMS OF SYSTEMS (SoS) MODEL FOR THE MULTIDIMENSIONAL COMPLEXITY ANALYSIS OF THE MODERN ENTERPRISES

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Abstract. *The present evolutions highlights that globalization represents a phenomenon that takes place at all levels of economic, social, technical, information etc. levels. In this context are developed complex and heterogeneous enterprises that must integrate different types of systems and individuals, and that must perform a global function as an expression of their requested behaviour in attaining the goals of competitiveness and survival on the markets. From this point of view, the paper proposes a model based on the systems of systems (SoS) concept for the multidimensional complexity analysis of the enterprises.*

The multidimensional complexity analysis of the modern enterprises in the context of the SoS-based model represents an important approach for the development of new competitive enterprises, and also for the growth of the existing ones in complex structures, with complex goals in a globalizing context.

Keywords: *systems of systems (SoS), multidimensional complexity, process map of SoS, competitiveness, SoS integration, SoS architectures*.

1. Introduction

The present enterprises became complex systems due to their needed responses relative to the external requests and constraints. Thus, their heterogeneity is a defining characteristic that determined the growth of their complexity at different levels as: communication with their external environment, internal processes, information processing and the internal information life-cycle, products and services development and implementation, research, management and control etc.

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Additional to this, a structural and functional growth of the modern enterprise takes place, such that they integrate different types of systems, that may be distinct enterprises, and individuals, that modify the old strong and weak points in a manner that should be known and controlled.

In these conditions, the management and development of the modern enterprises need models on which fundaments of their decisions will be built and analyzed. One of these models is based on the systems of systems (SoS) concept with the goal of representing the multidimensional complexity of the enterprise, and to offer a solution for the heterogeneous and complex enterprise's architecture as a SoS architecture.

2. The multidimensional complexity

Considering that every enterprise is characterized by its reality level as the internal environment governed by its specific set of laws, and that according to the functional and processes heterogeneity, corresponding reality under-levels can be determined based on their specific laws, results that the multidimensional complexity of an organization is defined as:

Definition 1. Let be an organization *Og* having the following specific reality under-levels: $L_1, L_2, ..., L_n$ that form the reality level *R*, and considering the complexity of each reality under-levels as $C_1, C_2, ..., C_n$ (Fig. 1), then the complexity *C* of the reality level *R* is determined by a relation of the type: $C = Int\left(C_1, C_2, ..., C_n\right)$, where *Int* is an integration operator, and represents the ***multidimensional complexity of the organization***, *MC*(*Og*).

The integration operator (see Fig. 1) can be of different types regarding the rules of interconnection in realizing the enterprise's level of reality as following:

a) the *Int* operator corresponds to the "maximum", such that:

$$C = Int\left(C_1, C_2, ..., C_n\right) = \max\left(C_1, C_2, ..., C_n\right) \tag{1}$$

b) the *Int* operator is determined by the weight of each complexity, such that:

$$C = Int\left(C_1, C_2, ..., C_n\right) = \mu_1 \cdot C_1 + \mu_2 \cdot C_2 + ... + \mu_n \cdot C_n, \tag{2}$$

where $\mu_i \in [0,1]$, $i = \overline{1,n}$, $\sum_{i=1}^{n} \mu_i = 1$.

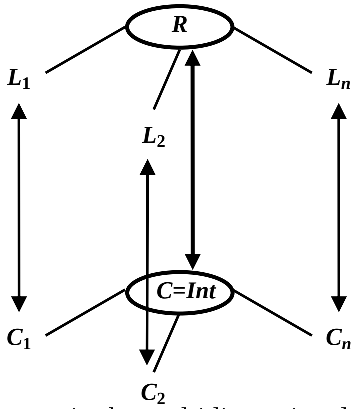


Figure 1. The *Int*operator in the multidimensional complexity analysis.

The reality under-levels of any enterprise are determined mainly by the following aspects of activity:

a) The life cycle structure of its products and services;

b) The competitiveness components;

c) The environmental and financial framework, all these related to the enterprise's objectives, polices, and strategies that consider the external environment.

For example, the reality under-levels for an enterprise are determined as in Table 1.

Table 1.

Types of reality under-levels in an enterprise

L_i	**Reality under-level**	**Specific laws**
L_1	Research and design	Laws of creativity, technical areas, standards, quality, reliability etc.
L_2	Production	Technological laws, productivity, specific standards etc.
L_3	Sales and distribution	Market and competition laws, clients behaviour etc.
L_4	Enterprise's quality	Laws of quality, quality management, leadership etc.
L_5	External relations with suppliers	Laws of collaboration, trust, agreements, etc.
L_6	Financial support	Financial laws of the external environment and the internal polices and strategies.
L_7	Economical support	Laws of economic efficiency, profit maximization, costs minimization etc.

