WHEN RICH GET RICHER THERE ARISES FINANCIAL CRISIS AND BOSE-EINSTEIN CONDENSATION IN A WILD ECONOMY

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Abstract. *We model one of the main laws of the wild market where rich get richer, while poor get poorer. The model reflects the main principle of the wild capitalism namely that richness attracts more richness. A fixed number of trading agents exchange and transfer money according to one main rule, when two agents are trading the money transfers from the poorer agent to the richer one. We show that in such wild market arises a new phenomenon analogous to a Bose-Einstein condensation (BEC), where many agents lose all their money. The remaining agents are fighting to become richer. With the time the condensation fraction increases; correspondingly, the number of rich or active agents decreases. The probability of the "excited" or active agents to have some money is well described by Bose-Einstein (BE) distribution. The fit is better when the trading time increases. When a majority of agents are losing all their money, the Bose-Condensation arises and the chemical potential vanishes. With growing time the condensation fraction increases. At the first stage the shape of the money distribution is well described by the Bose-Einstein one only for a low-energy/money region, while the high energy/money region is described by a Pareto power law with very large power coefficient β=12. At very later times the money distribution becomes dispersed, especially in the tail, and cannot be uniquely described by overall probability distribution. It rather covers the region in the probability-money plane, where many events happen with low probability. The boundaries of the money distribution dispersion may be also described by a Pareto power laws with different power law coefficients, larger and smaller than one. The Bose-Einstein condensation and condensate fraction growth describe a start and a development of the financial crisis, respecttively. We show that this crisis may arise in different market economies when the law the "rich get richer" dominates. We argue that to stop or to avoid the financial crisis one has to remove the BE condensate, where agents are frozen from any economic activity. This may occur due to the new money flow into the market. Then the condensed agents will be excited and take again an active part in the market activity. They can again compete obeying the rich get richer law and become*

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poor. Thus, the financial crisis can be avoided if the measures to stop the BEC or to remove the Bose-Condensed fraction will be taken. Thus we may define the financial crisis as the time when the majority of population are excluded from trading or from the money exchange process. Such a view dictates direct measures what to do to avoid any financial crisis, i.e. by making active all existing economic agents.

***Keywords**: Statistical mechanics, Bose-Einstein distribution, network, condensation, complexity, financial crisis, econometrics.*

1. Introduction

It was argued that the present huge financial crisis has been triggered by a shortage of money or by a liquidity crisis and will be followed by next collapses of large financial institutions around the world. The immediate bailout of banks by national governments was an attempt to save the situation. The massive forecloses have released the money for the rest of acting economic units as well. These actions have stimulated the economy for a while however the next downturns in stock markets around the world resulted in another series of numerous evictions and foreclosures. In the present paper we introduce a model which studies such a transient process of the financial crisis. The model addresses mainly the transient periods of the financial crisis development and describes well not only the massive series of forecloses and bankruptcy started with the US subprime mortgage crisis but indicates also on its possible causes.

Although experts are placing different views on the origin of the crisis everybody agrees that two measures can help. One is the money pumping in the market, the second one is a decreasing consumer debt levels. In both cases it is very important to understand how the incoming money is redistributed or how financial cuts influence such redistribution going on during the financial crisis. In original papers by Yakovenko et al (see, Refs [1,2] and references therein and other researchers [3,4]) analysing numerous real financial data of the income revenue statistics have found that during the crisis there is a transformation of money from the poorer to the richer part of the population. The deeper the crisis the more money is transferred from poor to the rich. This finding has attracted the attention of a broad scientific community. In turn that stimulated to reconsider main concepts, which are underlining modern financial econometrics. As main players two active competing classes such as employees and employers, or industrialists and workers, or poorer and richer agents taken part in a market have been identified.

In stream of papers the search for a cause of the crisis has been focused on studies of money distribution and on a role of a debt. [5-13].

The present paper is focusing on a main principle of the market economy where are two types of players, those who own the property and tools of the production (the industrialists) and those who are providing the services (the workers), who produce goods and receive a salary for the work they done. In the model introduced here at each trading event two representatives of the above classes are taken part, one from each class. The richer one is an industrialist, while the second one is a worker. The industrialist is selling goods produced and therewith, becomes richer with each such event. The worker is buying some goods and therewith paying money and becoming poorer. The detail flow of the goods is not taken into account, and the focus is on the time evolution of money distribution. Originally any agent has a start up money as if we would assume that the salary is given only ones. And any agent can be a worker or an industrialist depending on who is richer of two trading agents. The separation is instantaneous and arises only during the trading process when richer of two chosen trading agents becomes an industrialist while the other one becomes a worker. After the trading event the rich becomes always richer. At another trading event of two agents the separation into the classes is making again. So at each trading event there is a presence of the creature such as a Maxwell Demon[14]. The Demon separates rich and poor momentarily. His decision is based on an amount of money the two trading agents have at the moment of trading. After the Demon made his decision the principle “richer get even richer” works. This principle is playing a key role in the time evolution shape of the money distribution. We make a parallel of such an evolution with a transient period of a financial crisis. We show that after many laps or epochs of trading, there arises a transient co-existence of two states (or thermodynamic phases). One of them is similar to the celebrated Bose-Einstein condensate phase, while the other state is equivalent to a gas of free particles as those discussed in Refs [1,2] and many other papers[6-13]. The distribution of money for the second phase is described by the Bose-Einstein distribution with the vanishing chemical potential. With growing time there is a continuous transition of agents from the second excited phase into the condensate.

In our previous papers we have drawn a parallel between the financial crisis and a phenomenon of the Bose-Einstein condensation [5-7]. Such phenomenon is well studied in physics and associated with a formation of a Bose-condensate in a system of quantum gas particles known as bosons. When temperature lowers the particles are losing all their energy and dropped in a state without energy. This is well known as a formation of the Bose-Einstein condensate state, which is formed when the majority of particles have no energy. We define a Bose-Einstein condensate state of the market when the majority of economic agents have no money. This is exactly the state of the financial crisis because in this case the agents without money cannot take part into next trading and dead for the market and economy. We propose a simple model where such a state is formed and investigate a time evolution of a financial crisis.

