MEASURING THE GROWTH AND THE STRUCTURAL CHANGE OF A MULTI-SECTOR ECONOMY

Matti ESTOLA*

Abstract. *The production process of an n-sector economy is described by a vector-function analogously as the motion of an ideal particle. The annual aggregate Gross Value Added and the annual speed in production value are shown to be two norms of the same velocity vector of accumulated sectoral production values, and thus they measure the same matter. Structural change in production is analyzed in this framework too. The structure of production changes when the shares of sectoral productions change. Our first measure for structural change is constructed in this way. Another measure for structural change is constructed from the shape of the space curve an economy leaves in the production space. Empirical measures corresponding to the theoretical ones are calculated by Finnish data.*

Keywords: *Disaggregate macroeconomics, particle motion, economic growth, structural change. JEL: C02, E01, O11.*

1. Introduction

We define helpful concepts for analyzing the growth and the structural change process of an economy. Our approach is analogous to kinematics in physics: *Kinematics is the study of the geometry of motion; it deals with the mathematical description of motion in terms of position, velocity, and acceleration. Kinematics serves as a prelude to dynamics, which studies force as the cause of changes in motion* ([1] p. 25).

A change in production in an *n*-sector economy consists of *n* simultaneous one-dimensional changes. We describe this process by a vector-function illustrating the motion of an ideal particle − a body with no size and no internal structure. This ideal particle is represented by a point in the *n*-space of accumulated sectoral productions; the point illustrates the

* University of Eastern Finland, Faculty of Social Sciences and Business Studies, E-mail: matti.estola@uef.fi, Presentation for Econophysics, New Economy & Complexity, International Conference at 26-28 May, 2011, HYPERION UNIVERSITY and Hyperion Research & Development Institute, 169, Calea Calarasilor street, 030615, Bucharest 3rd District, ROMANIA. Thanks to an anonymous referee for improving comments.

position of the economy at a particular moment of time. Mathematically, the movement of a point with time in an *n*-dimensional space can be analyzed by a vector-function, see [2], pp. 21-27 and [3], pp. 517-550. The motion of an economy with time, illustrated by a change in the value of a vector-function, leaves a space curve in the *n*-space of accumulated sectoral productions. The moving economic object could as well be a multi-product firm or one industry with several producers, but here we study the disaggregated production process at economy level.

The advantage of modeling accumulated production, rather than annual flows of production, is that the production process becomes continuous. This allows us to measure the position, velocity, and acceleration of production in a fixed coordinate system. With these kinematic concepts we can study whether the assumption in the neo-classical theory − that firms produce at their profit maximizing equilibrium flows of production − is true. It appears that in every 3 main sector in Finland statistically significant non-zero acceleration in production is observed, see Section 6.

The value and volume of production and their growth are measured by the speed and the change of speed (not acceleration which is a vector quantity) of accumulated value and volume of production of an economy in *n*-dimensional space. The speed of production of an economy is one norm, and the Gross Value Added (*VA*) is another norm of the same velocity vector of accumulated production. Thus the speed of production of an economy and its change measure the aggregate flow of production and its growth, which analogy is often made in newspapers.

The advantage of analyzing growth in disaggregated framework − as compared with standard aggregated framework − is that changes in production structure can be analyzed in this framework too. Modeling accumulated sectoral productions separately, and combining these to model the aggregate *VA* rather than modeling the single time series of aggregate *VA*, hopefully gives us better forecasts. We introduce here the principle for this kind of analysis.

In a growing economy, the structure of production changes so that the share of primary production decreases and that of services increases, see e.g. [4]. This results among other things from Engel's law which postulates that a rise in living standard changes the consumption structure so that necessary goods are crowded out by luxury ones, see [5]. According to [6], the structural change in employment occurs due to changes in technology and demand while [7]-[8] stress the role of technologically leading industries in economic growth and structural change. Thus we have evidence that in a growing economy different sectors expand at varying rates, and this effect is concealed in aggregated analysis.

However, the concepts of structure and structural change in economic analysis have been understood in at least nine different ways, see [9]. Thus it is not clear what we mean by these concepts. In classical works, e.g. [10] and [11], it is clearly understood that the structure of an economy changes with its development. Formal approaches for this have been developed by Wassily Leontief, Luigi Pasinetti, and Richard Goodwin among others. These authors stress the difference between "horizontal" and "vertical" representation of economic structure. The former describes the circular structure of an economic system while the latter stresses the unidirectional process where individual agents cluster into a smaller number of classes, see [12].