Table 1 continued		
L_i	**Reality under-level**	**Specific laws**
L_8	Internal human factor	Behavioural laws, common and individual interests in the organization and outside it, fuzzy appurtenance to different contexts etc.
L_9	Natural external environment	Natural laws, evolution etc.
L_{10}	Social external environment	Social behavioural and evolution laws.
L_{11}	External financial context	Specific financial laws of the countries, regions, and their financial policies.
L_{12}	Cultural and educational context	Understanding capacities, resistance to changes, beliefs, quality requests etc.

Each reality under-level in table 1 is characterized by its complexity, such that the reality level of the considered enterprise / organization will be characterized by a multidimensional complexity according to definition 1.

3. The SoS-based model for the multidimensional complexity

Considering the organizational dimensions of the enterprise's development and implementation, results that the reality under-levels are structurally represented by the corresponding systems disciplinary oriented on specific technical, economical, financial, market, laws and settlements aspects. Thus, the reality level of the enterprise is implemented through the integration of these systems in a global organization. In this context, for every reality under-level can be specified the corresponding system according to the relation:

$$L_i \leftrightarrow S_i\ ,\ i = \overline{1,n}, \tag{3}$$

where L_i is a reality under-level of the enterprise, and S_i is the corresponding system.

If a reality under-level has more than one specific system, the relation (3) becomes:

$$L_i \leftrightarrow S_i = \{S_{1i}, S_{2i}, ..., S_{p,i}\},\ i = \overline{1,n}, \tag{4}$$

where $S_i = \{S_{1i}, S_{2i}, ..., S_{p,i}\}$ is the set of systems governed by the same assemble of laws, that implement L_i.

Let $L_1, L_2, ..., L_n$ be the reality under-levels of an enterprise, and $S_1, S_2, ..., S_n$ the corresponding systems or set of systems, such that relation (3) holds, then the reality level R results to be represented by the set $S = \{S_1, S_2, ..., S_n\}$, according to the relation:

$$R \leftrightarrow S = \{S_1, S_2, ..., S_n\}. \qquad (5)$$

According to the fact that in constructing reality level, interconnections between the corresponding reality under-levels are established, results that between the implementation systems of relation (5) are established also interconnecting relations, such that S becomes a system of systems (SoS) according to the following definition [1]:

Definition 2. A system of systems (SoS) is a set of systems that are following individual goals and functions in performing a global accepted goal.

According to definition 2 and relations (3) ÷ (5) results that the reality under-levels considered for an enterprise correspond to different goals (e.g., economical, financial, technical, social, etc.), and the resulted reality level must correspond to the global accepted goal of the enterprise.

In these conditions, let E be an enterprise, then its structure can be represented as a network of systems performing its global goal through specific functional dimensions viewed as reality under-levels, such that the enterprise E is a SoS according to the relation:

$$E = \langle S = \{S_1, S_2, ..., S_n\}, ICon, P \rangle, \qquad (6)$$

where: $S = \{S_1, S_2, ..., S_n\}$ represents the set of systems; *Icon* – the interconnection relation; P – the protocol of interconnection, as the assemble of rules according to which the systems of S will relate to each other in attaining the global goal.

Considering, in this context, the multidimensional complexity of the enterprise as an organization, according to definition 1, results that the corresponding measures are determined by the following relation:

$$C = Int(C_1, C_2, ..., C_n) = C(E) = C(S, ICon, P). \qquad (7)$$

Relation (7) represents the SoS-based model for the multidimensional complexity of the enterprise.

The SoS-based model of the enterprise and of its multidimensional complexity is applied for different types of structures:

a) *the individual enterprise*, where S is formed by the structural and organizational dimensions (e.g., departments, process oriented teams etc.);

b) *the extended enterprise*, where *S* is formed by a central enterprise and other ones specialized in different services and manufacturing areas;

c) *the virtual enterprise*, where *S* is formed temporarily by different individuals and/or independent enterprises that collaborate in attaining a goal through which are satisfied their individual goals;

d) *multidimensional complex enterprise*, where *S* is formed by a multitude of different organizations, enterprises, end even institutions, that collaborate to resolve complex and heterogeneous problems critical for a large area of interest, that can be extended at a global level.

