# FUZZY INFORMATION IN NATIONAL ECONOMICS

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***Abstract.*** *All models which are used in national economics – theory, practice and statistics – are based on precise numbers. In fact almost all information – which is used in national economics – becomes fuzzy to the uncertain conditions and parameters. If we think of "old" Keynesian economics all uncertainty in information has been ignored. Rational expectations theory assumes the availability of precise information – sometimes even stochastic components are ignored in it. In fact national economics run in a complex environment. So there must be a high degree of uncertainty about information and their relation. Enhancing all the models by fuzzy components enables the use of linguistic information such as "low", "medium", "high".*

*This paper discusses topics of the theory of national economics with focus to uncertain information and presents the results of so gained economical functions.*

***Keywords*:** *fuzzy, fuzzy number, fuzzy information, characterizing function, gross domestic product, unemployment rate, uncertainty, imprecision*.

## 1. It seems precise but.....

If we read books about microeconomics, macroeconomics or national economics it seems that all values – like:

- gross domestic product (GDP)
- price index
- unemployment rate
- inflation rate
- relative output gap
- etc.

which are used in all the mathematical or economical models are precise. If we have a closer look on those values or if we discuss them from a more critical point of view it becomes immediately obvious that almost all of

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them are not precise. Many of them are not precise because the necessary 100% availability is not feasible or cannot be guaranteed.

All the used information is based on data sampling or any kind of reporting. If we consider it on national level it is highly dependent if the information can be gained and on the quality of data which can be provided.

Finally based on these uncertainties we have to consider the impact on all of our calculation results because they are also uncertain and fuzzy.

**Example Bosnia-Herzegowina:** Consider a country in which a war took place. After the war it is separated from the original "mother" country. In this situation such a country lost all historical data. Almost no information about number of births, distribution of age of the population, unemployment, distribution of income, etc. is available for a certain period of time. If there is information it will be not complete, maybe partly wrong or is (based on political reasons) biased.

**Example “bank's data”:** do the bank really know how much money is available?

If we consider the case that a bank likes to know how much money is available we normally assume that someone can tell us yes we have EUR 45.567.234,13 on all accounts. But is this really the case? Of course every accountant will tell you that he knows the exact number. But this knowledge is based on a lot of digital systems. We all know that there might be some errors caused by erroneous data, mistakes done by "human interfaces", problems during data loads, different assumptions, etc. During my activities as data warehouse consultant I joined a lot of meetings where correctness of data and reports has been discussed. Correctness in huge systems or big companies is always a matter of tolerance. In some cases two reports are comparable if the values are equal up to ± EUR 50,00 in other cases they are seen as equal if the deviation is not more than EUR 500,00.

So it is obvious that in reality the question “How much money is available?” can only be answered using fuzzy information – even this is not convenient to everyone and has a touch of “being not serious”.

Assuming that a “perfect” reporting is available and the data collecting system is working in a perfect way we have to do rounding, e.g. calculating interest rates and interests – so the uncertainty or impreciseness is at least when we consider that unit Eurocent.

Based on the consideration that life is not precise we can use the technique of fuzzy number arithmetic. Each number will be described by a so called characterizing function.

*Definition of a characterizing function of a fuzzy number:*

A real function $\xi_{x^*}(.)$ is called a characterizing function of a fuzzy number $x^*$, if following conditions are fulfilled:

(1) $\xi_{x^*}: \mathrm{R} \to [0,1]$

(2) $\forall \delta \in (0,1]$ the $\delta$-cut $C_\delta(x^*) := \{x \in R : \xi_{x^*}(x) \geq \delta\}$ is a finite, non-empty and closed interval. So $C_\delta(x^*)$ is always a compact interval.

**How models will change**

Let us consider now some calculations which are often used in national economics. Based on the fact that the underlying information is not as precise as we assume in this model a change in the model happens and how it can be interpreted.

