AN EFFICIENT SOFT COMPUTING FOR A NEW SIMULATION APPROACH ON SEVERAL MODELS OF OPTIMAL STOCK MANAGEMENT IN THE DETERMINISTIC CASE

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Abstract. *The work is a generalization of three* ***Wilson's models*** *related with the gestion (management) of stocks in the deterministic case. A new variable r, called stock rate or* ***simulation rate****, appears* ***in all*** *these generalized models. This variable assures a better stock controlling by* ***deterministic simulation method****. For r = 1/2 we obtain all Wilson's results from the bibliography [4], [5]. For each economical model* ***the mathematical foundation*** *is given, together with several* ***numerical applications*** *and economical interpretations.*
The work contains: 3 economic models, 3 mathematical models, and 3 C++ programs.

Keywords: *Stock theory, optimal stock management, simulation rate, Wilson models, simulation models, C++ computer programs.*

1. Introduction

The work is a generalization of three **Wilson's models** related with stocks management in the deterministic case. A new variable *r*, called stock rate or **simulation rate**, appears **in all** these generalized models. This variable assures a better stock controlling by **deterministic simulation method**. For $r = 1/2$ we obtain all Wilson's results from the bibliography [4], [5]. For each economical model **the mathematical foundation** is given, together with several **numerical applications** and economical interpretations.

The work contains: 3 economic models
3 mathematical models
3 C++ programs
Each mathematical model generates **1) an informatics model** and **2) a C++ program**.

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So, the work contains three C++ valid programs:

a) source codification,

b) numerical output results and

c) print screen.

The C++ simulation programs have been validated by **supplementary techniques and independent computations**.

By simulation with various values of r a good manager has the possibility to choose **the best version** of his activity.

All models are based on **the same notations** with their specific meanings. So we used the following notations:

N = the total number of supplies.

h = the period length (the number of days, let us say) between two supplies; h is **constant**.

c_L = the launch cost for one demand of supply (the ordering cost per one order).

c_H = the holding cost (in warehouse) unit cost, per day.

c_P = the penalty cost per item, per day (when the stock s is less then the client's demand).

W = the production rate of the factory, on the unit time (the model 3).

D = the client's demand rate on the unit time (the model 3).

From time in time the manager has to stop the production activity, otherwise the whole quantity WT is two big, i.e. $WT >> Q$.

t_W = the working time (the production time) for the factory or Industrial unit.

t_S = the stop time (the factory doesn't work).

t_{WP} = the working time and the penalty time for the factory because the client's demands are not satisfied.

t_P = the penalty time and stop time.

Remark: Generally there exists the inequalities $c_L >> c_P > c_H > 0$, where $a>>b$ means a is much greater than b. When the supply order of magnitude q was done, the stock is augmented quite instantaneous.

Remark: Between the described elements there are some important relations i.e.

$Q = Nq,\ T = Nh$

$C_r(q)$ = the total cost (composed of the ordering and holding cost) for interval $[0,T]$, in model 1.

$C_r(q,s)$ = the total cost (composed of the ordering, holding and penalty cost) for interval $[0,T]$, in model 2.

$C_r(t_S,t_P)$ = the total cost (composed of the production cost and ordering, holding and penalty cost) for the interval $[0,T]$, in model 3.

In the above description the elements T,Q,W,D,c_L,c_H,c_P and r are **input data**.

The elements $q,s,N,h,t_S,t_P,t_W,t_{WP}$ are **unknown data** (positive real numbers). They must be found by using the mathematical models for the maintenance stock problem.

MODEL 1. The absence of Ware in the Warehouse is not allowed.

MODEL 2. The absence of Ware in the Warehouse is allowed.

MODEL 3. Production, Holding, Stop Production and Satisfy Client's Demands .

The aim of the stock theory is to determine the best values q^* (model 1), q^*, s^* (model 2), t_S^*, t_P^* (model 3) which minimize **the total maintenance costs**, respectively:

$$C_r(q^*) = \min_{q\in(0,\infty)} C_r(q)$$

$$C_r(q^*,s^*) = \min_{q\in(0,\infty),\ s\in(0,\infty)} C_r(q,s)$$

$$C_r(t_S^*,t_P^*) = \min_{t_S\in(0,\infty),t_P\in(0,\infty)} C_r(t_S,t_P)\,.$$

2. The Mathematical Model 1

For one period the cost is $c_L + rqc_H h$ and for N periods of length h the total cost is:

$$C_r(q) = c_L\frac{Q}{q} + rc_H Tq^2,\quad C_r'(q) = 0,\ \ q = \sqrt{\frac{c_L Q}{rTc_H}}\Bigg)$$

$$q^* = \sqrt{\frac{c_L Q}{rTc_H}},\ \ N^* = \frac{Q}{q^*},\ \ h^* = \frac{T}{N^*}\Bigg).$$