Thus, we found that any financial crisis is developing analogous to a formation and growth of the Bose-Einstein condensate. Such condensate corresponds to unemployed agents in very general sense that includes everything that lost income. In other words by unemployment fraction here we mean the fraction of all agents which have lost all their money and dropped out of the next trading processes. After the formation of such Bose-Einstein condensate associated with agents having nearly a zero money, the shape of the rest money distribution evolves slowly. After any time period such a distribution is well described by the Bose-Einstein formulae with vanishing chemical potential. At early stages the high money region is reminiscent the Pareto tail, ie the power law behaviour with very larger power coefficient, $\beta = 12$. At later stages it is dispersing so that the tail covers a whole region and therefore cannot be described by any probability function. The described transient period of the financial crisis can be compared with existing experimental data, see, for example, discussions presented in the Refs. [1,2,3,4,5,6].

2. Modelling of rich get richer trading processes

We consider a market consisting of N trading agents. At each trading event two interacting agents are chosen randomly. The choice who is the buyer and who is the seller is made with the use of the principle “rich get richer” also well known as a Law of Increasing Poverty. To implement this Law we introduce a presence of the Maxwell Demon, which decides who is a buyer and who is the seller at each trading event, where the buyer has always smaller money than the seller.

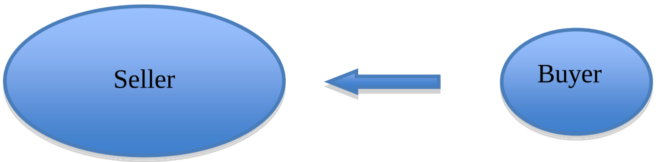


Figure 1. The scheme of each trading event performed according to the principle or Law of Increasing Poverty: “rich get richer”. This means that the richer agent is always chosen as the seller and after the trading his money increases.

Initially all agents are given nearly the same or a certain slightly distinct amount of money drawn from an arbitrary probability distribution P(t=0), while the total amount of money on a market is fixed and equal to *M*. This initial state is specifically not taken from a uniform probability distribution used in previous works [1,2,3,4] since from the start we have to identify two classes: the industrialists who sell and workers who buy.

At each trading event the Maxwell Demon indicates the poorer from two randomly chosen agents. Then, for example the poorer agent *i* acts as a buyer, while the other richer of two chosen agents, say the agent *j* acts as the seller. First we assume that the amount of the transaction is proportional to the actual funds of the buyer agent *i* which is a plausible assumption. However the results obtained are universal and do not depend on this parameter.

Thus, in this model the wealth of the poorer *i*-th agent decreases as:

$$m_i(t + 1) = m_i(t) - p\, m_i\,(t), \qquad (1)$$

while for the *j*-th richer agent the money increases as:

$$m_j(t + 1) = m_j(t) + pm_i\,(t), \qquad (2)$$

Where the parameter $p(0<p<1)$ indicates the fraction of the funds transferred during the purchase.

3. Concept of Bose-Einstein distribution of money and a condensation of agents without money

Next we study the time evolution of the money distribution. We call a time period of N interactions, where each agent has taken part in trading twice on average as one epoch. Interestingly that for each time period, after any fixed number of epochs, a probability of some agent to have the money, m, can be consistently described by the Bose-Einstein distribution which is naturally arises from an application of statistical mechanics to economics [5]. Such an application was based only on assumption that market is chaotic and therefore an ergodic hypothesis can be applied. This results in a next consideration of various ensembles consisting of time snapshots of the market taken after different time periods. The finding of the probability for the most optimal configuration of the market provided its statistical description. Considering a market as grand-canonical ensemble where number of active agent may change, for example, due to a bankruptcy of agents when their lost all their money and become inactive for the market. Within such an approach the most general distribution for the market has been emerged and it has a form very close to a Bose-Einstein distribution, which is conventionally used in Physics for a description of gas consisting of quantum Bose particles. What is a common between these two very different systems, in physics and economics?

The commonality is in fact that any agent may have the same amount of money or income as any other agent, that is, this is equivalent that any number Bose particles can occupy the same energy level. Moreover, agents having the same money economically indistinguishable as Bose particles having the same energy and this results in their statistical similarities (see, more details in Ref. [5]).

Note, that this is a pure statistical description and analogy that provides interesting consequences for the market. For example, at some conditions as very low temperatures the gas of Bose particles may have two states: 1) a BE condensate consisting of particles with zero energy and 2) a gas of the rest particles having non-zero energy. When temperature decreases there is a transformation of particles from the active part into the condensate.

Indeed, in the present model of the market we found that exactly such a situation arises. With the growing time the agents are separating into the condensate and active group consisting of trading agents. The condensate fraction is growing with time while the number of active agents decreases. In general, for any value of the parameter *p* the time evolution of the initial money distribution seems to relax very fast towards a time dependent probability density distribution, which is consisting of two parts: 1) the condensate and 2) the one having the form of the Bose-Einstein distribution:

$$P_{BE}(m) = \frac{n_0}{\exp\left(\frac{m-\mu}{T}\right)-1} \qquad (3)$$

where n_0 is a normalisation factor, μ is a chemical potential and T is a temperature. Our studies show that the time dependence for chemical potential, μ, and its value vanishes very fast with a few epochs from a start of the market operation, even for small values of *p*. Then, the BE condensate is formed, while the money distribution of the active agents ($m>0$) is described by the same equation (3) with $\mu=0$. When $m-\mu \ll T$ the distribution is simplified to the power law form:

$$P(m) = \frac{n_0 T}{m-\mu}. \qquad (4)$$

At initial stage, when the number of epochs is not very large, there, in the high energy/money part, arises a Pareto tail that can be approximated by the Pareto power law specified by the probability distribution of the form:

$$P_{Pareto}(m) = \frac{c_0}{m^{\beta}}, \tag{5}$$

where the parameter β specifies the power law coefficient, while c_0 is a normalisation constant. Interestingly that for the case when $m < T$ the equation (3) is reduced to a power low which is obtained by a Taylor expansion of the BE distribution function in the vicinity of the $m= 0$, *see the eq*. (4). In this case the power law coefficient is $\beta=1$ and $c_0=T\,n_0$.