## 2. Unemployment rate

Let us first start with an – as it seems on the first view – "simple" topic – the unemployment rate. It is defined by:

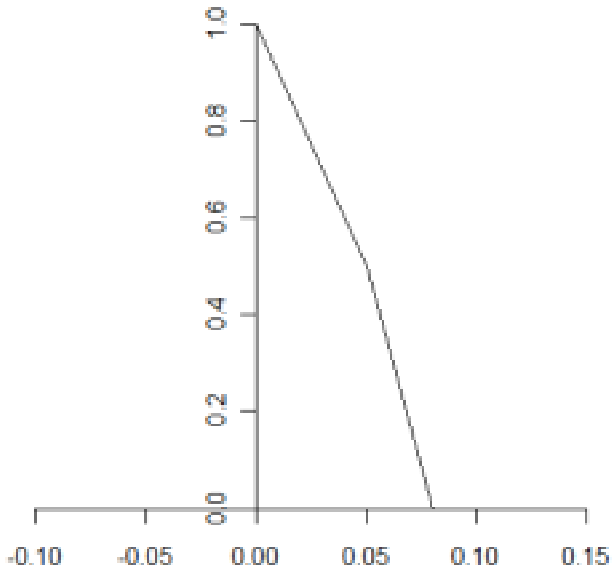
$$\text{Unemployment rate} = \frac{\text{Registered unemployed population}}{\text{Active population + registered unemployed population}}.$$

If we have a closer look on "active population" it means in national statistics that "self-employed" have not to be counted in the set of "active population". Also they cannot be seen as "registered unemployed" if they have no work (which means no or not enough orders). And of course there are some people which are employed and in parallel do their own business like a self-employed person.

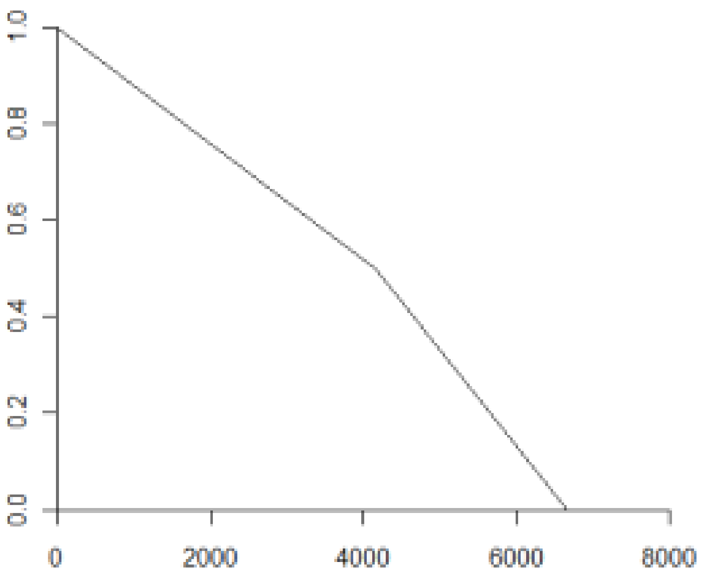
Last but not least we can asked the questions if really everyone who has no work is registered as "unemployed person".

In 2008 we had in Austria 8,318,592 of resident population, about 4,090,000 have been seen as active population and around 162,000 have been registered unemployed people. If we assume that these figures are "precise" we get an unemployment rate about 3.81%.

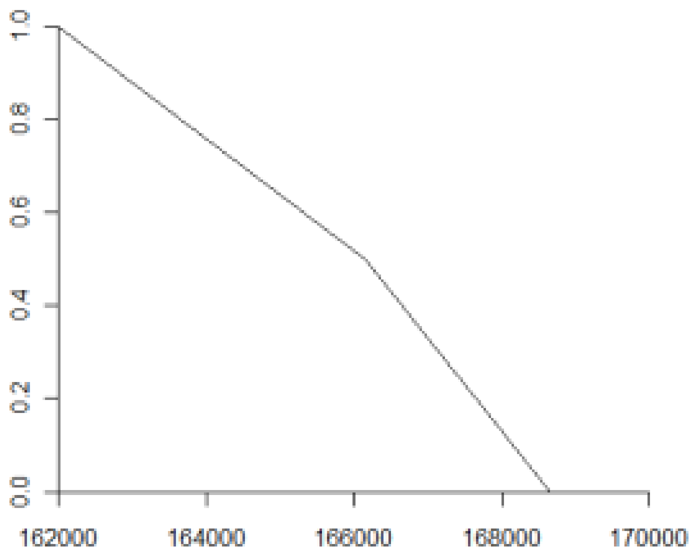
If we consider that 0.05% of the resident population cannot be registered as unemployed and are of course not self-employed and should be within the set of "active population" and we formulate it with the degree of believe of 0.5 and we say that it cannot be greater than 0.08% – we immediately get a fuzzy number of this factor of uncertainty with following characterizing function.



