ADJUSTMENT IN EXCHANGE RATE, INTEREST RATE, AND PRICE LEVEL BY ECONOMIC FORCES[1]

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Abstract. *We model the exchange rate of the currency of a relatively small country vis-à-vis to euro. To construct a complete model for the financial system, the dynamics of interest rate and average price level are modeled too. We show that the exchange rate changes due to imbalances in that part of the balance of payments of the home country, charged in Euros, if the central bank does not sterilize these impulses. The adjustment in the exchange rate, the interest rate, and average price level may be monotonous or oscillating, and the equilibrium may be stable or not. In our model, overshooting takes place in stable cases as compared with Dornbush's (1976) unstable equilibrium. The possible equilibrium exchange rate is shown to be conditional on investors' expectations of its future value. (JEL F31, F32, G15)*

Keywords: *Exchange rate, interest rate, price level, nonlinear dynamics.*

1. Introduction

Flood and Taylor (1996) classify the key theories of exchange rate movements as: 1) the Purchasing Power Parity (PPP) model; 2) the flexible-price monetary model; 3) the sticky price overshooting monetary model; 4) the portfolio balance model, and 5) the equilibrium model. According to the authors, simple macro fundamentals − relative inflations and interest rates − have poor explanatory power in exchange rate movements even over one-year horizon. Five-, ten-, and twenty-year averages, on the other hand, show a strong proportionality between exchange rate depreciation and movements in these fundamentals. A simple macro fundamentals based model thus outperforms the random-walk model at

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horizons of five years or longer. "…there are speculative forces at work in the foreign exchange market that are not reflected in the menu of macro-economic fundamentals..." (ibid. p. 285-6) "...exchange rate fundamentals change very slowly … while exchange rate time series show huge variation". (ibid. p. 287).

Meese & Rogoff (1983a) report that none of their tested macro-based exchange rate models outperforms the random-walk model. In (1983b), however, Meese & Rogoff and Koedijk & Schotman (1990) found models that outperform the random-walk model at time horizons beyond one year. Berben & de Jong (1999) report mean-reverting behavior of exchange rates toward the PPP equations and that the PPP theory does not explain the short-term movements. In Cappiello & De Santis (2005), the Uncovered Equity Return Parity theory beats the random walk model at two – and three-month forecasts in EUR/USD rate, but their specifications for the British pound did not beat the random walk model.

The role of expectations in modeling exchange rates has been studied e.g. by Chiarella et al. (2009) and Gourinchas & Tornell (2002). In Chiarella et al. the driving macro factor in exchange rates is the interest rate differential and the market microstructure element is described by the expectations of portfolio managers. Gouringas & Tornell claim that exchange rates are determined by two effects: an interest rate effect and a learning effect. Excess returns in currency markets result if agents misperceive interest rate shocks to be more transitory than they are.

We can summarize this review as follows. The exchange rate models have been concentrated on the international trade of goods by price competition (the PPP theory), on investors' speculative investments (the Interest Rate Parity (IRP) theory and the portfolio balance model), on macro-level money market behavior (the flexible and sticky price monetary models), or on general equilibrium in a two country model (the equilibrium model). The evidence supports the PPP theory in the long-term, but modeling the short-term fluctuations has been less successful. According to theoretical completeness and empirical success, none of the mentioned models alone fulfills the criteria for a complete theory of exchange rate movements. For that we need a theory that explains the short – and the long-term movements, where the international trade of goods and assets is present together with central bank's operations.

Here we introduce such model. Even though we operate with macro-level quantities, the proposed model is consistent with rational behavior of economic agents: physical investments depend on real GDP and interest rate, international trade of goods and assets depends on price and yield competition, and the demand of liquid funds depends on domestic and international trade. The modeled "prices" − exchange rate, interest rate, and average price level − are assumed to adjust according to excess demand in the corresponding market. We define the "economic forces" acting upon the three adjusting quantities which terminology has been used e.g. by Flood & Taylor (1996, p. 286) and Park (1997, p. 478). These forces contain parameters that allow the central bank to steer the process. The equilibrium values of the adjusting quantities define the steady-state of the model − the "zero-force" situation. We define inertial factors for the adjusting quantities and study the effect of inertia for the adjustment process.

The study is organized as follows. The current and the financial account of the home country are modeled in sections 2, 3. In sections 4, 5 is introduced a model for the exchange rate. Section 6 presents a model for the interest rate, and in section 7 exchange and interest rate are analyzed together. In section 8 is a model for average price level and in section 9 the complete model. Section 10 is a summary.

2. Modeling Current Account

We study the exchange rate of the currency of a small country vis-à-vis to euro. For simplicity, the home country is assumed to be Sweden but the modeling applies to any country with an own currency. The willingness to exchange euros to Swedish kronas due to the international trade of goods depends on the revenues of Swedish exporting firms in euros that have costs in kronas. Similarly, the willingness to exchange Swedish kronas to euros due to the international trade of goods depends on the revenues of Swedish importing firms in kronas that pay to their suppliers in euros. The part of the current account of Sweden, expressed in euros, is denoted as $R = X - M$, where $X(EUR/y)$ is export revenues[1] and $M(EUR/y)$ import costs of Swedish firms at time unit *y*. Firms' net willingness to change kronas to euros due to international trade then depends on R.