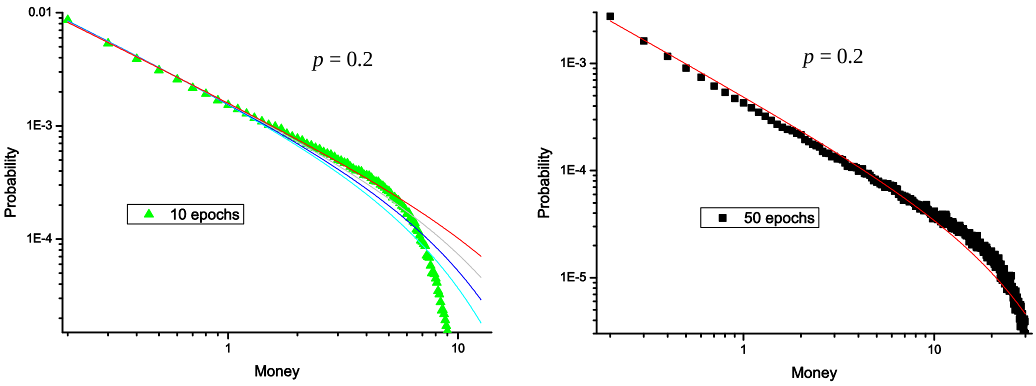
First we would like to consider the limiting case, when $p = 1$, which allows to obtain exact mathematical result illustrating the concept discussed. Here each buying agent (a worker or it can be even a poorer bank) is spending all his money in each transaction, while the money of an industrialist increases. As the result the worker (or poorer bank) is losing all his money and after that is expelled from the next trading processes, i.e. from the market. That is, the poorer bank has $m_i(t+1) = 0$ *and he is bankrupt* after the first time step. At the same time each seller (the industrialist or the richer bank) is gaining that is $m_j(t+1) = m_j(t)+ m_i(t)$. Thus, the seller absorbs the whole fortune of agent i. After $N/2$ trading events half of the agents will lose all their money. Following above we may call them a condensate. The rest, active agents will have some money that will depend on money initial distribution. When number of trading events increases and become equal to N, i.e. after one epoch all banks except one vanish and all the money will be in the hand of this one single agent. Note all other banks have no money, that is we may call they all are in Bose-condensed state, which in a physical system corresponds to the state where particles have no energy. The phenomenon of the formation such a state, where the majority of particles have no energy (or banks no money) is called the Bose-Einstein Condensation (BEC). Here, nearly complete (BEC) condensation happened during one time period – one epoch. There all agents are in the condensed state (except one richest bank- “the winner takes it all”).

Now, we show that the BEC is formed in other cases when the parameter $p<1$. In this case the formation of the BE condensate is significantly slow down. Then, practically all agents have many trading events before losing all their money and therefore there is enough time to form a BE money distribution of the rest active agents. Below we study numerically how the distribution of money evolves after some time period where all agents have taken part in the trading processes many times. The temporal evolution or the specific values of described parameters of the money distribution depend strongly on the rate of the money exchange, that is, on the value of the parameter *p*. They also depend on the topological structure of the financial trading network, but this issue we postpone to discuss to a later stage. We note that our trading network is equivalent to a gas of particles [1,2,3]. In this analogy each trading agent represents a particle. Then the trading processes of the economic agents are equivalent to the exchanging energy of the gas particles, which occurs in mutual collisions. However, the economic system differs from gas particles, by the presence of the Maxwell Demon which directs the energy/money transfer between two particles at their collision. In other words to implement the law "rich get richer" at each trading event we have to introduce the "creature" similar to the Maxwell Demon well known in thermodynamics. The creature has extra information which particle has higher energy/money and therewith indicates the direction of the energy/money transfer from the lower energy/money particle/agent to the one having higher energy/money and also controls the fraction of the energy transfer.

4. The fitting of distribution of money with the Bose-Einstein formulae

Thus, there are double reasons why the *BE* distribution should fit the data well. First because the same amount of money may have many agents and they are financially not distinguishable. The second is that the agents may leave the market when they lost all their money. Since condensed agents are not active the number of trading agents decreases and at the same time they become richer, in average. When *p* is very small the process is very slow. When the value *p* increases and becomes larger the condensate fraction and the temperature of the BE distribution's of active agents increase dramatically with growing number of epochs. At such transient process the Boltzmann exponential distribution practically does not appear. The money distribution can be described by universal fitting formula, as Bose-Einstein distribution, see, eq. (3).