The *Icon* relation can be of various types, determining corresponding network topologies of the enterprise, and the interconnection protocol, *P*, has different levels of integration capabilities, as is shown in Table 2.

Table 2.

The integration capabilities of the interconnection protocols.

Level of integration	Capabilities of the interconnection protocol
Interfacing	• The communication between the elements of *S* is established only at the level of data and information exchange, regarding the global goals. • The components of *S* preserve their functional, structural and disciplinary individuality specific to their homogeneity, quasi-homogeneity or heterogeneity. • An importance order of the elements can be established and accepted at the global level of the enterprise, *E*.
Interoperability	• Implements the communication between the systems of *S* that are considered as having the same weight / importance in the enterprise *E*. The information exchange is performed such that a cooperation context is formed at the level of the functional capabilities. • The *Icon* relation is implemented in a way that supports the interconnection relation of functional capabilities for the components of *S*. • The systems of *S* are functionally and disciplinary related and correlated. • The components of *S* preserve their individuality, but in a way that satisfies the cooperation conditions.

Interworking	• Implements the communication between the functions and tasks of the elements of *S*, and their coordination. Thus, the global goal is attained through cooperation, collaboration, and co-working, such that a level of performance is attained by the global structure, *E*. • The communication is performed both at the levels of information and functional states, such that the components of *S*, functionally converge in attaining the global goals. • The components of *S* are interconnected in a manner that creates, at the level of their global goals, an unifying bond. • Structurally, the components of *S* preserve their individuality, but the functional bonds in the context of *E* are much stronger and impose many constraints to each component.
Total integration	• The communication becomes an open context of information and states exchange through multi-dimensional channels, such that the *Icon* relation unifies, at the structural level, the components of *S*. • The components of *S* are 'regrouped' considering process-oriented or functional criteria, and thus their individuality is only functional, being not preserved at the organizational level. • The structure of the enterprise, *E*, becomes a new integrated global system, and appertains no more to the SoS class.

In these conditions, according to relation (7), the multidimensional complexity of an enterprise as SoS is determined by:

$$C = \left[C(S), C(ICon), C(P)\right], \tag{8}$$

where: $C(S)$ represents the complexity of the set of systems S; $C(ICon)$ – the complexity of the interconnections *ICon*; $C(P)$ – the complexity of the protocol P.

Thus, according to relation (8), results that the main metrics of the multidimensional complexity, C, correspond to the metrics of its main components (see Table 3).

Table 3.

Metrics of the multidimensional complexity of the enterprise.

1. Metrics for the complexity of the set of systems *C*(*S*)

1.1. The dimension of *S* as the cardinality of the set of systems, card(*S*), respectively the number of the component systems.

1.2. The goal heterogeneity of *S* (GHS) representing the goals of each systems structured in a corresponding table or database of the following form:

S_i/G_i	G_1	G_2	…	G_j	…	G_q	…	G_w
S_1	X		…		…	X	…	
S_2	X	X	…		…		…	X
…	…	…	…	…	…	…	…	…
S_k		X	…	X	…		…	
…	…	…	…	…	…	…	…	…
S_n	X		…	X	…	X	…	

1.3. The compatibility of the individual goals of the component enterprises with the global goal aspects of the structure *E*, viewed as points of: contradiction (●), non-contradiction (■), full compatibility (▲), or not related, in an compatibility table as:

$G_i/A_j(G)$	A_1	A_2	…	A_q	…	A_l	…	A_p
G_1	■	▲	…	●	…	▲	…	▲
G_2	▲	▲	…	■	…	■	…	▲
…	…	…	…	…	…	…	…	…
G_i	▲	●	…	▲	…	■	…	▲
…	…	…	…	…	…	…	…	…
G_n	●	■	…	▲	…	▲	…	●

2. Metrics for the complexity of the interconnections *C*(*ICon*)

If an interconnection is viewed as an edge between two nodes that represent two connected enterprises, then the structure *E* as a SoS is represented by a graph, and all the metrics of graphs are applied as metrics for *C*(*ICon*) as: number of nodes, number of edges, flows in graphs, edges capacities etc.

3. Metrics for the complexity of the protocol *C*(*P*)

Considering the role of the protocol, the following types of metrics can be used:

- the number of rules and/or procedures;
- the levels of activity considered by the protocol's rules and procedures;
- the depth at which the protocol's rules and/or procedures influence the behaviour of each component enterprise;
- the degree of autonomy left to the individual enterprises etc.