[1] Measurement units are in braces after the quantities; see de Jong (1967). Unit *y* can be any time unit.

We denote the price of Swedish export good i by $p_i(SEK/kg)$ and the euro price of the corresponding foreign good by $p_{fi}(EUR/kg)$. The international price competitiveness of Sweden is measured by H_R:

$$H_R = \sum_{i=1}^{n} w_i[S(t)p_{fi} - p_i] = S(t)\sum_{i=1}^{n} w_1 p_{fi} - \sum_{i=1}^{n} w_i p_i(t) = S(t)\bar{p}_f - \bar{p}(t), \quad (1)$$

where $0 \le w_i \le 1, \sum_{i=1}^{n} w_i = 1,$ are the weights of the goods in the measure, n the number of possible export goods in Sweden, $\bar{p}_f, \bar{p}$ weighted averages of prices abroad and in Sweden, and $S(SEK/EUR)$ the exchange rate of the currencies; $S, \bar{p}$ are set as functions of time t because later we model their dynamics. The greater H_R is, the better the price competitiveness of Sweden.

Our literature review shows that the short-term changes in exchange rates are caused by financial investments, and in the long-term exchange rates adjust so that the PPP equations hold. This is understandable because PPP equations express equalities between purchasing powers of currencies. Suppose the law of one price holds for all n goods between Sweden and the euro area. Then:

$$H_R = 0 \Leftrightarrow S\sum_{i=1}^{n} w_i p_{fi} = \sum_{i=1}^{n} w_i p_i \Leftrightarrow S\bar{p}_f = \bar{p} \Leftrightarrow \frac{1}{S\bar{p}_f} = \frac{1}{\bar{p}},$$

where $\bar{p}_f, \bar{p}$ are as in (1). The last form of the equation shows equality in purchasing powers of the currencies in units kg/SEK. If this does not hold, people begin to exchange the less powerful currency to the more powerful one until exchange rate balances the purchasing powers. However, because goods' prices are rigid, other explanations are needed for short-term movements in exchange rates.

Because the quality of goods is difficult to measure, the international trade of goods and services is assumed to be determined according to price competitiveness. We can thus write:

$$X = f(H_R),\ f'(H_R) > 0,\ f(0) = 0;\ M = g(H_R),\ g'(H_R) < 0,\ g(0) = 0.$$

The model for R becomes then:

$$R(H_R) = f(H_R) - g(H_R),\ R'(H_R) = f'(H_R) - g'(H_R) > 0,\ R(0) = 0. \quad (2)$$

From (2) we get: $\partial R/\partial \bar{p}_f > 0$, $\partial R/\partial \bar{p} < 0$, and $\partial R/\partial S > 0$. Devaluation of Swedish krona vis-à-vis to euro thus has a positive effect on the price competitiveness and also on the current account of Sweden.

3. Modeling Financial Account

International investments are assumed to be made according to the highest expected yield, and we study directly net investment flows. In continuous time, the uncovered interest rate parity equation is:

$$E_t[S(t_1)] = S(t)e^{\int_t^{t_1}(r(s)-r_f(s))ds}, \ t_1 > t,$$

see the Appendix. By r with unit $1/y$ is denoted the Swedish interest rate, by r_f with unit $1/y$ the interest rate of euro, and $E_t[S(t_1)]$ is investors' expectation at time moment t of the spot rate at time t_1. The net investment flow of euros to Sweden at time moment t then depends on quantity H_F :

$$H_F = S(t)e^{\int_t^{t_1}(r(s)-r_f(s))ds} - E_t[S(t_1)]. \tag{3}$$

If $H_F > 0$, investors on the average consider that investing in Sweden is more profitable than in the euro area and vice versa, see the Appendix. Thus H_F measures the international yield competitiveness of Swedish assets; the greater H_F is, the better the yield competitiveness of Swedish assets.

We denote that part of the financial account of Sweden, charged in euros, by $B(EUR/y)$, at time t:

$$B = h(H_F), \ h'(H_F) > 0, \ h(0) = 0.$$

If $B > 0$, a positive net investment flow of euros takes place to Sweden and vice versa. By taking account (3) the following results are obtained from (4): $\partial B/\partial S > 0, \partial B/\partial r > 0, \partial B/\partial r_f < 0,$ and $\partial B/\partial E_t[S(t_1)] < 0.$ The devaluation of Swedish krona vis-à-vis to euro and an increase in Swedish interest rate thus have a positive, and an increase in the interest rate of euro and an expectation of the future devaluation of Swedish krona have a negative effect on the financial account of Sweden.