In the last 20 years emerged a literature called evolutionary or "neo-Schumpeterian" economics, see e.g. [13] or [14]. This framework stresses disharmony in the growth process and the role of micro and industrial level changes on macro dynamics because technological innovations spread in economies at different sectoral speeds, see [15]. Our framework, too, belongs in evolutionary economics. However, we do not study the reasons for structural change in an economy. We only measure the process by two different ways that give similar results. We treat structural change as a continuous process and show that during 1975-2002 Finland faced one exceptional period in this process at depression years 1990-92.

The only reference we found that analyzes structural change by using vector space methods like we is [16]. The measures for structural change introduced in [16] are based on the angle between two vectors, while our measures are based on a change of a vector-function. The coordinates of the vector in [16] are sectoral productions while we use the shares of sectoral productions of total production. Thus our measures are not directly comparable.

Other ways to measure structural changes have been concentrated on testing the stability of parameters in regression models; see e.g. [17]-[19]. These tests, no doubt, document changes in the production structure, but they do not inform about the causes of these changes. The advantage of our framework, as compared with e.g. [17], is that our measures can be used as endogenous variables in models that explain the structural change process.

This study is organized as follows. The data is described in Section 2. In Section 3 the production process of a n-sector economy is studied by the motion of an ideal particle in the n-space of accumulated sectoral productions. The framework introduced in Section 3 is applied in Sections

4 and 5 in measuring the structural change in production in an economy. The acceleration of sectoral productions is measured in Section 6, empirical results from structural change are given in Section 7, and Section 8 is a summary.

2. The data used in the study

We use annual sectoral production values at current prices and at year 2000 prices from Finnish economy. The latter are used to represent production volumes. The data contains the following 9 main sectors at 1975-2002 ([20]): 1 = A (Agriculture, forestry and hunting) + B (Fishing), 2 = C (Mining and quarrying) + D (Manufacturing) + E (Electricity, gas, and water supply), 3 = F (Construction), 4 = G (Trade, repair of motor vehicles and household goods) + H (Hotels and restaurants), 5 = I (Transport, storage and communication), 6 = J (Financial intermediation and insurance), 7 = K (Real estate and business activities), 8 = L (Administration, compulsory social security) + M (Education), 9 = N (Health and social work) + O (Other community, social and personal services) + P (Household service activities) – Financial intermediation services indirectly measured (FISIM). These 9 sectors cover the total production in Finland. In some of the graphical demonstrations, the 9 main sectors are aggregated to 3 as follows: X_1 (primary production) = A+B, X_2 (secondary production) = C+D+E, and X_3 (private and public services) = F+G+H+I+J+K+L+M+N+O+P - FISIM.

In Finland, manufacturing is divided in 13 sectors: DA: Food products, beverages and tobacco, DB+DC: Textiles, textile products, leather and leather products, DD: Wood and wood products, DE: Pulp, paper and paper products, publishing and printing, DF: Refined petroleum products, coke and nuclear fuel, DG: Chemicals and chemical products, DH: Rubber and plastic products, DI: Other non-metallic mineral products, DJ: Basic metals and fabricated metal products, DK: Machinery and equipment, DL: Electrical and optical equipment, DM: Transport equipment, DN: Other manufacturing and recycling. These sectors cover the whole Finnish manufacturing and we have annual data of these from years 1975-2008.

3. Production according to particle motion

We describe the production in an *n*-sector economy by a vector-function in a coordinate system. “*A coordinate grid, together with a set of* *synchronized clocks at every point in space, is called a reference frame*", [1] p. 5. Let time unit (t_0, t_1), where $t_0 < t_1$ are two time moments, be partitioned in intervals Δs. The accumulated value of production at sector *k* during (t_0, t_1) is then:

$$X_k(t_0, t_1) = \sum_{s=t_0}^{t_1} Q_k(s)\,\Delta s$$

and letting $\Delta s \to 0$ we get:

$$X_k(t_0,t_1) = \lim_{\Delta s \to 0} \sum_{s=t_0}^{t_1} Q_K(s)\,\Delta s = \int_{t_0}^{t_2} Q_k(s)\,\mathrm{d}s.$$

Now, $\int_{t_0}^{t_1} Q_k(s)\,\mathrm{d}s$ with unit € measures the accumulated value of production at sector *k* during (t_0, t_1). This is analogous as in physics. If, for instance, Q_k is the velocity of a body with unit km/hour, $\int_{t_0}^{t_1} Q_k(s)\,\mathrm{d}s$ with unit *km* measures the movement of the body during time $t_1 - t_0$.