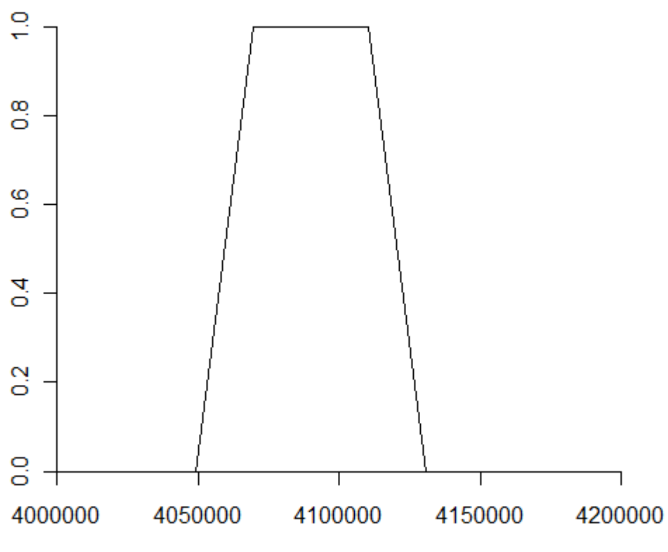

Based on this the number of “not considered” people has following characterizing function.



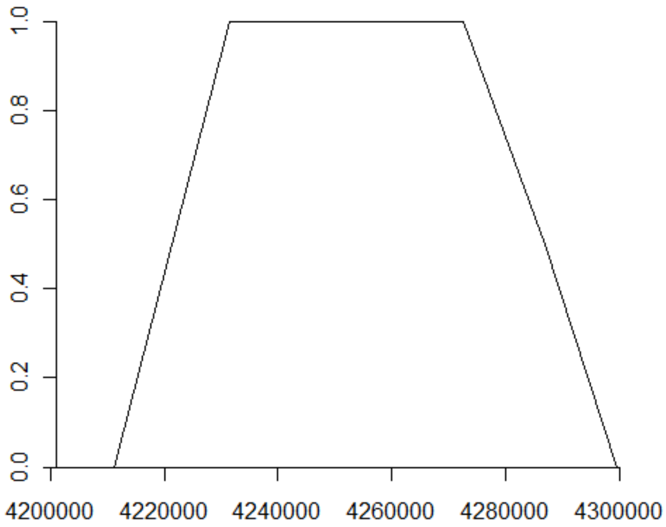
And immediately we get for “registered unemployed population” the fuzzy “unemployed population”.



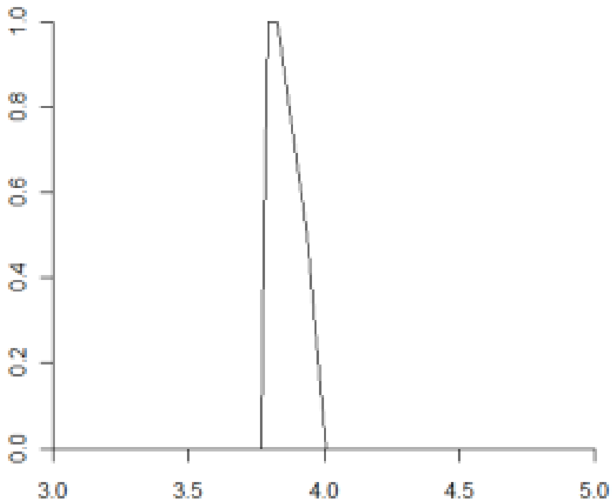

Let us assume that “active population” is also a fuzzy number with characterizing function shown below.




So we get for the sum of “active population” and “unemployed population” following characterizing function.




As result we get the characterizing function of the fuzzy number "unemployment rate" as shown below.