4. Modeling Exchange Rate Dynamics

Daily changes in exchange rates occur due to people's daily willingness to exchange currencies. We do not know, however, whether these daily demands and supplies are caused by transactions in goods or asset markets. Due to this, both these reasons for exchanging currencies must be included in a theory attempted as a complete description for the determination of exchange rates. The balance of payments contains both these items, however. The part of the balance of payments of Sweden, charged in euros $N(EUR/y)$, can be formulated roughly by adding the corresponding current and financial accounts:

$$N = X - M + B = f(H_R) - g(H_R) + h(H_F). \qquad (5)$$

Now $S'(t)$ with unit $SEK/(EUR \times y)$ measures the instantaneous flow of the exchange rate; if $S'(t) > 0.$ Swedish krona is devaluing against euro and vice versa. The equation of motion for the exchange rate is:

$$S'(t) = \Phi_s(F_s),\ F_s = A_1(t) - (1-a)N(t),\ \Phi'(F_s) > 0,\ \Phi_s(0) = 0, \qquad (6)$$

where A_1 with unit EUR/y is Swedish central bank's net buying of euros by kronas at time unit *y*; if $A_1 > 0,$ Swedish central bank buys EUR with SEK and vice versa; $0 < a < 1$ is the share of the balance of payments of Sweden expressed in euros that is left on currency accounts in banks and is not exchanged to kronas when N is positive (from kronas to euros when N is negative). For simplicity, a is assumed to be a constant that reflects the payment routines of firms. According to Eq. (6), $S'(t) > 0$ if $F_s = A_1 - (1-a)N > 0,$ and vice versa. We name F_s as the "force" acting upon the exchange rate S. Eq. (6) gives a clear policy rule for the central bank if it aims to keep the exchange rate fixed: $S'(t) = 0$ if $F_s = 0 \Leftrightarrow A_1 = (1-a)N,$ Thus if $N > 0,$ the central bank must buy EUR by SEK for SEK not to revalue, and vice versa.

Because of the general functional forms in Eq. (6), we do not try to solve the equation but only study its economic implications. From (6) we get $\partial S'(t)/\partial S < 0,$ and if this relation is strong enough so that $S'(t)$ changes its sign with large changes in $S(t),$ the process is stable. Results $\partial S'(t)/\partial i < 0, i = r, \overline{p}_f,$ and $\partial S'(t)/\partial j > 0,\ j = r_f, \overline{p}, E_t[S(t_1)], A_1,$ show that increases in domestic prices increase the speed of devaluation of krona etc. Investors' expectations of future devaluation of Swedish krona vis-à-vis to euro increase its current speed of devaluation via international investments, and if Swedish central bank increases its buying of euros by kronas, this increases the speed of devaluation of krona.

5. Equilibrium Exchange Rate

The equilibrium exchange rate S^* is obtained from Eq. (6) by setting $F_s = 0 \Leftrightarrow S'(t) = 0$. This gives:

$$A_1(t) = (1-a)[R(H_R) + h(H_F)]. \qquad (7)$$

From (7) we get by implicit differentiation: $\partial S^* / \partial i < 0, i = r, \bar{p}_f$, and $\partial S^* / \partial j > 0,\ j = r_f, \bar{p}, A_1, E_t[S(t_1)]$. These results support the common assumptions of factors affecting exchange rates, and $\partial S^* / \partial E_t[S(t_1)] > 0$ shows that expectations fulfill themselves, that is, expected future devaluation has an immediate devaluing effect on krona. To get a preliminary formula for the equilibrium exchange rate, we assume all behavioral functions in (6) linear and passing through the origin. Eq. (6) takes then the form:

$$m_s S'(t) =$$

$$= A_1(t) - (1-a)\left[\alpha(S(t)\bar{p}_f - \bar{p}) + \beta\left(S(t)e^{\int_t^{t_1}(r(s)-r_f(s))ds} - E_t[S(t_1)]\right)\right], \qquad (8)$$

where $R(H_R) = \alpha H_R, h(H_F) = \beta H_F$ and $\alpha, \beta > 0$ are constants with units $(kg \times EUR)/(SEK \times y)$ and $EUR^2 / SEK \times y)$, respectively. Constant $m_s > 0$ with unit EUR^2 / SEK has the same role as inertial mass in Newtonian mechanics, $m_s = F_s / S'(t), F_s = A_1 - (1-a)N$. The greater m_s is, the less sensitive $S(t)$ is to changes in F_S. Parameters α, β measure consumers' and investors' sensitivity to H_R, H_F, respectively.

The equilibrium exchange rate S^* is obtained from (8) as follows:

$$S'(t) = 0 \Leftrightarrow S^* = \frac{A_1(t) + (1-a)(\alpha\bar{p} + \beta E_t[S(t_1)])}{(1-a)\left[\alpha\bar{p}_f + \beta e^{\int_t^{t_1}(r(s)-r_f(s))ds}\right]},$$

Because S^* is a time-dependent random variable, this explains the vulnerability of exchange rates as compared with goods' prices. Eq. (8) can be presented in Error-Correction form, too, as:

$$S'(t) = -z[S(t) - S^*],\ z = \frac{(1-a)}{m_s}\left(\alpha\bar{p}_f + \beta e^{\int_t^{t_1}(r(s)-r_f(s))ds}\right) > 0. \qquad (9)$$

where z is the Error-Correction "coefficient". According to (9), $S'(t) > 0$ if $S(t) < S^*$, and vice versa.

