III-rd Section:

COMPLEXITY SCIENCE

COEXISTENCE OF OPPORTUNISTS, CONTRARIANS AND INCONVINCIBLES IN BINARY OPINION NETWORKS

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Abstract. *We study a model for the emergence of collective decision making heuristics, consisting of four different classes of interacting agents, whose opinions are described by binary state variables. In particular, a subtle interplay between opportunist, contrarians and inconvincibles sticking stubbornly on their opinion, leads to phase transitions which abruptly can change the outcome of a public debate. At a critical density the inconvincibles reduce the fraction of an initial majority such that the initial minority wins the debate.*

Keywords: *social networks, opinion spreading, dynamical phase transitions, majority/minority interaction, opportunistic-contrarian behaviour.*

1. Introduction

Applications of methods earlier used in the area of statistical physics to social phenomena such as opinion formation, socio – and economic dynamics have been extremely successful during the last few years. [1, 2, 3] The present study extends the analysis of the dynamics of randomly assembled threshold networks to social phenomena [4]. In the context of socio physics this may help to understand under which conditions special shocking events or propaganda are able to influence the results of public debates. In this model, each agent i is assigned K randomly chosen agents who discuss a topic in order to convince agent i arrive at a final decision, in favour or against.

The dynamical decision making process is based on the majority – and minority rule such that each individual may adopt the opinion of the majority or minority, respectively. In this report the heterogeneous model is extended to two other groups of agents, who never change their opinion, the so called inconvincibles who either always stick to the YES or always

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stick to the NO state. The model thus describes how agents with different backgrounds try to convince each other, in either their or the opposite direction expressing their desire to identify with one of four distinct social groups or to differentiate themselves from them.

In particular, we study the effects of opportunists and contrarians as well as both sided inconvincibles with interactions based on only four behaviour schemes expressed by simple cellular automata rules which are able to generate rather complex behaviour. One focus of this study is the determination of critical network parameters, where the network undergoes transitions from the existence of two attractors, the YES and NO attractor, to one single stable attractor [5, 6].

2. Four prominent cellular automata rules

The model network consists of a population of N agents of a democratic community who are characterized by a specific opinion on a particular subject. The opinion of agent i is described by an Ising spin variable σ_i, only capable to take the value +1 or 0, i.e. to say YES and NO, to buy or not to buy, or to vote Party A or Party B, respectively. We assume that each agent i can be influenced by K other agents of the community with $1 \leq K \leq N$. These so-called "neighbours" are chosen randomly according to a uniform probability density distribution.

The basic principle of the majority/minority rule model often applicable in the real life context was originally proposed by Galam [3]. The model mimics public debates, where at each time a discussion group of K agents step is selected at random and all agents take the opinion of the majority inside the group. If K is odd, there is always a majority in favour of either opinion. If K is even instead, there is the possibility of a tie. For simplicity we will further concentrate on K odd, although the formalism for K even will be similar. Agents which follow the majority rule can be considered as opportunists who always adopt the opinion of the majority with respect to their neighbourhood. The second group of individuals the so-called contrarians always adopt the opposite opinion of the majority and follow the minority rule. They usually have no own opinion, they simply want to be opposite in order to be opposite. Agents which follow the minority rule can be considered as contrarians who always adopt the opinion of the minority. The third and fourth groups of agents consist of inconvincibles who always stick stubbornly on their opinion, supporting either the YES or the NO opinion [6]. Accordingly they follow the tautology rule which says always YES, and the contradiction rule which says always NO independent of their input stimulus. These Boolean behaviour functions can be given in terms of cellular automata rules. The representation for $K = 3$ input units is shown in table 1 where we represent majority (232), minority (023), contradiction (000) and tautology (255).

Table 1.
Cellular automata rules according to the Wolfram notation.

in	232	023	000	255
000	0	1	0	1
001	0	1	0	1
010	0	1	0	1
011	1	0	0	1
100	0	1	0	1
101	1	0	0	1
110	1	0	0	1
111	1	0	0	1

Note that the dynamical evolution for the majority/minority rule could also be written in terms of a linear threshold network widely used in models of neural and genetic networks [7, 8, 9], where the interactions are ferro – or anti-ferromagnetic, respectively and take the form:

$$\sigma_i(t+1) = \pm\theta\left(\sum_{(j)} \sigma_j(t)\right) \quad i = 1,\dots,N. \tag{1}$$

A suitable order parameter, the macroscopic variable, the public opinion at time *t*, which defines the degree of acceptance of the YES or NO state, can be defined by the total magnetization:

$$m(t) = \frac{1}{N}\sum_{i=1}^{N} \sigma_i(t). \tag{2}$$

The time dependent magnetization describes the degree of support of the YES state before a debate, one of the competing states YES or NO are monotonically approached after a short transient. The system relaxes to $m=1$ or $m=0$ in the case of consensus, or to $m=\frac{1}{2}$ in the case of a tie.