This shows us that the unemployment rate is for sure between 3.79% and 3.83% but e.g. with a degree of believe of 0.5 it is between 3.78% and 3.94%.

Calculation of gross domestic product (GDP)

The gross domestic product is calculated by:

$$Y_{marker\ rates}^{gross} = C^{private} + C^{public} + I^{gross} + (Ex - \mathrm{Im})$$

where:

$Y^{gross}_{market\ \ rates}$ is the gross domestic product at market rates

$C^{private}$ is the private consumption

$C^{public}$ is the consumption of the public sector

$I^{gross}$ is the investment

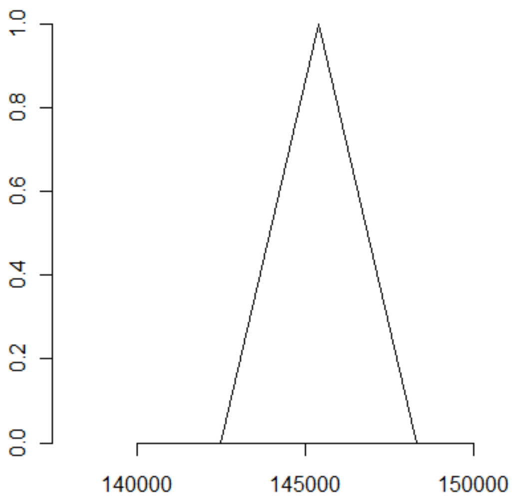
$Ex$ is the export

$Im$ is the import.

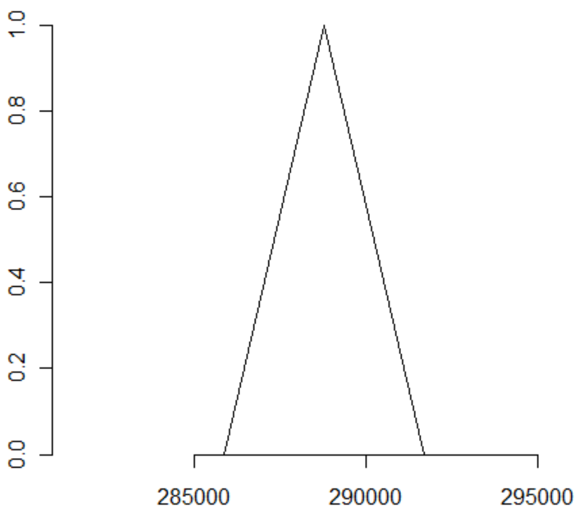
If we use the classical approach we get e.g. for Austria following figures for the year 2009 (in EUR).

| | |
|---|---|
| $C^{private}$ | 145,408,500,000 EUR |
| $C^{public}$ | 143,364,000,000 EUR |
| $I^{gross}$ | 58,149,200000 EUR |
| *Ex_IM* | –75,462,300,000 EUR |
| $Y^{gross}_{market\ \ rates}$ | **271,459,400,000 EUR.** |

Let us now consider that $C^{private}$ is not a precise number and we see it as a fuzzy number which has a triangular characterizing function. Assuming that with a degree of believe of 0.5 we have a deviation from the reported value of ±1% we get following characterizing function.



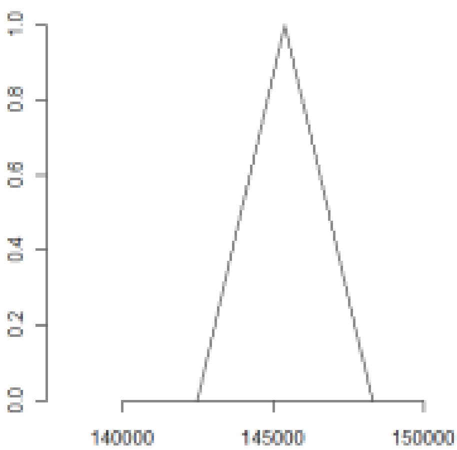

If we now calculate the gross domestic product we will get a fuzzy number which has a characterizing function shown in the next picture.