6. Dynamics of Interest Rate

In section 5 the exchange adjustment was monotonous, but we have evidence that exchange rates may adjust in an overshooting manner too. To model this kind of behavior, we assume the time unit short enough for the price level to stay fixed (i.e. less than a month). In short-term, the adjusting variables in money and currency markets are the exchange and the interest rates while goods' prices are more rigid.

Domestic interest rate is assumed to adjust in the money market according to the excess demand of Swedish kronas. A stable demand function for Swedish kronas − independent of international trade − is assumed and called the "domestic" component of the demand. This depends on the Swedish nominal GDP (the transaction motive) denoted by $Y(t) = \bar{p}(t)Q(SEK/y,$ where $Q(kg/y)$ is the real GDP valued by the average price level $\bar{p}(SEK/kg)$. For simplicity, Q is assumed constant. The speculative component of domestic demand of Swedish kronas negatively depends on interest rate due to the alternative cost for liquid wealth. The international demand component of Swedish kronas depends on the surplus in the balance of payments of Sweden of which N is a part. The total balance of payments of Sweden is denoted as $N_T = N + N_R$ where N_R is the rest of the balance of payments charged in other currencies than euro (N_R is though expressed in euros with unit EUR/y). For simplicity, N_R is assumed constant.

The demand of Swedish kronas M_D with unit SEK at time moment t is then:

$$M_D(t) = \left[L\big(Y(t), r(t)\big) + (1-a)S(t)[N(t) + N_R]\right]\Delta t, \ \frac{\partial L}{\partial Y} > 0, \ \frac{\partial L}{\partial r} < 0,$$

where $L(Y(t), r(t))$ is the domestic component of the demand, $\Delta t = y$ the length of the time unit that transforms the measurement units to those required, and constant $0 < \alpha < 1$ was defined earlier. The amount of Swedish kronas circulating in the economy at moment t, $M_s(t)$ with unit SEK, is then:

$$M_s(t) = M_s(t-y) + A_2(t) + S(t)A_1(t)\Delta t,$$

where $M_s(t-y)$ is the money in circulation at time $t-y,$ and $A_2(SEK)$ the extra supply of Swedish kronas by Swedish Central bank during time unit y. If $A_1, A_2 > 0,$ Swedish central bank increases its supply of Swedish kronas in the market, and vice versa. The excess demand of kronas at time t is then:

$$M_D - M_S =$$
$$= [L(Y(t), r(t)) + ((1-a)[N(t) + N_R] - A_1(t))S(t)]\Delta t - M_S(t-y) - A_2(t).$$

If $N < 0,$ people buy euros by kronas more than they do the opposite, and then $(1-\alpha)\ SN\Delta t < 0,$ is a part of supply of kronas. However, if $A_1 < 0,$ the term $-A_1 S\Delta t > 0$ is a part of the demand of kronas by euros.

The money market in Sweden is dynamized as follows:

$$r'(t) = \Phi_r(F_r), \quad \Phi'_r(F_r) > 0, \quad \Phi_r(0) = 0, \quad F_r = M_D - M_S, \quad (10)$$

where F_R is the "force" acting upon the interest rate. This force contains central bank's policy variables A_1, A_2; setting $A_1 = (1-a)[N + N_R],$ $A_2 = L(Y,r)\ \Delta t - M_S(t-y),$ the central bank can eliminate the force.

By taking account (5) we get from (10): $\partial r'(t)/\partial i > 0, i = Q, \overline{p}_f$, and $\partial r'(t)/\partial j < 0,\ j = A_1, A_2, M_S(t-y), r_f, E_t[S(t_1)],$ while the signs of $\partial r'(t)\partial k,\ k = r, S, \overline{p},$ are ambiguous. To save space we analyze only the ambiguous results. An increase in $\overline{p}$ may increase or decrease $r'(t)$. The domestic demand component causes a positive effect (increasing $\overline{p}$ raises the need of liquid funds), and the international demand component causes a negative effect (increasing $\overline{p}$ reduces the current account). An increase in r has a negative effect on the domestic demand thus decreasing $r'(t),$ and a positive effect on the international demand. An increase in S improves the balance of payments of Sweden thus negatively affecting $r'(t)$. However, only if $(1-a)[N + N_R] - A_1 > 0,$ we can uniquely sign the partial as $\partial r'(t)/\partial S > 0.$

The equilibrium state in the money market, $F_r = 0 \Leftrightarrow r'(t) = 0,$ gives:

$$[L(Y(t), r(t)) + (1-a)S(t)[N(t) + N_R] - A_1(t)S(t)]\Delta t + A_2(t) = M_S(t-y). \quad (11)$$

From (11) we can derive results concerning the equilibrium interest rate r^*. All these partials turn out to be of ambiguous sign, however, and so we do not present them.