3. Mean-field approach

In the thermodynamic limit of asymptotically large *N*, statistical predictions for the time evolution of the magnetization $m(t)$ Equation (2), as well as the time evolution of the Hamming distance $d(t)$ [7, 8, 9] can be derived analytically, provided that:

(i) Every agent has exactly *K* incoming connections from *K* distinct input agents *j* chosen with uniform probability among the other $N-1$ agents.

(ii) The interaction or rules of behaviour are chosen randomly according to a given probability density distribution.

(iii) The network is sparsely connected ($K \sim \log N$).

Eventually, for $K = 3$ input units, the magnetization, i.e. the probability that a randomly chosen agent is in the YES state in the next time step is given by the iterative map:

$$m(t+1)=3m^2(t)-2m^3(t) \tag{3}$$

for the opportunists and:

$$m(t+1)=1-(3m^2(t)-2m^3(t)) \tag{4}$$

for the contrarians. Eventually, the time evolution of the magnetization for the third group, the inconvincibles ist trivially:

$$m(t+1)\equiv 1 \text{ or } m(t+1)\equiv 0 \tag{5}$$

for those who are in favour of the YES, and the NO, respectively.

In all cases, for the majority and the minority interaction rule as well as for tautology and contradiction damage spreading remains confined and it is easy to show that an arbitrary initial distance $d(t)$ decreases monotonically to zero [4].

4. Mixtures of all four classes of agents

Be p_O the fraction of opportunists, p_C the fraction of contrarians, and p_A and $p_B=p_A+\varepsilon$ are the fractions of the inconvincibles with respect to the YES and the NO state. Here $\varepsilon > 0$ is in favour the NO state, while $\varepsilon < 0$ favours the YES state of the inconvincibles. The magnetization $m(t+1)$ is a superposition of all contributions to the YES state and takes the form:

$$m(t+1) = p_O(3m^2(t) - 2m^3(t)) + p_C(3m^2(t) - 2m^3(t)) + p_A. \tag{6}$$

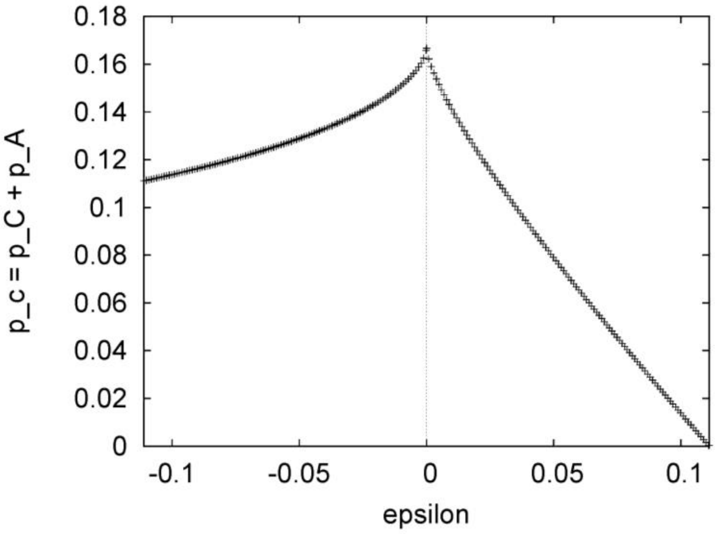

Figure 1. Critical p_C as a function of ε for $-\frac{1}{9} \le \varepsilon \le \frac{1}{9}$.

Making use of the normalization condition and eliminating the quantity p_O :

$$p_O + p_C = 2p_A + \varepsilon = 1 \tag{7}$$

we have:

$$m(t+1) = (1 - 2p - \varepsilon))(3m^2(t) - 2m^3(t)) + p \tag{8}$$

where the quantities $p = p_C + p_A$ and ε serve as control parameters. The long time behaviour is determined by potential stable fixed points of the cubic equation in m^*:

$$m^{*3} - \frac{3}{2}m^{*2} + \frac{i}{2(1-2p-\varepsilon)}m^* - \frac{p}{2(1-2p-\varepsilon)} = 0. \tag{9}$$

According to the Cardano formula the nature of the solutions depends uniquely on the sign of the discriminant D which specifies the number of real solutions. For $D > 0$ we have two stable attractors separated by the unstable solution which specifies the basins of attraction of the attractive fixed points. For $D < 0$ one finds one unique stable solution. Figure 1 shows the critical line p_C associated with $D = 0$ as a function of ε. Note that $p_C = 0$ for $\varepsilon = \frac{1}{9}$ and $p_C = \frac{1}{9}$ for $\varepsilon = -\frac{1}{9}$. This critical line is of crucial importance, since the existence of only one unique attractor leads to the victory of the YES or NO decision independent of the initial condition.