4. The SoS architectures in building modern enterprises

The main problem in designing and implementing an enterprise as a SoS is to establish its architecture according to the constructive relation (6).

As any architecture, the SoS architecture considers functional blocks, but these will represent, in fact, the individual enterprises that form *E* through *S*, *ICon*, and *P*. Thus, taking into account that any SoS is a network of systems, results that the network topologies (e.g., hierarchical, star, ring, etc.) represent architectures that can be considered.

Also, another type of architectures can be developed, that are based on the interconnection of the component enterprises / systems, as in table 4[2, 3]. These interconnected – oriented architectures correspond to the following relation $\forall p = \overline{1,n}$, at least two indices $u, v \in \{1, ..., n\}$ exist, such that:

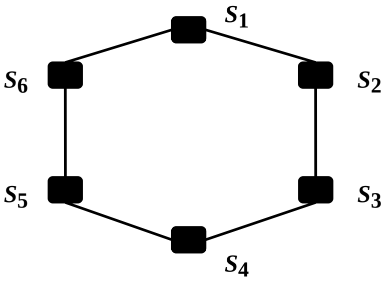
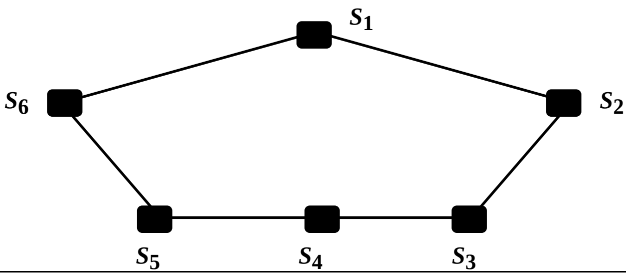
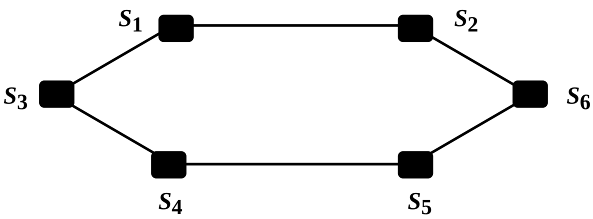
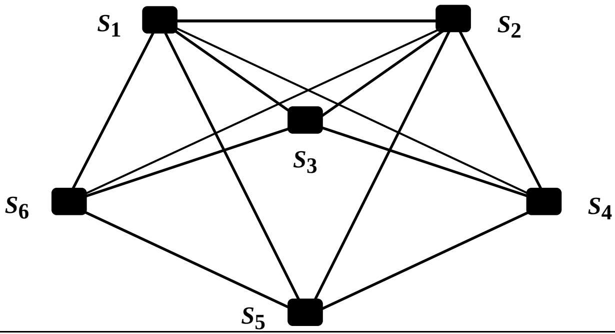
$$\begin{aligned}
&[e_{pu} \in ICon \text{ and } e_{pv} \in ICon] \text{ or} \\
&[e_{up} \in ICon \text{ and } e_{vp} \in ICon] \text{ or} \\
&[e_{up} \in ICon \text{ and } e_{pv} \in ICon] \text{ or} \\
&[e_{pu} \in ICon \text{ and } e_{vp} \in ICon] \text{ ,}
\end{aligned} \tag{9}$$

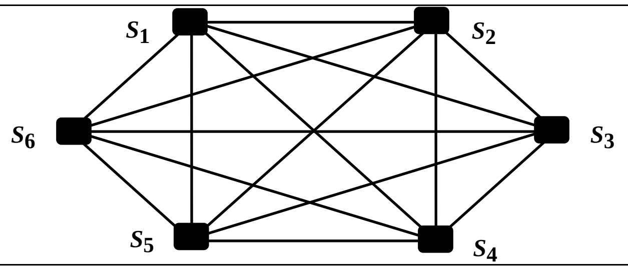
where $e_{\alpha\beta}$, $\alpha, \beta = p, u, v$ are oriented or not oriented edges that connect the nodes $S_1, S_2, ..., S_n$ representing the component enterprises.

Considering the architectures presented in table 4, other types can be derived.

The interconnection – oriented architectures are recommended due to their capabilities to support all the characteristics of the interconnection protocols, from interfacing to total integration, being also a support for the synergistic integration of the SoS when all the edges represent the information set (see [2, 3]) of the SoS, respectively of the enterprise *S* as a SoS.