7. Dynamics of Money and Currency Markets

Due to the general functional forms in Eq. (6) and (10) we study the system by phase diagrams, see the Appendix. From (7) and (11) we get the slopes of curves $S'(t) = 0$ and $r'(t) = 0$ in coordinates (r, S) as:

$$\left.\frac{\partial S^*}{\partial r}\right|_{S'(t)=0} = -\frac{h'(H_F)\dfrac{\partial H_F}{\partial r}}{R'(H_R)\dfrac{\partial H_R}{\partial S} + h'(H_F)\dfrac{\partial H_F}{\partial S}} < 0,$$

$$\left.\frac{\partial r^*}{\partial S}\right|_{r'(t)=0} = \frac{A_1 - (1-a)\left[N + N_R + S\dfrac{\partial N}{\partial S}\right]}{\dfrac{\partial L}{\partial r} + (1-a)S\dfrac{\partial N}{\partial r}}.$$

The attracting or repelling character of the demarcation lines depends on the signs of the partials:

$$\frac{\partial S'(t)}{\partial S} = -\Phi'_S\,(F_S)(1-a)\left[R'(H_R)\frac{\partial H_R}{\partial S} + h'(H_F)\frac{\partial H_F}{\partial S}\right] < 0,$$

$$\frac{\partial r'(t)}{\partial r} = \Phi'_r\,(F_r)\left[\frac{\partial L}{\partial r} + (1-a)S\frac{\partial N}{\partial r}\right] = \Delta t.$$

Line $S'(t) = 0$ is thus an attractor in the direction of S, and line $r'(t) = 0$ is an attractor in the direction of r if $\partial L/\partial r + (1-a)\partial N/\partial r < 0$, and vice versa. Now, we get six cases depending on the sign of $\partial r'(t)\partial r$ and whether the slope of curve $r'(t) = 0$ is positive or negative, or steeper than that of curve $S'(t) = 0$ in coordinates (r, S). The non-plausible cases of exactly horizontal and vertical slopes are omitted. If the central bank does not overreact when it tries to stabilize the balance of payments, then $|A_1| \le |(1-a)[N + N_R]|$. Assuming this we see that $\partial r^*/\partial S > 0$ if $N + N_R < 0$ and $\partial N/\partial S, |\partial L/\partial r|$ are small, and $\partial N/\partial r$ is large. However, $\partial r^*/\partial S > 0$ holds also if $N + N_R > 0$ and $|\partial L/\partial r| > (1-a)\,S\,\partial N/\partial r$.

The system is stable in Figures 1-3 and unstable in Figures 4-6; The adjustment is cyclic in Figures 1, 2 and monotonous in Figure 3 (the initial points are assumed to be on the demarcation lines). Overshooting takes place in Figures 1, 2, but contrary to Dornbush's (1976) and Lyons' (1990) models, here overshooting takes place in stable cases. We thus do not have to make the strong assumptions of these authors that investors in the market rightly guess the location of the saddle-path and that the investors operate together so that the exchange rate exactly jumps on the saddle-path. Such information requirements question the reliability of these models in explaining real world behavior.

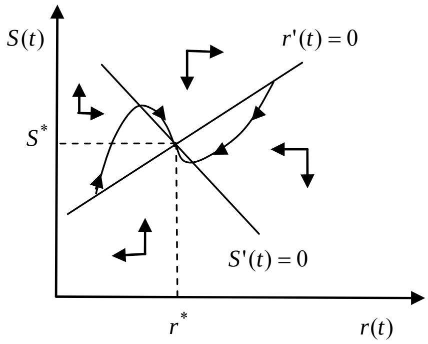


Figure 1.

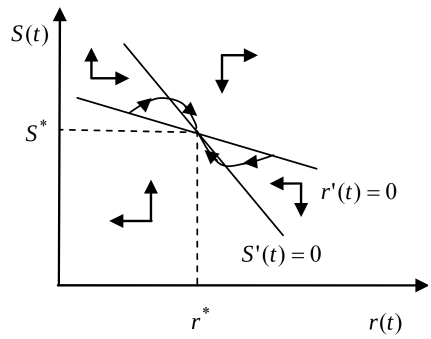


Figure 2.

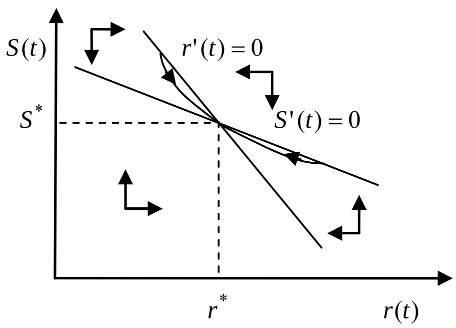
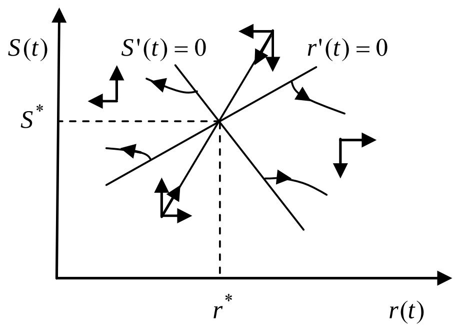


Figure 3.



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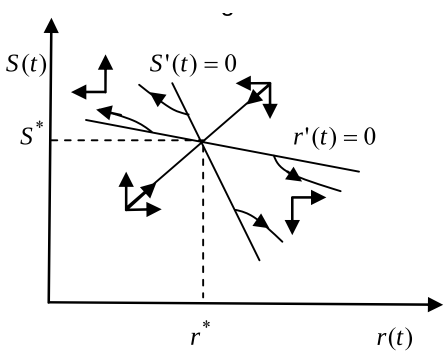

Figure 5.