The total ordering and holding cost for interval $[0,T]$, in model 1 is:

$$C_r(q^*) = 2\sqrt{r\,c_L c_H QT}\,.$$

Because $\dfrac{\partial C_r(q^*)}{\partial r} = \dfrac{\sqrt{c_L c_H QT}}{\sqrt{r}} > 0$ then the function $C_r(q^*)$ is increasing with *r*.

3. The Mathematical Model 2

For one period *h* (for one supply) the cost is $c_L + rsc_H h_1 + (1-r)(q-s)c_P h_2$ and for *N* periods of constant length *h* the total cost is $[c_L + rsc_H h_1 + (1-r)(q-s)c_P h_2]N$. We denote it by:

$$C_r(q,s) = [c_L + rsc_H h_1 + (1-r)(q-s)c_P h_2]N. \quad (8)$$

If we take into account the relations between variables the total cost is:

$$Nq = Q,\ \ Nh = T,\ \ \frac{h_1}{h} = \frac{s}{q},\ \ \frac{h_2}{h} = \frac{q-s}{q},\ \ h = h_1 + h_2,$$

the relation (8) becomes (9):

$$C_r(q,s) = c_L \frac{Q}{q} + r\,c_H T\frac{s^2}{q} + (1-r)\,c_P T\frac{(q-s)^2}{q}.$$

The extreme points for the function $C_r(q,s)$ in *q* and *s* are given by:

$$\frac{\partial C_r(q,s)}{\partial q} = 0,\ \ \frac{\partial C_r(q,s)}{\partial s} = 0,$$

and we obtain successively:

$$s = \frac{(1-r)c_P}{rc_H + (1-r)c_P}q;$$

Notation: $\rho(r) = \dfrac{(1-r)c_P}{rc_H + (1-r)c_P},\ \ 0 < \rho(r) < 1,\ \ \rho(1) = 0$

$$q^* = \sqrt{\frac{c_L}{rc_H}\frac{Q}{T}\frac{1}{\rho(r)}},\ \ s^* = \sqrt{\frac{c_L}{rc_H}\frac{Q}{T}\rho(r)}$$

$$C_r(q^*,s^*) = 2\sqrt{r\rho(r)c_L c_H QT} \text{ and } N^* = \frac{Q}{q^*},\ h^* = \frac{T}{N^*}.$$

Remark. For $r = \frac{1}{2}$ we obtain all the results from Wilson's model, with $\rho\left(\frac{1}{2}\right) = \frac{c_P}{c_H + c_P}$.

4. The Mathematical Model 3

The total cost per unit time is denoted C_r where:

$$C_r = \frac{c_L + c_H rs(t_W + t_S) + c_P(1-r)(q-s)(t_P + t_{WP})}{t_W + t_S + t_P + t_{WP}}. \qquad (*)$$

Between the variables $q, s, t_W, t_S, t_P, t_{WP}$ and the constant elements T, W, D there are a lot of relations.

We suppose that the client's demand D satisfies the assumption $D = Q/T$.

Now we have to decide what variables are the main variables in formula (*).

Related with W and D there are three cases: a) $W - D < 0$ impossible problem; b) $W - D = 0$ uninterested problem; c) $W - D > 0$ interested problem.

Version 1: We keep the variables t_W, t_S and eliminate t_P, t_{WP}. The method doesn't work.

Version 2: We keep the variables t_S, t_P and eliminate t_W, t_{WP} together with q and s. The method works and we obtain successively:

$$s = Dt_S,\ q - s = Dt_P,\ t_W = \frac{Dt_S}{W-D},\ t_{WP} = \frac{Dt_P}{W-D},\ W - D > 0$$

$$C_r(t_S, t_P) = \frac{c_L(W-D) + DW[c_H r t_S^2 + c_P(1-r)t_P^2]}{W(t_S + t_P)}.$$

The main variables are t_S, t_P.

Remark: For $r = 1/2$ in (22) we obtain the Wilson's result from references.

