MULTIFRACTAL STRUCTURE IN INDIAN STOCK MARKET INDICES

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Abstract. *In this paper we perform a detailed multifractal analysis on NSE Nifty and BSE Sensex indices over a period of eight years using the partition function approach. US S&P 500 index series is also analyzed for the purpose of comparison. In all the three cases the partition function $\chi_q(\varepsilon)$ is found to have a power law dependence on the size of partition ε. ln $\chi_q(\varepsilon)$ vs ln ε graphs for all the three index series are plotted for q= –10 to q = +10, stepped by 1. Values of the mass exponent τ(q) for different values of q are obtained by the linear fits to the ln $\chi_q(\varepsilon)$ vs ln ε graphs. We have also plotted τ(q) vs q graphs and the mass exponent τ(q) is found to have a non-linear dependence on q for all the three index series. All the three graphs are concave downwards, again signaling the presence of multifractality in the three index series. Finally, we plot the singularity spectra f(α) for the three index series. The widths of the spectra for Nifty and Sensex are smaller than that for S&P 500, indicating that Indian markets are inefficient compared to the US market.*

Keywords: *Multifractality, stock market, partition function, singularity spectrum, market inefficiency.*

1. Introduction

Time series generated by the financial systems e.g. stock markets, *fx* markets, commodity markets, etc. carry useful information about the underlying mechanisms that govern these complex systems. But the traditional analysis techniques of Econometrics fail to extract the useful information from the time series. Consequently, analysis techniques from other fields, especially physical sciences, are being used to obtain a better description of financial markets. Multifractal analysis, which was initially introduced to investigate the intermittent nature of turbulence, has been extensively employed to study the multifractal properties of financial time series [1-3]. Different methods have been developed for the multifractal

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analysis of the time series. Some of these methods are: (a) partition function method [4], (b) rescaled range analysis [5], (c) wavelet transform method [6] (d) detrended fluctuation ayalysis (DFA) method [7] and (e) multifractal detrended fluctuation ayalysis (MDFA) method [8,9].

In this paper we use the partition function method to study the multi-fractality in Indian stock market indices Nifty and Sensex. We also study the multifractality in the US market index S&P 500 for the purpose of comparison. The paper is organized as follows: In section 2, we give a brief description of the partition function method. Section 3 gives details of the data analyzed in the paper. Empirical results and their discussion are presented in section 4 and finally we conclude in section 5.

2. Methodology

The multifractal structure of financial time series can be investigated using the partition function method. The method is described briefly as follows.

Let $\{x\ (t)\colon t=1,2,3\ldots\ldots\ldots\ldots..T\}$ be the closing value of index series of a stock market. T is the total length of the series. The series is divided into N parts of equal length $\varepsilon = T/N$. The mass probability of the ith part of the series (i =1, 2, 3………N) is defined as:

$$P_i(\varepsilon) = \frac{I_i(\varepsilon)}{\sum_{i=1}^{N} I_i(\varepsilon)}, \tag{1}$$

where $I_i(s)$ is the sum of indices in ith part of the series. The partition function can be calculated as:

$$\chi_q(\varepsilon) = \sum_{i=1}^{N} P_i^q(\varepsilon). \tag{2}$$

In the presence of fractality the partition function scales as:

$$\chi_{q(s)} \alpha s^{\tau(q)}, \tag{3}$$

where τq is called the mass exponent. If τ_q is found to depend non-linearly on q, the series under analysis is said to be multifractal. The singularity exponent $\alpha(q)$ and its singularity spectrum $f(\alpha)$ are related to $\tau(\alpha)$ through the following relations:

$$\alpha = \frac{d\tau(q)}{dq} \tag{4}$$

$$f(\alpha) = \alpha\, q - \tau(q). \tag{5}$$

Generally two types of multifractality are present in the financial series:

(a) – multifractality due to a broad fat-tailed probability density distribution of the financial series. This type of multifractality cannot be removed by shuffling the series and

(b) – multifractality due to long range correlations of small and large fluctuations which can be removed by shuffling the financial series.

So if both types of multifractality are present in the financial series, shuffling of the series will remove multifractality due to long range correlations and as a result the shuffled series will exhibit a weaker multifractality as compared to the original one.

3. Data

To study multifractality in the indices of Indian markets, we used daily closing values of Nifty and Sensex. As US market is a widely studied developed market, we also studied multifractality in its index (S&P 500 index) for the purpose of comparison. The data period of each index series is until 30/31 December, 2010 and the starting date is chosen such that the total length of each series is 2048 trading days.

4. Empirical results

Values of the partition function $\chi_q(\varepsilon)$ for different values of ε were calculated using Eq. 3. Figure 1 shows $\ln \chi_q(\varepsilon)$ vs. $\ln\varepsilon$ plots for $q = -10$ to $q = 10$, stepped by 1 for the three series. Slopes $\tau(q)$ of these plots for different values of q were determined by linear fits to these plots. Figure 2 displays $\tau(q)$ vs. q plots for all the three series. A mono-fractal series is characterized by a linear dependence of $\tau(q)$ on q, while a multifractal series exhibits a non-linear dependence of $\tau(q)$ on q. All these plots, shown in figure 2, are concave downwards, implying presence of multifractality in all these series. The highest non-linearity or the strongest multifractality is observed in case of Sensex series, while the smallest non-linearity or weakest multifractality is observed in case of S&P series.