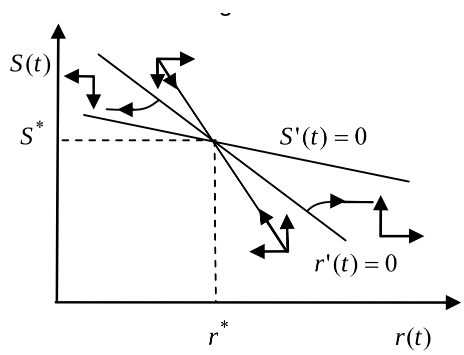

Figure 6.

The proposed model raises also another critical comment concerning the strong form of rational expectations hypothesis in economics. Suppose (6) is the true model for exchange rate behavior. Then none of the agents in the market has enough information to accurately forecast the exchange rate because the model contains unspecified functions. Thus we should not use an exact theory in modeling expectations unless we know that the theory is the true one and all agents are aware of this.

In the model, any of the quantities $A_1, A_2, Q, E_t[S(t_1)]$, $M_S(t-y), r_f, \overline{p}, \overline{p}_f$ may cause an impulse in the system, and S, r either converge into their equilibrium values (Figures 1-3, 7-8), or diverge away from them (Figures 4-6, 9).

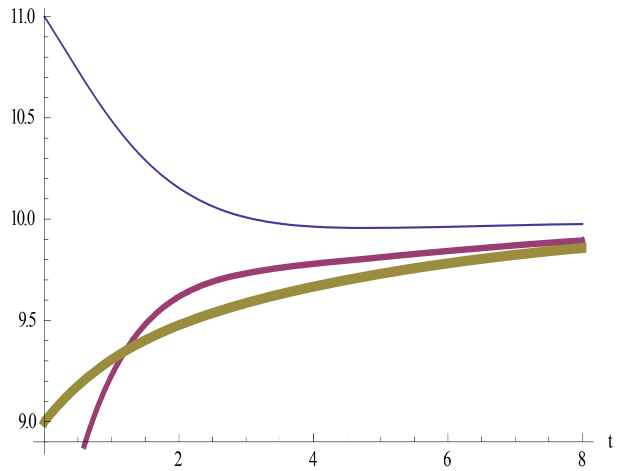

Figure 7.

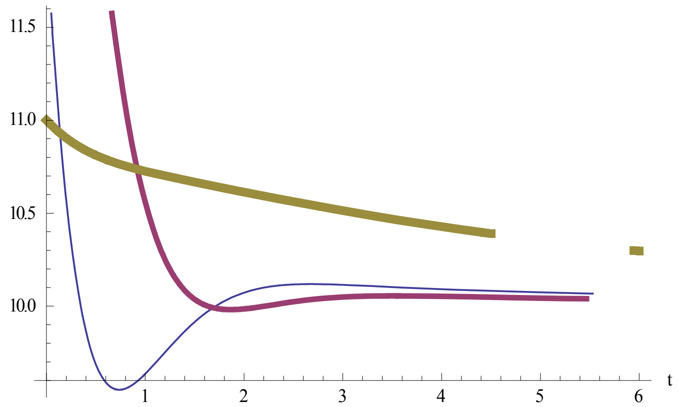

Figure 8.

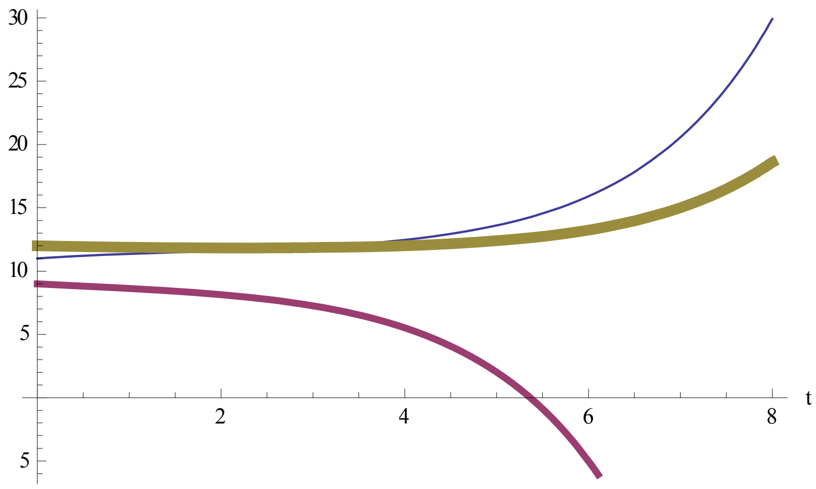

Figure 9.