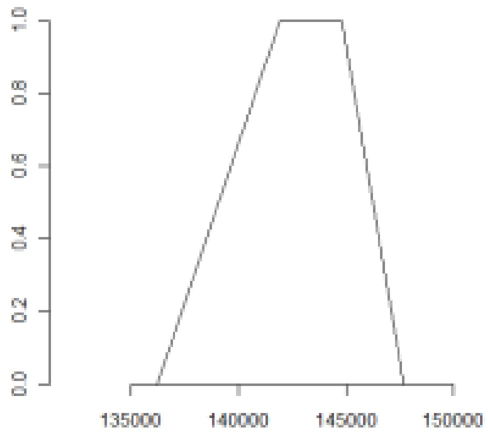
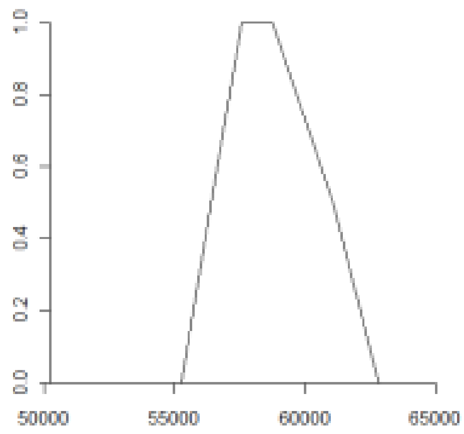
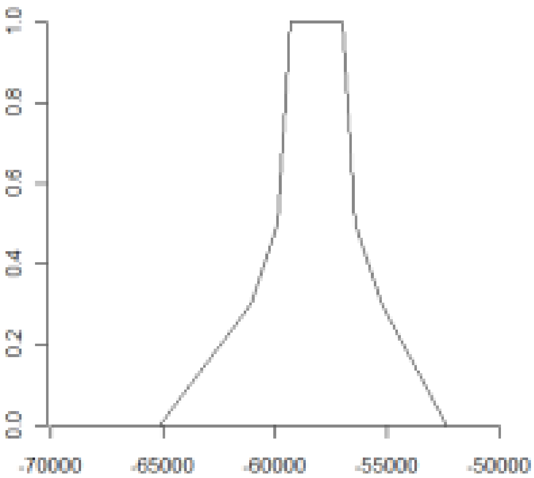


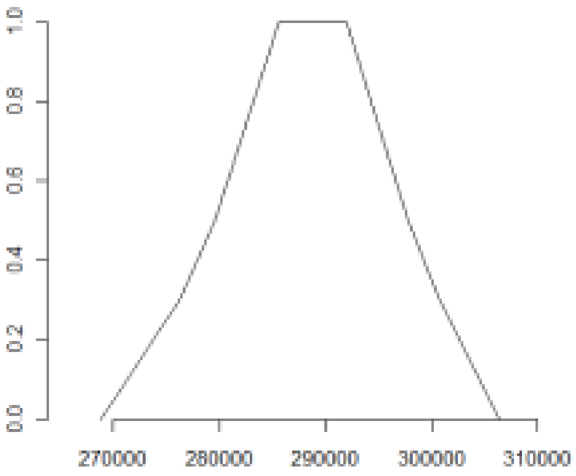

It has the same shape as the characterizing function of $C^{private}$.

Considering a degree of believe of 0.5 it is between 287,318,400,000 EUR and 290,226,600,000 EUR which is a deviation of 5.52% to the lower bound and 6.91% to the upper bound from the original reported value.

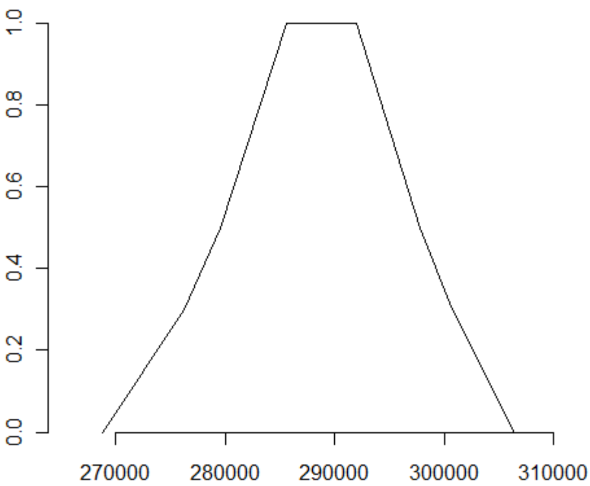
In the next step we make this model more complex taking into consideration that none of the input values are precise. We define the characterizing functions as shown in the table below.