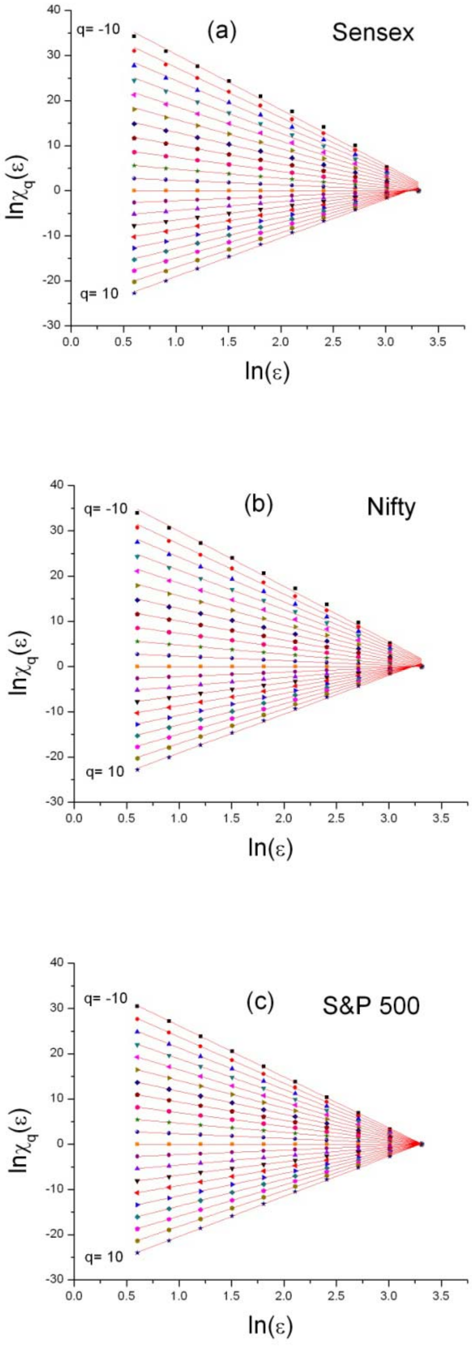

Figure 1. $\ln\chi_q(\varepsilon)$ vs $\ln(\varepsilon)$ plots.

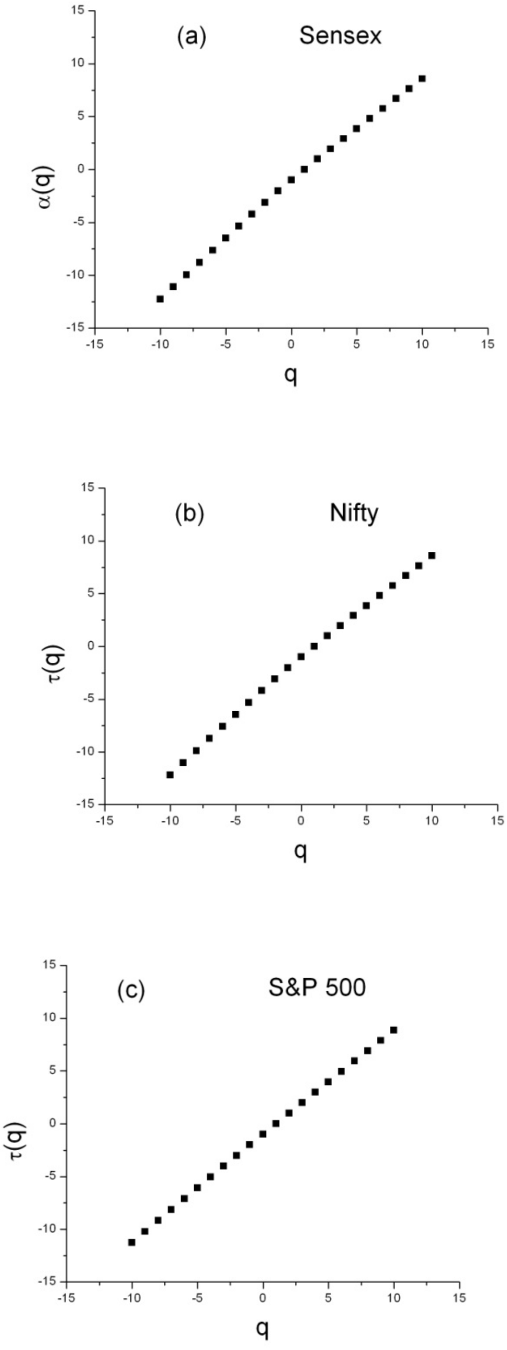

Figure 2. $\tau(q)$ vs q plots.