This critically depends on quantities: $\partial N / \partial S$, $\partial N / \partial r$, and $\partial L / \partial r$. Unstable cases are examples of financial panics. Suppose the domestic interest rate decreases due to some reason. Such nominal reasons could be an expected future devaluation of krona $(\Delta E_t[S(t_1)] > 0)$, or that the central bank increases its supply of kronas $(\Delta A_i > 0, i = 1,2)$. A real reason could be a decrease in Swedish real GDP $(\Delta Q < 0)$. A decrease in r diminishes the financial account of Sweden which decreases the international demand of kronas thus further decreasing r. Decreasing financial account has a devaluing effect on krona, which may stabilize the balance of payments. However, if the devaluation is not strong enough to turn the balance of payments positive, decreasing international demand of kronas further decreases r. The process continues with decreasing r, balance of payments deficit, and the devaluation of krona vis-à-vis to euro. For the same reasons, we may also observe increasing r, surplus in balance of payments and a continuous revaluation of krona if the revaluation does not stabilize the balance of payments.

8. Dynamics of Price Level

To study the adjustment of the whole financial system, we introduce a model for the average price level. We set $\overline{p}$ to adjust according to the excess demand in the goods' market. The domestic demand of goods depends on real consumption $C(Q)$, real investment $I(Q,r)$ and real public demand G; these all have unit kg/y. The international demand of Swedish goods consists of real exports $S \times X/\overline{p} = S \times f(H_R)/\overline{p}$ with unit kg/y. The supply in the goods' market consists of domestic real GDP $Q(kg/y)$ and real imports $M/\overline{p}_f = g(H_R)/\overline{p}_f$ with unit kg/y. The equation of motion for $\overline{p}$ is then:

$$\overline{p}'(t) = \Phi_{\overline{p}}(F_{\overline{p}}),\ F_{\overline{p}} = C(Q) + l(Q, r(t)) + G + \frac{S(t)f(H_R)}{\overline{p}(t)} - \frac{g(H_R)}{\overline{p}_f} - Q,$$

$$\Phi'_p(F_{\overline{p}}) > 0,\ \Phi_{\overline{p}}(0) = 0. \tag{12}$$

The excess demand in the goods' market $F_{\overline{p}}$ is the "force acting upon $\overline{p}$". Eq. (12) is the macro-level analogue for the law of demand and supply for a single good in Samuelson (1941). From (12) we get: $\partial\overline{p}'(t)/\partial i > 0$, $i = S, \overline{p}_f, G$, $\partial\overline{p}'(t)/\partial j < 0$, $j = \overline{p}, r$, and $\partial\overline{p}'(t)/\partial Q > 0$ if $C'(Q) + \partial l/\partial Q > 1$, and vice versa. The equilibrium average price level $\overline{p}^*$ is obtained from (12) by setting $F_{\overline{p}} = 0 \Leftrightarrow \overline{p}'(t) = 0$. From the resulting equation we get: $\partial\overline{p}^*/\partial i > 0, i = S, \overline{p}_f, G$; $\partial\overline{p}^*/\partial r < 0$, while $\partial\overline{p}^*/\partial Q$ is of ambiguous sign.

9. Simulations with the Model

In order to study the whole model, we take the first order Taylor series approximation of the system in the neighborhood of the equilibrium point $(S^*, r^*, \overline{p}^*)$. This assumption is supported by Meese & Rose (1991) who report no evidence of nonlinearities in exchange rate models. We introduce three nonnegative inertial factors: $m_s = 1/\Phi'_s(F_s)$ with unit EUR^2/SEK (already discussed in the context of Eq. (6)), $m_r = 1/\Phi'_r(F_r)$ with unit $SEK \times y^2$, and $m_{\overline{p}} = 1/\Phi'_p(F_{\overline{p}})$ with unit kg^2/SEK. For example, $m_{\overline{p}}$ measures the sensitivity of $\overline{p}'(t)$ with respect to the force $F_{\overline{p}}$, similarly as mass measures inertia in Newton's equation $F = ma$. This way we can study the role of inertia of the adjusting quantities for the dynamics. Now $F_s, F_r, F_{\overline{p}}$ are functions of $Z^*, Z^* = (S^*, r^*, \overline{p}^*, Z_0)$, where Z_0 is a fixed vector of the exogenous quantities. Because $F_s(Z^*) = F_r(Z^*) =$ $= F_{\overline{p}}(Z^*) = 0$, the linearized form of the model takes the form:

$$\begin{pmatrix} m_s S'(t) \\ m_r r'(t) \\ m_{\bar{p}} \bar{p}'(t) \end{pmatrix} = \begin{pmatrix} \frac{\partial F_s}{\partial S}(Z^*) & \frac{\partial F_s}{\partial r}(Z^*) & \frac{\partial F_s}{\partial \bar{p}}(Z^*) \\ \frac{\partial F_r}{\partial S}(Z^*) & \frac{\partial F_r}{\partial r}(Z^*) & \frac{\partial F_r}{\partial \bar{p}}(Z^*) \\ \frac{\partial F_{\bar{p}}}{\partial S}(Z^*) & \frac{\partial F_{\bar{p}}}{\partial r}(Z^*) & \frac{\partial F_{\bar{p}}}{\partial \bar{p}}(Z^*) \end{pmatrix} \begin{pmatrix} S(t) - S^* \\ r(t) - r^* \\ \bar{p}(t) - \bar{p}^* \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{pmatrix}$$

where $\varepsilon_i, i = 1,2,3$ are constants the numerical values of which depend on the degree of nonlinearity of functions $\Phi_i, i = S, r, \bar{p},$ and on the values of the exogenous variables.