For example, for a market consisting of 1million trading agents the money distribution recorded after 10 and 50 epochs is presented in Figure 2, where the trading fraction was taken as, $p = 0.2$. For the case of 10 epochs we present 4 different fitting curves, in cyan, blue, grey and red colours, obtained with the use of eq. (3) with the usage of the fitting parameters: the temperature, T =3.8, 4.9, 6.8, 11.1 and the constants, $10^7 \cdot n_0$ = 45,35,25,15, respectively. The fitting value of the chemical potential is equal to μ=0 in all cases. Note that for all these curves the product Tn_0 is fixed and is approximately equal to $17 \cdot 10^{-6}$. When the fitting temperature T rises from T =11.1 the fitting curve is transformed in a straight line associated with the power law behaviour, $P(m)= 17 \cdot 10^{-6}/m$, that perfectly fits the upper part of the data. The similar situation arises for the data recorded after 50 epochs. However, already in this case we are able to find such values of fitting parameters that all data can be described by a single analytic curve associated with the Bose-Einstein distribution, eq. (3), see, for example, the red curve presented on the Figure 2*b*. There, the values of fitting parameter are taken as T =13.6 and $n_0 = 37 \cdot 10^{-8}$. With larger number of epochs and larger values of *p, all money distribution data can be always fitted by a single BE distribution.* Of course, the condensate point is excluded from the distribution and therefore it has not been shown on the Figure 2.



a) *b)*

Figure 2. The various fits of the money distribution for a state of a market consisting of 1million trading agents recorded after a) 10 and b) 50 epochs, when the trading fraction, p = 0.2. All solid coloured lines correspond to the Bose-Einstein distribution function (see, eq. (3)) for different values of parameters, T and n_0 but for the fixed value of the chemical potential equal to μ = 0. For a) the coloured fitting curves (cyan, blue, grey and red) correspond to the temperatures, T = 3.8, 4.9, 6.8, 11.1 and the constants, $10^7 \cdot n_0$ = = 45,35,25,15, respectively. Note that for each of this curve the product Tn_0 is fixed and is approximately equal to $17 \cdot 10^{-6}$. For b) the red curve corresponds to T =13.6 and $n_0=37 \cdot 10^{-8}$. The comparison a) and b) indicates that with increasing number of epochs all money distribution data can be fit with one Bose-Einstein distribution function with μ = 0. The point associated with the Bose-condensate fraction, which is inherent to the case, has not been shown on the Figure.

If we compare the found distribution with analogous model in physics, say a gas, then, this gas must be considered as a quantum gas and will consist of boson particles at temperature *T*. Its spatial dimension is unknown. The parameter n_0 will correspond to a normalisation factor that can be chosen to fit the BE distribution function for a quantum Bose gas particles. In this analogy the parameter μ will correspond to the chemical potential of the system consisting of these bosons. In the discussed model the value of μ vanishes very fast with the time and the condensate fraction formed. The detail results of numerical experiments are presented in the next section.

5. Analysis of numerical experiments and simulations of the wild market evolution

In numerical modelling presented below we have considered a market with $N = 10^5$ and 10^6 trading agents.

The results obtained in both cases are similar and universal. For the first case, when the value *p* is small all agents spend only a small fraction of the capital they have, i.e. they are very cautious to spend too much. As a result with number of epochs the distribution of money is changing slowly, one may say adiabatically. Specifically, when the parameter $p = 0.2$ after the time periods of 10, 20, 50 and 100 epochs the simulated money distribution is presented in the Figure 3, below.

The lower energy/money part is also well fitted by the Bose-Einstein (BE) distribution, $P(m)= n_0/(exp\,(m/T) -1)$ provided that *T* is larger than the crossing temperature T_{cross}, ($T > T_{cross}$). In this case the BE function can be approximated a power law with a power 1: $P(m) \approx Tn_0/m$. While the product Tn_0 is a constant for each set of data, inversely proportional to the number of epochs, N_{epochs}, the value of one of the two parameters, *T* or n_0, can be chosen arbitrarily, provided that *T* is larger than T_{cross}. The value of crossing temperature, T_{cross}, *turns out* to be roughly proportional to the number of epochs, N_{epochs}. The data of the high-energy/money tail is strongly dispersed. The dispersion increases with the value of the parameter *p* and the number of epochs. However when teh dispersion is not very large for small number of epoch as and p they may be equally well described by the celebrated Pareto Power Law, which is presented by straight lines with very steep slope, where the power coefficient, β~12.

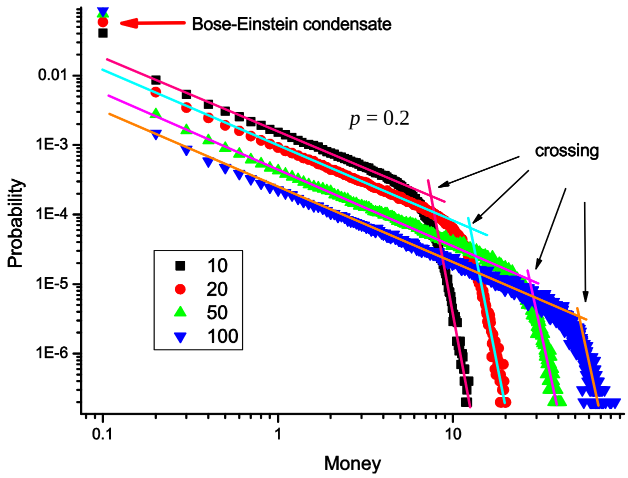


Figure 3. The time evolution of money distribution recorded after 10, 20, 50 and 100 epochs of the free market, where 1 Mln. agents are trading the fraction, p= 0.2*, of their money. The data are* presented by black, red, green and blue symbols for 10, 20, 50 and 100 epochs, correspondingly. The vertical and horizontal axes correspond to the frequency (or the probability) and the amount of money the agents have, respectively. Each set of data may be conventionally separated into three groups, the lowest or "zero", low and high energy/money parts. The first bin is zero energy/money. It covers those agents having the smallest amount of money, close to zero. For the most of the evolution time the majority of agents belongs this bin. The other two parts (low and high energy/money groups, of the data can be fitted by straight lines in the logarithmic scales that specify power law distributions. These two groups are separated by the value of money which we call as the crossing temperature, T_{cross}, which is equal to the value of money associated with the crossing of two lines associated with the same number of epochs and is indicated by black thin arrows with notation as "crossing".