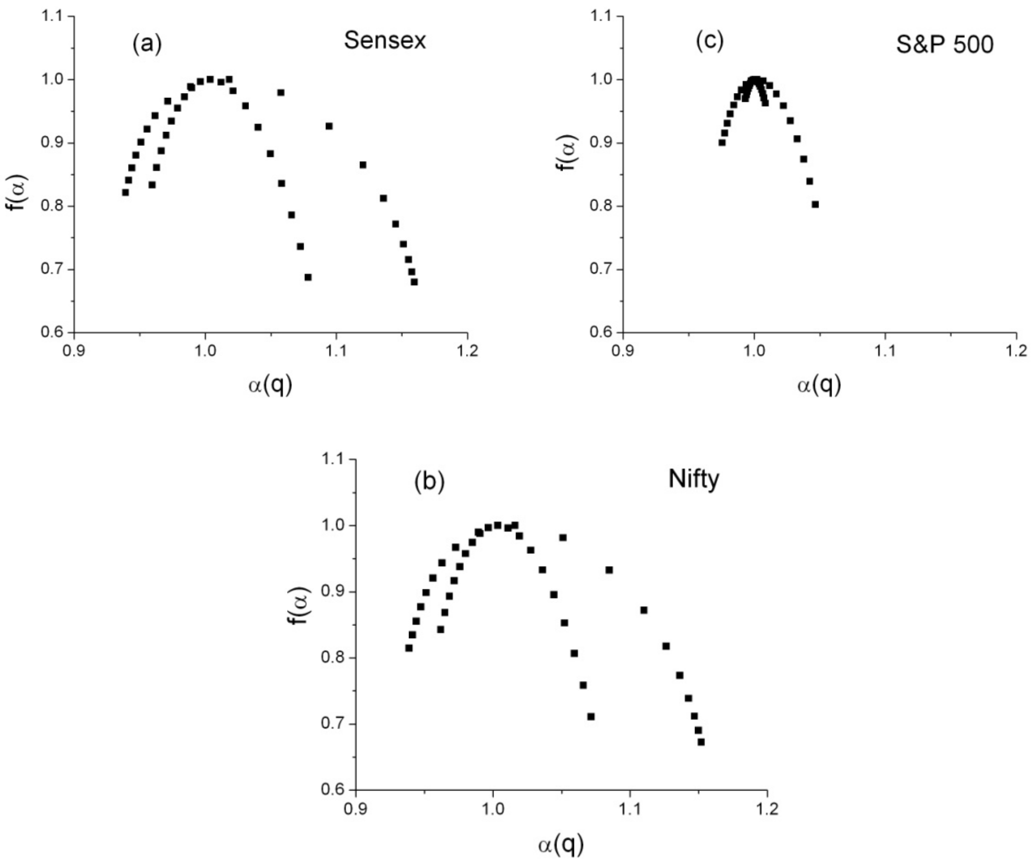

Figure 3. Singularity spectra for the three series.

A better way to visualize the multifractal nature of the time series is to plot the singularity spectrum $f(\alpha)$. It is a more effective method because here one can easily assess the strength of multifractality in different series as the width of the singularity spectrum is proportional to the strength of multifractality. Figure 3 displays singularity spectra for all the three series under study. The inverse parabolic shapes of the singularity spectra $f(\alpha)$ confirm the presence of multifractality in all cases. As can be seen from figure 3, the size and shape of the singularity spectra are different for different series and it appears that the singularity spectra may contain some useful statistical information about price movements and the degree of development of markets [10, 11]. As observed earlier, the strongest multifractality or the largest width of the spectrum ($\Delta\alpha = 0.220$) is observed in case of Sensex series, while weakest multifractality or the smallest width ($\Delta\alpha = 0.071$) is observed in case of S&P series. For Nifty

($\Delta\alpha$ = 0.213). Smaller values of $\Delta\alpha$ for Sensex and Nifty suggests that Indian markets are less efficient compared to US market.

As mentioned earlier, there are two main sources of multifractality in the financial series: (a) Long range correlations and (b) fat-tailed probability distribution. Long range correlations can be removed by shuffling the series. So in order to assess the contribution of long range correlations to the observed multifractality in different series, we shuffled all the three series 100 times. Figure 3 also displays the singularity spectra for the shuffled series. It can be seen from figure 3 that the width of $f(\alpha)$ for the shuffled series is less than that of the original series in all cases, implying the presence of both types of multifractality in all the three series.

5. Conclusions

We investigated the multifractality in the Indian stock market indices Nifty and Sensex using the partition function method. Our results show that the partition function $\chi_q(\varepsilon)$ for all three series scales as a power law with box size ε for each order q, implying that all series have multifractal structure. By comparing the singularity spectrum $f(\alpha)$ of the original series with that of the shuffled series, we also found that the multifractality observed in all time series is due to the presence of long range correlations of small and large fluctuations and also due to broad fat-tailed probability distribution. Finally a comparison of the singularity spectrum $f(\alpha)$ of S&P 500 series with those of Nifty and Sensex series showed that the width of the spectra for the developed US market is less than those for the Indian market. This result suggests that the Indian market is less efficient than the US market.

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