For the simulations we set $\varepsilon_i = 0, i = 1,2,3$ and $S^* = r^* = \bar{p}^* = 10.$ Interest rate is thus expressed in per cent, average price level in proper units and $10(SEK / EUR)$ is close to the prevailing rate. For $\partial F_s / \partial S$ etc. we set values the signs of which match the results derived, and $m_{\bar{p}}$ is assumed to be greater than m_s, m_r; producers and consumers are thus assumed to be more rigid than investors. The results with certain initial conditions are shown in Figures 7-9; the thinnest curve represents $S(t)$ and the thickest $\bar{p}(t)$. In all cases, $m_s = m_r = 1,$ $m_{\bar{p}} = 3,$ and the other parameters are as follows. **Figure 7:** $\partial F_s / \partial S = -1,$ $\partial F_s / \partial r = -0.5,$ $\partial F_s / \partial \bar{p} = 0.5;$ $\partial F_r / \partial S = 0.5, \partial F_r \partial r = -1,$ $\partial F_r / \partial \bar{p} = 0.5;$ $\partial F_{\bar{p}} / \partial S = 0.5, \partial F_{\bar{p}} / \partial r = -0.5;$ $\partial F_{\bar{p}} / \partial \bar{p} = -1,$ **Figure 8:** $\partial F_s / \partial S = -2;$ $\partial F_S / \partial r = -1,$ $\partial F_S / \partial \bar{p} = 0.5,$ $\partial F_r / \partial S = 2,$ $\partial F_r / \partial r = -2,$ $\partial F_r / \partial \bar{p} = -0.2,$ $\partial F_{\bar{p}} / \partial S = -0.3;$ $\partial F_{\bar{p}} / \partial r = -0.3,$ $\partial F_{\bar{p}} / \partial \bar{p} = -1,$ **Figure9:** $\partial F_S / \partial S = -1,$ $\partial F_S / \partial r = -0.5,$ $\partial F_S / \partial \bar{p} = 0.5;$ $\partial F_r / \partial S = 1, \partial F_r / \partial r = 1,$ $\partial F_r / \partial \bar{p} = -0.2;$ $\partial F_{\bar{p}} / \partial S = 0.5,$ $\partial F_{\bar{p}} / \partial r = -0.5,$ $\partial F_{\bar{p}} / \partial \bar{p} = -1.$

In Figure 7, the adjustment is smooth while in Figure 8 the exchange and the interest rate overshoot their equilibrium values. Figure 9 shows an unstable case where the money and the currency market collapse: krona devalues, interest rate decreases, and hyperinflation takes place in goods' market.

10. Conclusions

We modeled the exchange rate of the currency of a small country on the basis of the international competitiveness of the country in terms of relative prices and expected yields in investments. The model produces the common forecasts how various macro variables affect the exchange rate, it explains the mean-reverting behavior of exchange rates and also their vulnerability: the equilibrium exchange rate was shown to be a random variable that depends on investors' expectations of its future value. In order to model the observed nonlinear behavior of exchange rates, an equation for interest rate was added in the model. By the dynamic model of two equations, overshooting and currency crises were shown to occur in certain cases. Adding an equation for price dynamics in the model, the simultaneous adjustment of the three quantities was shown to be either monotonous or cyclic, and either stable or non-stable.

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Appendix

Let $x(t)$ be kronas to be invested in Sweden and $z(t)$ euros to be invested in a euro country at time *t*. Continuous time interest rates r, r_f in the two countries are defined as growth rates of the investments,

$$\frac{x'(t)}{x(t)} = r(t), \qquad \frac{z'(t)}{z(t)} r_f(t).$$

The left hand sides have unit $1/y(SEK/y$ divided by SEK and EUR/y divided by $EUR)$ and so have the right hand sides. Solving these equations gives:

$$x(t_1) = x(t)e^{\int_t^{t_1} r(s)ds}, \quad z(t_1) = z(t)e^{\int_t^{t_1} r_f(s)ds}, \quad t \le s \le t_1.$$

where $x(t_1)$ are the kronas and $z(t_1)$ the euros to be obtained at time t_1. Let $S(t)$ (SEK/EUR) be the spot rate at time *t* and $E_t[S(t_1)]$ investors' expectation at time *t* of the spot rate at time t_1. Investing in Sweden is expected to be more profitable if $E_t[(t_1)]z(t_1) < x(t_1)$ holds with $x(t) = S(t)z(t)$ (equal amounts are invested and more kronas are expected from Sweden). Money is thus flowing into Sweden, if:

$$\frac{x(t_1)}{z(t_1)} > E_t[S(t_1)] \Leftrightarrow \frac{S(t)z(t)e^{\int_t^{t_1} r(s)ds}}{z(t)e^{\int_t^{t_1} r_f(s)ds}} > E_t[S(t_1)] \Leftrightarrow$$

$$\Leftrightarrow S(t)e^{\int_t^{t_1} (r(s) - r_f(s)ds)} - E_t[S(t_1)] > 0.$$