From the Figure 3 we see that for all different time periods the numerical data are grouped into three sets: the "zero" or lowest money point and two other groups which may be characterised by straight lines with different slopes. The group of "zero" money points is indicated by arrow as Bose-Einstein condensate. In the double logarithmic plot the straight lines correspond to the power law distributions. The different coloured lines of low money region correspond to different times of the market expressed in a number of epochs. For all times the money distribution is described by the same BE equation which can be expressed in the power low distributions with the power coefficient, β=1 and β=12, for low and high money region respectively. At very large number of epochs, N_{epochs} >> 100, the separation into two power law distributions vanishes.

The first group of "zero" money points corresponds to agents who lost all their money and it is located separately. With the time the probability to find more and more agents in this bin increases (see, the Figure 3) where the first point (bin) corresponds to the condensate. This point describes a probability of agent's bankruptcy or a probability of agents to lose all their money. With growing number of epochs this probability rises (see, Figure 3). Very consistently that at the presence of the condensate the money distribution of rest active agents is well described by the BE eq. (3), when the chemical potential vanishes. In low money region the BE eq. (3) may be well replaced by equivalent power law distributions, see the straight coloured lines on the Figs 3 and 4, where the money distribution data for the cases with $p = 0.2$ and $p = 0.5$ are fitted, respectively.

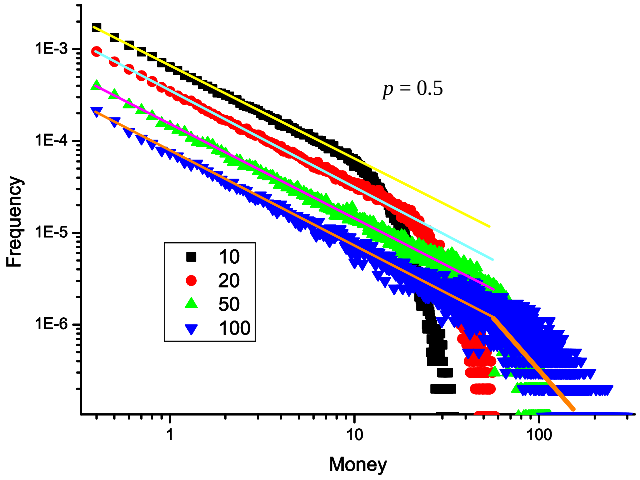


Figure 4. The distribution of money for the market where 1 million agents are trading a fraction of p= 0.5 of their asset in each event. The data have been recorded after 10, 20,50 and 100 epochs and noted by black, red, green and blue symbols respectively (see, Fig. 3, for a comparison, where the case with p=0.2 presented). Here, the first group of zero points, associated with the BE condensate, have been excluded from the graph. The lower energy/money part is well described by BE distribution, the eq. (3), with the activity parameter $a= exp(\mu)/T$=1.00 at any arbitrary temperature provided T that $T > T_{cross}$. For this low money region the BE equation may be approximated by the power law with the fitting constant – the product Tn_0. Then the frequency (or a probability) of agents to have the amount of money m is obeyed the power law: $P(m) = Tn_0$ /m, where the value, Tn_0, depends on a number of epochs N_{epochs}. It is inversely proportional to N_{epochs}. The second high energy/money part may be well described by a Boltzmann distribution or a Pareto power law, where $P(m) \sim 1/m^{12}$ (see, the coloured straight lines with steeper slope). Plotted in the logarithmic scale one may clearly see the changes of the distribution shape arising in the region of large money. The lower energy part is well described by BE distribution associated with the state with the BEC, which are fitted by yellow, cyan, magenda and orange lines, described by the eq. (3) with μ=0 and the temperature $T > T_{cross}$.

When the value of the parameter, p, increases further the evolution of the money distribution and the formation of the Bose-Einstein condensate is going faster, see, for a comparison, the changes in the money distribution for p=0.2 and p=0.5 presented in Figs 3 and 4, respectively, where the money distributions of active agents recorded after 10, 20, 50 and 100 epochs are given. The behaviour of the money distribution, for p= 0.5, is very similar to one described above for the case p = 0.2. At all times they are well described by Bose-Einstein distribution with μ= 0. In addition there arises a high-money tail, which at small values of p and small number of epochs may be equally well described by Pareto power law or by exponential (or the Boltzmann) distribution. The condensate fraction grows here (for $p = 0.5$) faster than in the case, $p = 0.2$ (see, Figure 5). In the low money region the BE equation may be replaced by its Taylor expansion expressed as the power law, $P(m) = Tn_0/m$. The results are universal and do not depend on the size of the market. For example, for the smaller market consisting of 10^5 agents the distribution of money recorded after 100 epochs is also described by BE or the power law with the fitting constant, Tn_0=0.006.

5. The growth of the Bose-Einstein condensate fraction with time

In general we noticed that when the parameter p increases the formation of the Bose-Einstein condensate is going faster and faster, see Figure 5. First question arises: what is the origin of this BE condensate. Such condensate is nothing but a set of unemployed people or bankrupt banks.

We have shown on the Figure 5 that the growth of the condensate fraction is well described by the power law as:

$$n_{condensate} = 1 - \frac{n_0}{(t + t_0)^{\alpha}}. \qquad (6)$$

The values of all the parameters, n_0, t_0 and α, depend on the parameter p, which control the amount of the money fractions transferred at each trading event. The found power law has an intimate relation with the Pareto distribution arising in the last income region. Obviously the power law as $n_0/(t+t_0)^{\alpha}$ indicates on a very slow growth of the zero income population associated with the long term evolution of the market. The condensate corresponds to agents who lost all their money and therefore without money they are not active and dead for the economy. The rest, who left, are intensively trading to become rich. But here the law rich get richer is working and unavoidably some agents losing constantly their money, i.e.

they are eventually end up in the condensate. Effectively this law "rich get richer" means that there is a serious battle to stay "alive" and become richer and richer. The final destination of this game is complete condensation, where all agents except one (the "winner takes it all") are collected in the dead condensate.