| | | |
|---|---|---|
| $C^{private}$ | 145,408,500,000 EUR |   |

| | | |
|---|---|---|
| $C^{public}$ | 143,364,000,000 EUR |  |
| $I^{gross}$ | 58,149,200000 EUR |  |
| *Ex_Im* | –75,462,300,000 EUR |  |

| | | |
|---|---|---|
| $Y^{gross}_{market\ rates}$ | **271,459,400,000 EUR** |  |

If we have a look now on the resulting characterizing function of $Y^{gross}_{market\ rates}$ we see that it is mostly influenced by the shape of the trapezoidal characterizing functions of the input values and also a small bit of the characterizing function of *Ex_Im*.




The values of the $Y^{gross}_{market\ rates}$ show us now that we cannot exclude that the gross domestic product is between 285,594,400,000 EUR and 291,950,600,000 EUR. Considering a degree of believe of 0.5 it is between 279,528,500,000 EUR and 297,745,800,000 EUR.

Out of this you can see that the interpretation is much more complex as it has been assuming that the underlying information is an exact one.

## 3. Comparison problems

Economical thinking means always taking decisions. A single decision is considered as a good or optimal decision if at least one of the following criteria are fulfilled:

- minimized costs
- maximized profit
- minimized loss
- maximized utility.

If a decision has to be taken out of 2 options it will be based on a comparison which might be e.g.

- less costs
- greater profit
- less loss
- greater utility.

If all the calculated or measured values are precise the comparison can be done in the classical way and the decision finding is "simple".

In the case of calculations or measurement of fuzzy number it has to be considered what means "less" or "greater".

One method can be to compare δ-cuts for each δ-cut level. But at first let us define the "δ-cut" of a fuzzy number $x^*$.

*Definition of δ-cut:*

$\forall \delta \in (0,1]$ the δ-cut is defined by $C_\delta(x^*) := \{x \in R : \xi_{x^*}(x) \geq \delta\}$ where $\xi_{x^*}(x)$ is the characterizing function of the fuzzy number $x^*$.

If we define:

$$\underline{x}^\delta := \min\{x \in R : \xi_{x^*}(x) \geq \delta\}$$

and

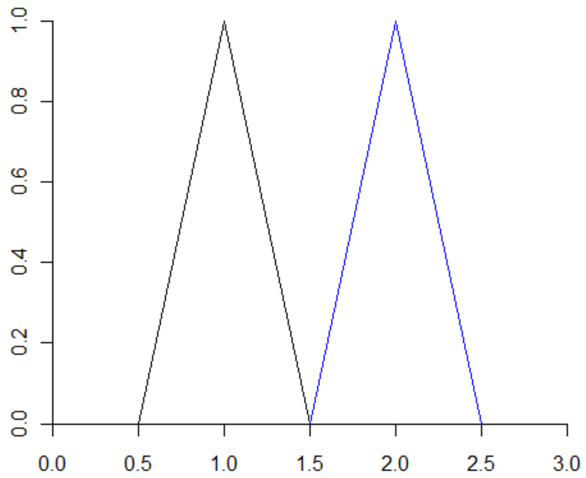
$$\overline{x}^\delta := \max\{x \in R : \xi_{x^*}(x) \geq \delta\}$$

we get:

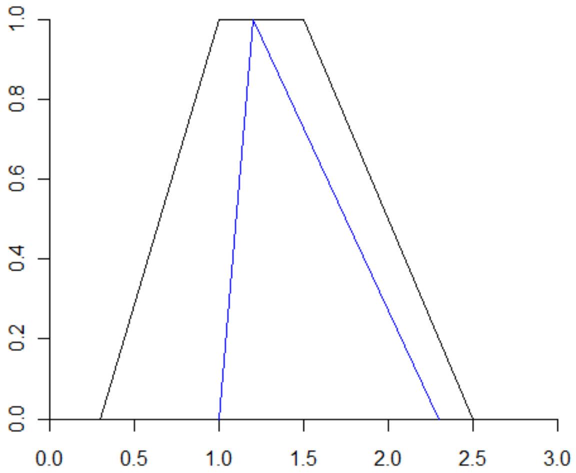
$$C_\delta(x^*) = [\underline{x}^\delta; \overline{x}^\delta]. \ .$$