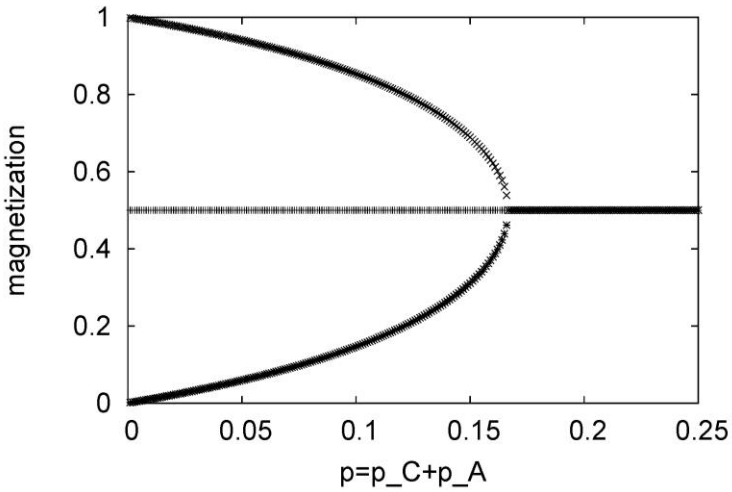

Figure 2. Time evolution of the magnetization $m(t)$ for an equal fraction of inconvincibles $p_A = p_B$.

Provided that $\varepsilon = 0$ we find the critical $p_c = p_C + p_A = \frac{1}{6}$, where the two stable fixed points undergo a backward bifurcation with the three fixed points:

$$m_1 = \frac{1}{2}\left(1 + \sqrt{\frac{1-6p}{1-2p}}\right) \quad m_2 \equiv \frac{1}{2} \quad m_3 = \frac{1}{2}\left(1 - \sqrt{\frac{1-6p}{1-2p}}\right). \tag{10}$$

Figure 2 shows that essentially we recover the familiar results of the model studied earlier [4], where for $p_c < \frac{1}{6}$ we find two attractors for the YES and NO states (top and bottom curve), while for $p_c > \frac{1}{6}$ we have the tie situation, where we have a fifty-fifty equilibrium with a unique attractor $m^* = \frac{1}{2}$ (middle curve).

4.1. The case $\varepsilon \neq 0$: Unbalanced fractions of inconvincibles

Let us now consider the case, where more inconvincibles are in favour of the YES state than in the NO state, or vice versa. It is quite obvious that provided that the fraction of the inconvincibles in favour of the YES the NO will be able to win the debate even with an initial minority since the critical separating line will be shifted above or below the values $\frac{1}{2}$. Indeed a low order expansion of the corresponding fixed point for small p and small ε yields:

$$m^* \approx \frac{1}{2} + \varepsilon(1 + 6p + 36p^2). \qquad (11)$$

In principle, this expansion reflects that the outcome is already explicitly put into the model as an assumption. We have three fixed points, where the middle one, which is unstable, plays the role of the separator. Figure 3 depicts the stable as well as the unstable fixed points of Equation (9) for $\varepsilon > 0$, where the NO opinion is favoured. The lower curve, which signals the victory of the NO opinion, is stable over the whole range. The separator (middle line) is always unstable. The upper line which coalesces with the separator at the critical p_c signals the victory of the YES opinion and is only stable for subcritical values of p.

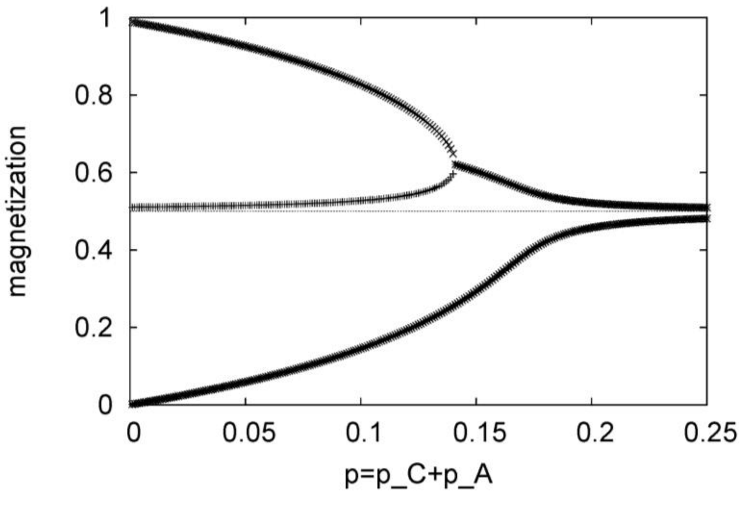

Figure 3. Magnetization m^* as a function of the concentration $p = p_C + p_A$ for $\varepsilon = 0.01$.