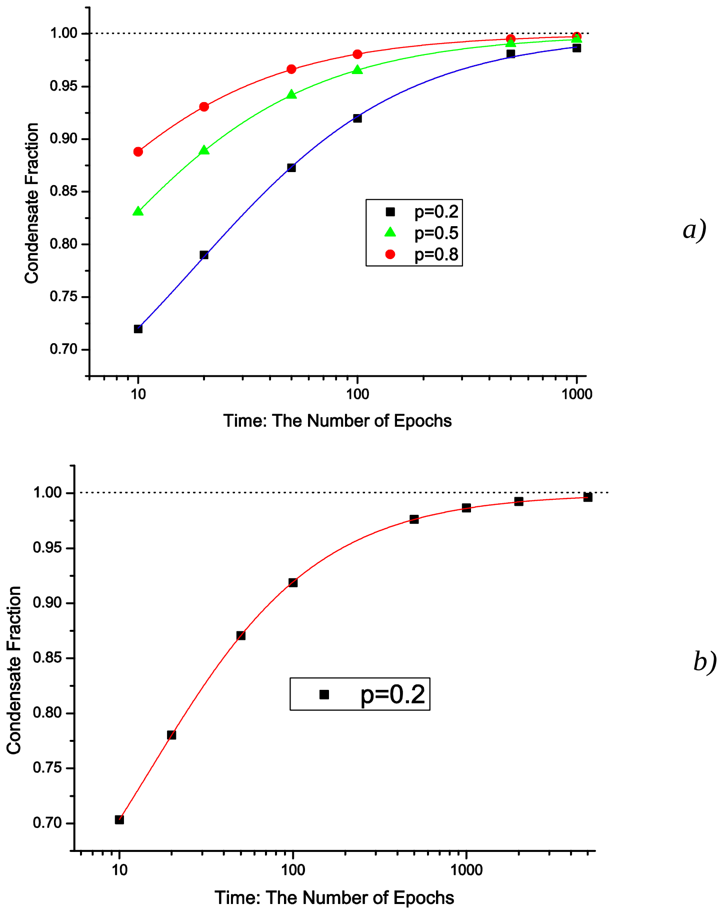
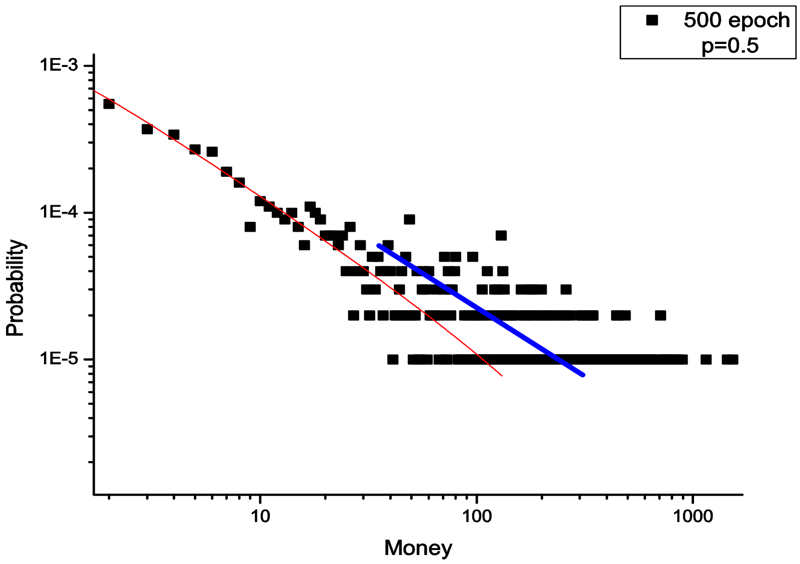


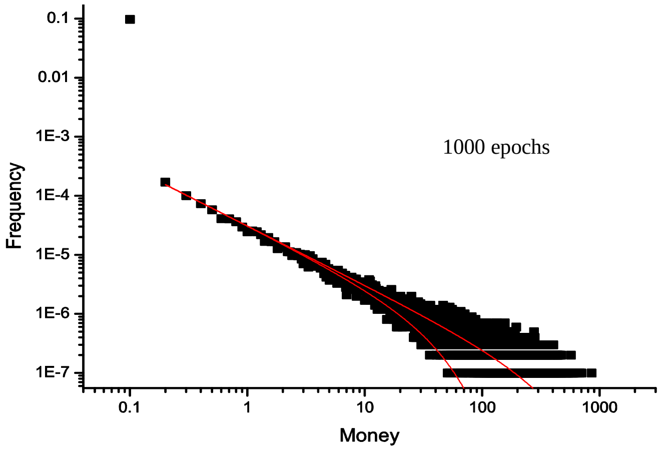
Figure 5. a) The evolution of the condensate fraction with the time for three models associated with the parameters $p = 0.8$ (black), $p = 0.5$ (red) and $p = 0.2$ (blue). The numerical data are presented by points. The smooth curve is a fitting with the function $m_{fraction}(t)=1-n_0/(t+t_0)^{\alpha}$, where the parameter α is weakly depend on p. It is equal to 0.8 for $p = 0.2$, 0.81 for $p= 0.5$ and 0.86 for $p= 0.8$. The dependence on p other fitting parameters as t_0 and n_0 is also not very strong. So we have here t_0 =12 and n_0=3.55 for $p = 0.2$, t_0 = 4.84 and n_0 = 1.5 for p= 0.5 and t_0 =3.4 and n_0 = 1.05 for $p = 0.8$. So the growth of condensate fraction with time is going on via a power law, which is very slow. The approaching to a complete condensation may happen only at infinite long time, where the condensate fraction $n_{condensate}(t = \infty) = 1$. b) The growth of a condensate fraction for the case $p = 0.2$ presented on a longer time scale, up to 5000 epochs, indicating a perfect fit with the given equation.