Notation:

$$\rho(r) = \frac{(1-r)c_P}{r c_H + (1-r)c_P},\ 0 < \rho(r) < 1,\ \rho(1) = 0,\ \rho\left(\frac{1}{2}\right) = \frac{c_P}{c_H + c_P}$$

$$t_P = \sqrt{\frac{W-D}{DW}\frac{c_L c_H}{c_P^2}\frac{r}{(1-r)^2}\rho(r)} = t_P^*,\ t_S = \sqrt{\frac{W-D}{DW}\frac{c_L}{c_H}\frac{\rho(r)}{r}} = t_S^*$$

$$t_W = \sqrt{\frac{D}{W(W-D)}\frac{c_L}{c_H}\frac{\rho(r)}{r}} = t_W^*,\ t_{WP} = \sqrt{\frac{D}{W(W-D)}\frac{c_L c_H}{c_P^2}\frac{r}{(1-r)^2}\rho(r)} = t_{WP}^*$$

$$s = \sqrt{\frac{D(W-D)}{W}\frac{c_L}{c_H}\frac{\rho(r)}{r}} = s^*,$$

$$q = \sqrt{\frac{D(W-D)}{W}c_L c_H \rho(r)}\left(\frac{1}{c_H}\sqrt{\frac{1}{r}} + \frac{1}{c_P}\sqrt{\frac{r}{(1-r)^2}}\right).$$

Now we compute the total cost C_r for the best main values t_S^*, t_P^* and obtain successively:

$$C_r(t_S^*, t_P^*) = \frac{2c_L(W-D)}{W(t_S^* + t_P^*)},\ C_r(t_S^*, t_P^*) = 2\sqrt{\frac{D(W-D)}{W}c_L c_H r\rho(r)}.$$

5. The importance of holding rate $r \in (0,1)$

Compute the partial derivative $\frac{\partial C_r(t_S^*, t_P^*)}{\partial r}$ and put the condition $\frac{\partial C_r(t_S^*, t_P^*)}{\partial r} = 0$. We obtain the algebraic equation in variable *r:*

$$(c_P - c_H)r^2 - 2c_P r + c_P = 0,\ c_P - c_S > 0$$

$$\Delta = 4c_P c_H > 0,\ r_1 = \frac{c_P + \sqrt{c_P c_H}}{c_P - c_H} > 1,\ 0 < r_2 = \frac{c_P - \sqrt{c_P c_H}}{c_P - c_H} < 1.$$

For $r \in (0, r_2]$ the function $C_r(t_S^*, t_P^*)$ is increasing function; for $r \in (r_2, 1)$ the function $C_r(t_S^*, t_P^*)$ is decreasing function. Generally the stock manager has to choose the value $r \in (0, r_2]$.

Remark: For the manager it is very important to know the best value of working time t_W^* (formula (28)) and the best value of stop time t_S^* (formula (27)).

We denote:

$t_W + t_{WP}$ = the work cycle; $t_S + t_P$ = the stop work time;

$\frac{T}{t_W + t_{WP} + t_S + t_P}$ = the total number of work-stop cycles over the period $[0, T]$. Generally this is a real number, but the manager can take the integer part.

6. Validation of C++ program by independent computations

We can verify the correctitude of numerical computations by the following tests:

$$t_W^*(W - D) = s^*; \ \ (t_W^* + t_{WP}^*)(W - D) = q^*.$$

Finally we denote $t_W + t_{WP}$ = the work cycle.

7. The Informatics Models. C++ Codification and Program Results

Now we write a C++ program for each mode 1, model 2 and mode 3.

For model 1 we use the input data T, Q, c_L, c_H, r and the formulas (6) and (7).

For model 2 we use input data T, Q, c_L, c_H, c_P, r and the formulas (11), (12), (13) and (17).

For model 3 we use input data $T, Q, W, c_L, c_H, c_P, r$ and the formulas (28), (27), (26), (29), (30), (31) and (39).

The numerical simulation is done by the values of variable r hold in vector rn[21]. The dimension 21 could be changed by the user. In C++ program the identifiers of variables look like in the theory, with small modifications. For example:

c_L = cL, c_H = cH, c_P = cP, q^* = qs, s^* = ss etc.

All the input data are introduced by computer keyboard. The program asks for input data step by step.