If we try to compare the two fuzzy numbers $x^*$ and $y^*$ by using the corresponding δ-cuts we can say that $x^*$ is less or equal than $y^*$ if for each δ-cut of $C_\delta(x^*) = [\underline{x}^\delta; \overline{x}^\delta]$ and $C_\delta(y^*) = [\underline{y}^\delta; y^\delta]$ we get $\underline{x}^\delta \leq \underline{y}^\delta$ and $\overline{x}^\delta \leq \overline{y}^\delta$.

This is valid if we try to compare fuzzy numbers $x^*$ and $y^*$ with characterizing functions as shown below.




Unfortunately in case of fuzzy numbers with characterizing function shown in the next picture this approach will not work.




So another approach is to use the method of "Steiner point".

*Definition of "Steiner point":*

The "Steiner point" is defined by:

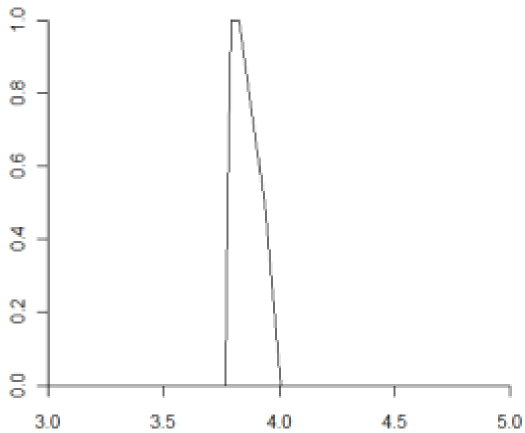
$$\delta_{x^*} = \int_0^1 \frac{\underline{x}^\delta + \overline{x}^\delta}{2} \, d\delta,$$

where:

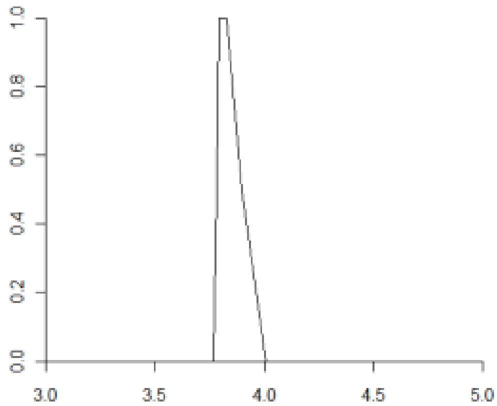
$$C_\delta(x^*) = [\underline{x}^\delta; \overline{x}^\delta].$$

## 4. Unemployment rate

Let us now consider the unemployment rate of Austria in 2008. If we assume that the measurement could be improved in 2009. Now assuming 0.03% of the resident population (instead of 0.05%) cannot be registered as unemployed and are not self-employed and should be in the set of “active population”. We get following characterizing functions.

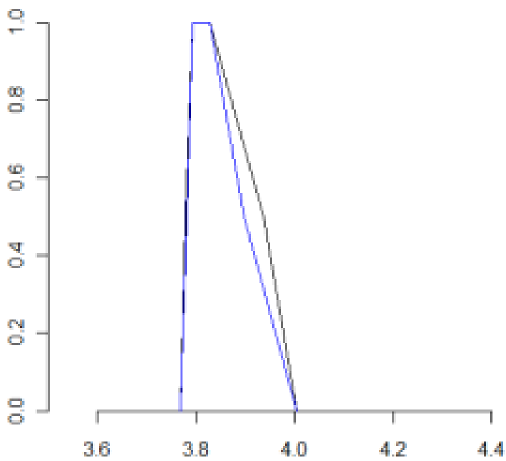


2008



2009

Answering the questions which of these two unemployment rates is the better one – in the sense of “which is lower?” – we expect that it is the unemployment rate of year 2009 if we have a look on the picture below.