7. A dispersion of the money distribution at long time scales

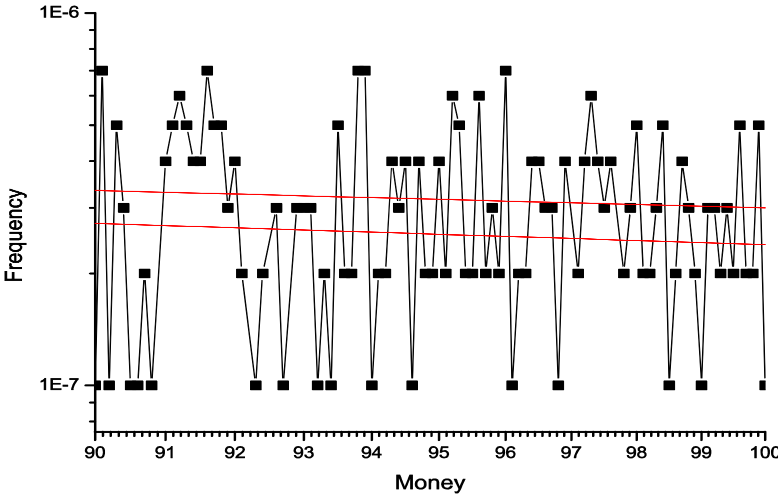
The financial market is chaotic. This forms a basis for an application of statistical methods and statistical mechanics [3] that provides a specific form for its money distribution. Indeed, in a line with these ideas we found that money distributions in many financial systems can be described by appropriate distribution functions, see for example Refs[1-6]. However, studying here the market evolution where a *Law of Increasing Poverty* operates, which we connected with the state of the market at a financial crisis, we found another new phenomenon – this is the dispersion of the money distribution. Numerical experiments indicate that at very long time scales, after many epochs $N_{epochs} >> 100$, the frequency at which agents may have a smaller and larger money may be equal, ie this happen with the same probability. For example, for the market with $p = 0.5$ after 500 epochs with a frequency one from a set of one million agents have the money in a range from 50 to 5000 relative units, see Figure 6. This effectively indicates that many rare events accumulating in the money distribution are responsible for the dispersion. From other side agents may have also large money, with large and very small probability. In other words the distribution function occupies a whole region in the probability-money or frequency money plane (see, the Figure 6 where the money distribution data recorded after 500 and 1000 epochs are presented).



a)



b)



c)

Figure 6. The distribution of money recorded after 500 epochs for the market when the parameter, p = 0.5. In the logarithmic scale used here one may clearly see the rise of fluctuation towards the region of larger money. In spite these fluctuations or an appearance of rare events one may see that the whole energy/money spectrum is well described by the BE distribution associated with the formation of the BEC state (see, the red curves, described by the eq. (3) with μ=0). a) The used fitting constant is $n_0 = 6 \cdot 10^{-6}$ and the temperature is $T = 237$. All data may be also well described by the Pareto power law, $P(m) = Tn_0/m$, that coincides with the same red line, where the fitting constant is the product: $Tn_0 \approx 0.001422$. It may be equally described by other larger fitting constant (see, the blue straight line). These fluctuations in fact correspond to some rare events happen with the frequency equal to one in a whole market history. Due to such rare events it is too difficult to give a definite answer about precise money distributions when the number of epochs is very large as 1000. Due to these rare events the data are strongly dispersed and therefore it is impossible to fit them by a single ambiguous curve. We believe that this dispersion is an intrinsic feature of any market associated with its chaotic behaviour where any agent

may have any amount of money although it may be a very rare event. Comparison of the distributions recorded after a) 500 epochs for the $p = 0.5$ and b) 1000 epochs for $p = 0.2$ indicates that they are not much different have a very strong dispersion in both cases. We also present two BE curves (in red) which correspond to the same product $Tn_0 \approx 2{\cdot}10^{-4}$, but different temperatures, $T = 20.5$ and $T = 205$, for the left and right red solid curves on Fig. 6I), respectively; c) The same as on b) but zoomed in the region where 90<m<100 and line is connecting the data points just for a guide for eyes. One sees here that the distribution in the tail cannot be approximated by analytic function. The red lines are the Pareto power law fits.

Thus the money distribution recorded after a very long time as 1000 epochs occupies a region in the frequency money plane, see Figure 6. The results have been obtained with the use of numerical experiments. Moreover we found that after such a long time the whole money distribution (or the whole dispersed region) is consistent with the same Bose-Einstein form where the chemical potential vanishes. Here the largest part of the agents has no money, i.e. they form the BE condensate state. However the money distribution of active agents is strongly dispersed. The dispersion increases when the amount of money increases. Effectively this means that some large amount of money may occur very rare but some other nearly the same money with very high probability, see c) where some small region in the money tail, $90 < m < 100$, is zoomed. In other words the data accumulate many very rare events and effectively correspond to some chaotic dynamics of the market. From one hand this justifies the usage of ergodic hypothesis and methods of statistical mechanics which predicted the Bose-Einstein distribution of money [3]. On the other hand the growing dispersion indicates that the tail of the money distribution cannot be fitted by a single unambiguous curve but rather a region in the money-frequency plane.

In summary we found that when the parameter p describing a fraction of the buyer money used in a financial transaction increases the value of the chemical potential vanishes faster with each epoch and faster growing the condensed fraction. Then more economic agents are losing all their money and form the Bose condensate. Agents in the Bose-condensate are not active and practically dead for the next market evolution. This is exactly the state of financial crisis. Note that this condensation of agents is different from a condensation of links or connections, discussed previously in a context of complex networks [15-17]. The rest, the active agents, who still have money, do continue trading, while their number decreases significantly with each new epoch of the crisis evolution.