Table 4.
Interconnection – oriented architectures of the enterprises as SoS (for n = 6).

1.	**Serial SoS architecture**
	
2.	**Closed serial SoS architecture**
	
3.	**1 – Centred SoS with open serial interconnection**
	 S_1 – central system; $S_2 \div S_6$ – open serial under-connection.
4.	**2 – Centred SoS with open serial under-connection**
	 S_1, S_2 – central system; $S_3 \div S_6$ – open serial under-connection.
5.	**2 – Centred SoS with closed serial under-connection**
	 S_1, S_2 – central system; $S_3 \div S_6$ – closed serial under-connection.

6.	**Total connected SoS or complete SoS**
	

5. The process map of the enterprise developed as SoS

In order to develop or re-engineer an enterprise as a SoS, the main dominant processes are should be considered as following:

P1 – communication;

P2 – collaboration, co-working;

P3 – SoS control;

P4 – configuration and reconfiguration;

P5 – analysis and identification;

P6 – decision and planning;

P7 – production / service delivery;

P8 – learning and experience accumulation.

The corresponding process map of the enterprise developed as a SoS is represented in figure 2.

The above processes must support the global function of the enterprise in the conditions of:

- individual missions of the component systems / enterprises;
- specific internal structures and organizations;
- individual resources and capabilities;
- individual management of the component systems / enterprises.

Accordingly, the design of each process is critical for the development of the enterprise as a SoS in the context of multidimensional complexity.

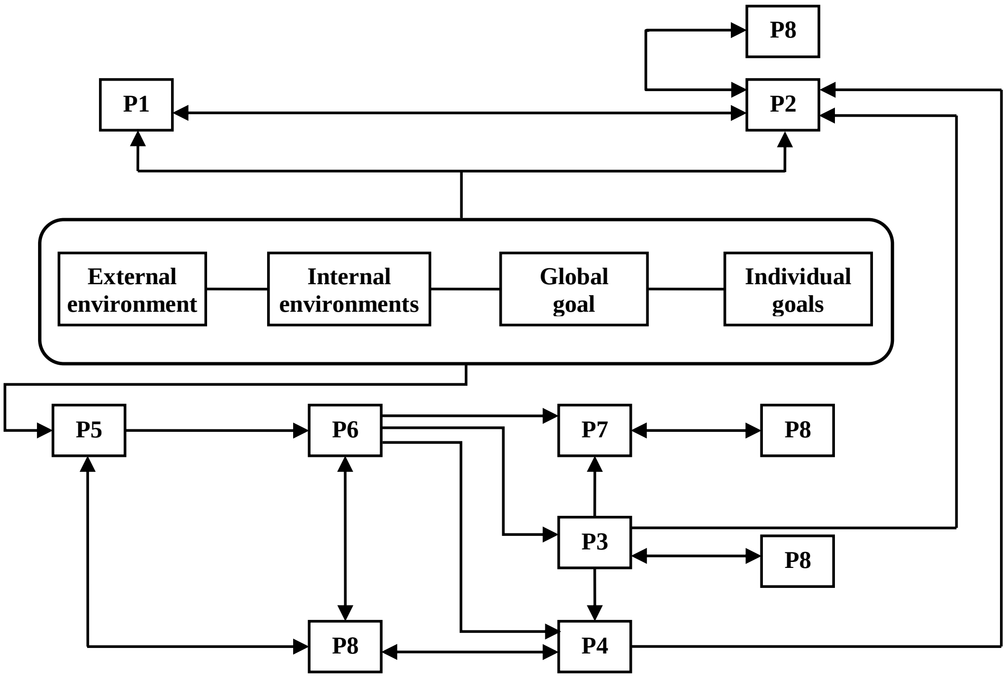


Figure 2. The process map of the enterprise as a SoS.

6. Conclusions

The present economical, technological, and business environment is characterized by heterogeneity and complexity that determine specific dynamics in the enterprises' behaviour.

Thus, regarding the complexity of the modern individual enterprises, and the modern business structures, result two main aspects of the problem: the multidimensional complexity of the enterprises, and the development of the enterprises towards complex structures as systems of systems (SoS).

The present paper considers both aspects in defining the multi-dimensional complexity concept and its SoS corresponding aspects. In this context are developed the SoS-based model of the multidimensional complexity of the enterprise, the SoS architectures of the enterprises built as SoS, and the process map for the enterprises with SoS structures.

The resulted framework represents an approach both for the analysis and the development of the modern enterprises in the context of the SoS concept and multidimensional complexity.

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