8. The correspondence between theoretical notations and C++ notations and meanings

Theory	C++	Theory	C++
T	T	W	W
Q	Q	D	D
q^*	qs	t_W^*	tws
s^*	ss	t_S^*	tss
N^*	Ns	t_{WP}^*	twps
$[N^*]$	Nsi	t_P^*	tps
h^*	hs	$C_r(q^*)$	Crs
$[h^*]$	hsi	$C_r(q^*, s^*)$	Crs
c_L	cL	$C_r(t_S^*, t_P^*)$	Crs
c_H	cH	r	r
c_P	cP	$\rho(r)$	ro

Theory	C++
$t_W + t_{WP} + t_S + t_P$	sumti
$t_W + t_{WP}$	work cycle
$t_S + t_P$	stop cycle
$r_1, r_2, \cdots, r_n$	rn[21], n < = 20
q_1^* (first supply)	qs1
$q_{N^*}^*$ (last supply)	qsL
Q	Qcontrol
Q	Qcontroli (integer)
$\frac{T}{t_W^* + t_{WP}^* + t_S^* + t_P^*}$	Nworkstopcycle.

9. C++ Programs

C++ program for Model 1
C++ program for Model 2
C++ program for Model 3.

10. Applications for model 1

$Q = 1800\,000$ Kg (of flour, let us say, in a bred factory).

T = one month; $T = 30$ days.

$c_L = 3000$ units (of money).

$c_H = 1$ unit / 600 Kg / day, $c_H = 0.00166666$

$r = \frac{1}{4}$; $r = \frac{1}{3}$; $r = \frac{1}{2}$ (3 cases for r).

Find the optimal values $q^*, N^*, h^*, C_r(q^*)$ and draw a supply plan.

Solution. We arrange the results in the table 1. The value of stock rate r is at the manager's disposal. The numerical results shows the best version.

Table 1

	$r = \frac{1}{4}$	$r = \frac{1}{3}$	$r = \frac{1}{2}$
q^*	657 267	569 209	464 757
N^*	2.738	3.16	3.87
$\overline{N}^*$	3	3	4
h^*	10.95	9.419	7.75
$\overline{h}^*$	10	10	7.5
$C_r(q^*)$	16432	18974	23274

The supply plan is:

$q_1^* = 657\,267$	$q_1^* = 569\,209$	$q_1^* = 464\,757$
$q_2^* = 657\,267$	$q_2^* = 569\,209$	$q_2^* = 464\,757$
$q_3^* = 485\,466$	$q_3^* = 661\,582$	$q_3^* = 464\,757$
	$q_4^* = 405\,729$	

Verification $Q = 1800\,000$ $Q = 1800\,000$ $Q = 1800\,000$.

11. Applications for model 2

$Q = 30\,000$ TV sets (let us say, in a supermarket);

T = one year; $T = 360$ days;

$c_L = 500$ units (of money); $c_L = 500$;

$c_H = 2$ units / 1 TV set / day; $c_H = 2$;
$c_P = 10$ units / 1 TV set / day; $c_P = 10$;
$r = \frac{1}{3}$; $r = \frac{1}{2}$; $r = \frac{c_P - \sqrt{c_P c_H}}{c_P - c_H} = 0.69 < 1$;
$r = \frac{3}{4}$ (we simulate 4 cases related with r).

Find the optimal values $q^*, N^*, h^*, C_r(q^*, s^*)$ and draw a supply plan.

Solution. We arrange the results in the table 2.

Table 2

	$r = \frac{1}{3}$	$r = \frac{1}{2}$	$r = 0.69$	$r = \frac{3}{4}$
$\rho(r)$	$\frac{10}{11}$	$\frac{5}{6}$	0.691	$\frac{5}{8}$
q^*	262.20	223.62	208.26	210.82
s^*	238.26	186.33	144.44	131.75
N^*	114.41	134.16	144.05	142.30
$\overline{N}^*$	114	134	144	142
h^*	3.14	2.68	2.5	2.52
$\overline{h}^*$	3	3	3	3
C_r^*	114415	134164	143413	142302 the max cost

The supplying plan is the following:

$q_1^* = 262$	$q_1^* = 223$	$q_1^* = 208$	$q_1^* = 211$
$q_2^* = 262$	$q_2^* = 223$	$q_2^* = 208$	$q_2^* = 211$
…… ……	……. ……		
$q_{113}^* = 262$	$q_{133}^* = 223$	$q_{143}^* = 208$	$q_{141}^* = 211$
$q_{114}^* = 394$	$q_{134}^* = 341$	$q_{144}^* = 256$	$q_{142}^* = 249$.

Verification $Q = 30\,000$ $Q = 30\,000$ $Q = 30\,000$ $Q = 30\,000$ (on each colon of r).

For the value $r = r_2 = 0.69$ the total cost has the maxim value 143413.

12. Conclusions

All the three economical models together with the C++ appropriate programs are useful tool for a manager wishing to realize an optimal activity. The programs make an economical mini-library.

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