8. Financial crisis and the condensate formation

In the present work we have studied a simple model of the financial crisis considering a fixed number of *N* economic agents interacting and trading in pairs. We have implemented the law of the "rich get richer" and considered the evolution of such a market. For a starting, original distribution of agent's money, we assumed that all agents have slightly different amounts of money with equal probability. The different initial distributions of money give the same results. There with the growing time arises a separation of agents into groups or classes. The first largest group of the financial crisis is condensed agents who have no money. Their number increases with time. The second group is active agents whose number decreases with each new epoch of the market evolution. With the time the probability of active agents to have an amount of money *m*, the quantity *P*(*m*), named as the money distribution reaches some universal shape, which may be well described by the Bose-Einstein distribution. The financial transactions between agents have been described by the parameter *p*, which effectively characterises a money turnover of the market for a certain time period. We studied the time evolution of the money distribution between agents when the parameter, *p* was fixed to different values, between 0 and 1. The most interesting finding is that the development of the financial crisis is equivalent to a growth of the Bose-Einstein condensate. The existence of the described phenomenon does not depend on a specific value of *p*, while the rate with which the Bose-condensate grows does.

The small values of p correspond to a market where trading is limited by transactions associated with small amounts of money in a comparison with an amount money which each buying agent is spending in a trading process. In such a case of the "safe market economy", even although agents are very cautious to use all their money in a single trading event they still are losing their money because of the power of the "rich get richer" law. At large *p* agents use the largest part of their money in their individual single trading processes. There the amount of the condensate increases with time very fast. But in all cases when the condensate has been formed we found that the shape of the money distribution does not depend on anything and takes the universal Bose-Einstein distribution form with vanishing chemical potential.

For small values of *p*, as $p = 0.2$, at early stage of the market, we noticed another new phenomenon – a separation of the active part of the market (the active agents) into two classes, of low and of high incomes.

The Bose-Einstein distribution emerges with the market evolution from the low class. It fits well along a very long time, although the parameters of the distribution changes. The money distribution however, seems, is splitting, explicitly down into two regions or two scales: the region associated with small money values and the second one – the high money region. Thus, the system is effectively decomposed into two classes having the lower and the higher incomes even if we have not taken into account the Bose condensate. The low class is described well by traditional Bose-Einstein distribution, where the associated market temperature, T, changes with time while the chemical potential μ remains zero. The high class is described by Pareto power (see, also the Refs [1-4]). With the time the tail in the money distribution is dispersed and the boundary threshold separating money of both classes vanishes. There a qualitative crossover of the Bose-Einstein distribution and the scale free one emerges. The creation of the Bose-Einstein distribution of money has been recently derived on the basis of an application of statistical mechanics economics and reveal in USA revenue income data [5].

There the free economy market was considered as a complex system displaying a chaotic behaviour. Taking into account this fact an ergodic hypothesis has been used. As a result statistical ensembles of the market snapshots have been built up. Then the most optimal distribution of money on such a market has been revealed to have a very general Bose-Einstein distribution form. When the average money of agents is much larger than the chemical potential of the market, the distribution takes the celebrated Boltzmann form. Analogous, although different applications of statistical mechanics and models with Boltzmann form of money distribution have been presented in a series of papers, recently [19-29].

The decreasing ratio of the fitting parameters $|\mu|/T$ means the motion of the economic system towards financial crisis [5]. Here, we have shown that the development of financial crisis is a time dependent formation of the Bose-Einstein condensation. Although it was noticed earlier that the Bose-Einstein Condensation phenomenon could arise during the financial crisis [5], its development and consequences have not been studied. The formation and evolution of such condensate consisting of non-active ("dead") economic agents as a form of financial crisis has been described here for the first time. Contrary to the financial crisis associated with a formation of the (BEC) condensate and an economic growth of human society and an exit out-of-recession can correspond to an activation of the "dead" agents. In a process of such activation their number in the condensate decreases and they again have a new life to trade and to support the economy. Thus, we have here shown that for the “rich get richer” principle known as a Law of Increasing Poverty, the crisis arises very fast, in a few epochs from the start of the market operation. During the crisis most of the agents are in the BEC ground state, which is different from the active trading state in which the rest of the agent, do exist. The condensed agents with zero money may be activated by debt, although this may lead to other problem or by luck or fortune when they will sell some product successfully.

Summary

We have described a wild market economy where the majority of economic agents are losing their money and dropped in a state without money. We call such a state of economic agents as Bose-Einstein “condensate” that reminiscent BEC state existing in physical systems. When this happens the number of agents who have no money, i.e. in the BE condensate, is of the order of the total number of agents, *N*. There is no here a strict phase transition into the BE condensate, but rather a crossover. The money distribution of the rest active agents has been identified by fitting as a Bose-Einstein one with zero chemical potential. Such a condensed state has been created after some period of time expressed as a fixed number of epochs.

The excellent fitting of the BE money distribution and the formation of the BE condensate on a market where a financial crisis developing confirm the general idea that market should be described with the use of statistical mechanics and with BE distributions.

In conclusion, we have considered a simple model of a free market economy which operates according to a rich get richer law. We have demonstrated that such market leads directly to a financial crisis associated with a formation of a new condensed state. In such a state a numerous agents lost all their asset and dead for the next market operations. We shown that for any market parameters chosen such as a number of agents, the amount of money or the fraction p of money used in the financial transaction the trading on this market is very risky. The market evolution unavoidably leads to a formation of the Bose-condensed state where a significant fraction of economic agents lost their money forming the Bose-condensate. The next participation of these agents in the market is prohibited unless they will loan the money and receive the debt or be very lucky to inherit wealth. In such a market poor people get poorer and poorer. At the long term there are always left a few rich people on the market, i.e. having a lot of money, who are trading between each other- finally "the winner takes it all". While the majority of population forms the condensate who is not trading because it has no money. Because the condensate has nothing to do financially it can become politically or otherwise active and violent as recent riots in London. The government should take care to stop the growth of the condensate, this is a very important.

In next we plan to consider the generalisation of this model market with debt and salary which awake dead agents and increase a population of the active ones.

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