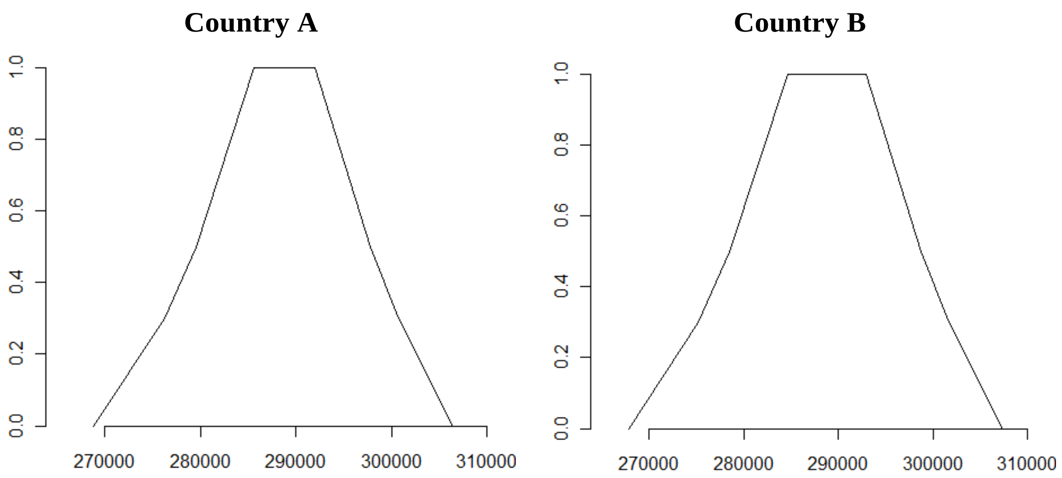



The calculation of the Steiner point shows
Steiner point of unemployment rate 2008 is 3.85.
Steiner point of unemployment rate 2009 is 3.84.

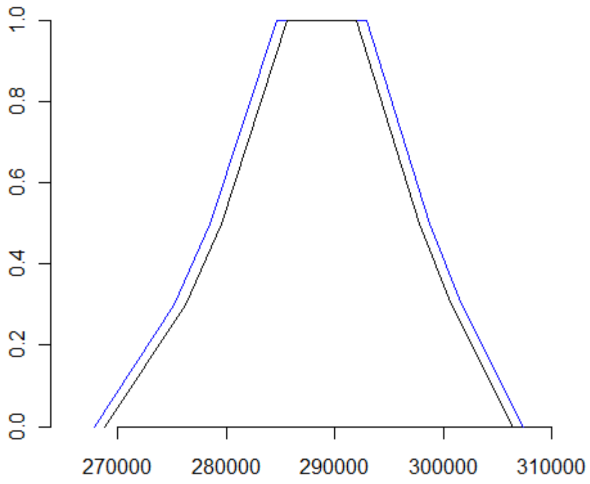
So the unemployment rate of 2009 is lower than the unemployment rate of 2008.

**GDP of 2 different countries**

Let us assume that the GDP of two different countries are almost equal. The figures of country B are more “uncertain” or “imprecise” than the figures of country A.




If we print both characterizing function in the same graph it can be easily seen that GDP of country B is more imprecise (black...A, blue...B).




If we calculate now the Steiner points of the two GDPs we get the same value 288,477.30. This is because of the fact that the characterizing function of country *B* has the same shape as the characterizing function of country *A*. It has been shown that two fuzzy numbers are “equal” although their degree of uncertainty is different. Maybe this can be seen as the weakness of the Steiner point method.

## 5. Conclusion

Using the power of characterizing functions to describe fuzzy information and the underlying calculation methods the reality can described in a better and more realistic way. Maybe it is not the most convenient method for decision finding – as we have seen in the example of comparison of 2 “similar” GDPs – but in future the modelling using fuzzy information techniques will increase.

In many cases we have to deal with imprecise information, imprecise figures and uncertainty. In classical calculation methods this uncertainty is ignored.

In the next years methods of calculation, analyzing and comparing fuzzy information have to be improved. Of course there are many applications in reality for this topic but at the moment most of them are ignored and classical or stochastic methods are used instead. Not every uncertainty can be explained by stochastic methods and it is not a proper way to ignore it in mathematical models.

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