With increasing values of ε the bifurcation point moves to the left until it reaches $m^* = \frac{1}{4}$ at $p = 0$ for $\varepsilon_c = \frac{1}{9}$. For $\varepsilon < 0$ the situation is analogous, however with the restriction:

$$p = p_C + p_A > |\varepsilon| \tag{12}$$

as depicted in Figure 4, where the role of the upper and lower fixed point is reversed. Note however that the relevant critical line begins at $p = p_C + p_A > |\varepsilon|$ with the upper $m^*= 1$. In analogy to the case $\varepsilon > 0$ the bifurcation point moves to the left with increasing absolute value of ε until it reaches $m^*=$ at $p = \frac{1}{9}$ for $\varepsilon = -\frac{1}{9}$.

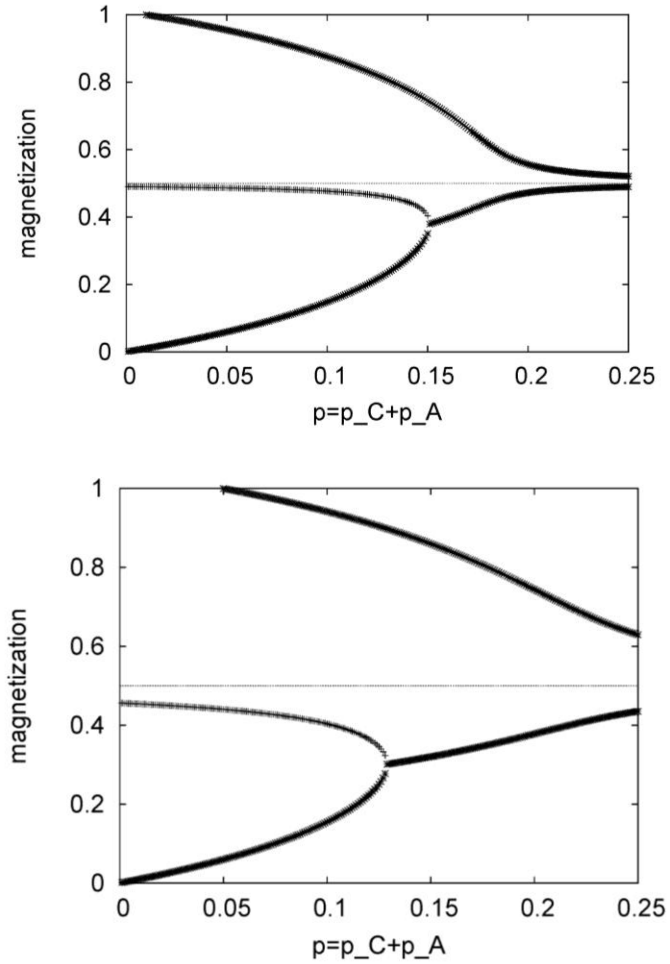

Figure 4. Magnetization m^* as a function of the concentration $p= p_C + p_A$ for $\varepsilon = -0.01$ (upper figure) and $\varepsilon = -0.05$ (lower figure).

5. Discussion

We have presented an agent-based model consisting of four prototypes of characters in order to study the subtle balance of opportunistic, contrarian as well as inconvincible behaviour in the dynamics of opinion spreading. We determined a critical parameter $p_c = p_C + p_A$ specifying the fraction of contrarian and inconvincible agents as a function of the control parameter ε which specifies the fraction of the inconvincibles in favour to the NO state. At low concentrations of the inconvincibles we find two stable fixed point configurations with a more or less clear YES or NO majority. Beyond the critical point we find a unique stable YES or NO decision depending on the balance between the YES and NO inconvincibles. We can conclude: increasing the fraction of the inconvincibles which are on the respective side is quite more advantageous than convincing the opportunists. Even the smallest difference in inconvincibles can turn the system over by shifting the separator to a lower or higher value in order to push the YES or NO decision, respectively. In any case, one should not try to convince the inconvincibles. On the other hand, once the inconvincibles have reached a critical threshold the victory to the YES or NO becomes certain.

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