With cumulative sectoral production values as the coordinate axes, our reference frame is:

$$X_1 = \int Q_1(s)\,\mathrm{d}s, \quad \ldots, \quad X_n = \int Q_n(s)\,\mathrm{d}s. \tag{1}$$

Cumulative sectoral production values in the coordinate axes make the measurement units of speed €*/y* and acceleration €/*y*^{2} of accumulated value of production similar to those found in mechanics; *y* is a time unit like a year or a month. In this coordinate system, the point describing the position of the economy moves away from the origin all the time, and it does not change its direction of motion toward the origin. In particle motion in physics, analogously, a coordinate axis measures the length the particle has moved in the direction in a time unit. An analogous coordinate system for (1) is devised for production volumes too. Annual value of production at sector *k* at year t is denoted as:

$$Q_k(t)\,(€/y)\,,\;\; Q_k(t) = p_k(t)\,q_k(t) \;\text{ where }\; p_k(€/unit)$$

is the price and $q_k(unit/y)$ the annual volume of production. The empirical correspondence for annual volume of production at year t is

$Q_{kr}(t)$, $Q_{kr}(t) = p_k(t_0)\, q_k(t)$, where t_0 is fixed (in our case, year 2000). In empirical analysis, $Q_k'(t)$ is approximated by $\Delta Q_k / \Delta t$ with:

$$\Delta t = 1(year).$$

Designating the initial time moment as 0, the position of the economy in the *n*-space of accumulated production values at time moment *t* can be denoted by the following vector-function:

$$X(t) = (X_1(t), \dots, X_n(t)) = \left(\int_0^t Q_1(s)\mathrm{d}s, \dots, \int_0^t Q_n(s)\,\mathrm{d}s \right). \quad (2)$$

In time $(0, t)$, Function (2) can be graphed as a space curve in coordinate system (1). In Figures 1 and 2 are three-dimensional realizations of Function (2) of the Finnish economy at 1975-2002 with accumulated values and volumes of sectors X_1, X_2, and X_3 as the coordinate functions, see Section 2. Every observation represents the position of the Finnish economy at the corresponding year. In Figure 2, the observed points are modeled by a second order time polynomial (the continuous graph). The graph is constructed from the models for the Sectors X_1, X_2, and X_3 shown in Figures 3-5; see Section 6. Figures 1 and 2 show how the Finnish economy "moves" along the curve with time, which establishes our analogy to particle motion.

The average velocity vector of the economy along space curve (2) is:

$$\frac{\Delta X(t)}{\Delta t} = \left(\frac{\Delta X_1(t)}{\Delta t}, \dots, \frac{\Delta X_n(t)}{\Delta t} \right), \quad (3)$$

and the instantaneous velocity vector of the economy at point $X(t)$ is:

$$X'(t) = (X_1'(t), \dots, X_n'(t)) = (Q_1(t), \dots, Q_n(t)). \quad (4)$$

Geometrically, $X'(t)$ is the tangent vector of the space curve (2) at point $X(t)$. Instantaneous acceleration vector of accumulated value of production in the economy is defined analogously:

$$X''(t) = (Q_1'(t), \dots, Q_n'(t)).$$

In Newtonian mechanics, the three-dimensional real physical world is approximated by the three-dimensional Euclidean space. Assuming that the above-described production space is approximately Euclidean, we can calculate the Euclidean norm of vector $X'(t)$ in the *n*-space as:

$$Speed(t) = \| X'(t) \| = (Q_1^2(t) + \cdots + Q_n^2(t))^{1/2}. \qquad (5)$$

Scalar *Speed* with unit $€/y$ measures the instantaneous velocity of accumulated value of production in the economy in reference frame (1) at point $X(t)$. An example of calculating the *Speed* of accumulated value of production in two-dimensional case is given in Figure 6, where X_i is the accumulated value of production at sector $i, i = 1, 2.$ The length of vector *AB* is $\|\Delta X\|,$ where $\|\Delta X\| = X(t_1) - X(t_0)$ and $X(t_j) = (X_1(t_j), X_2(t_j)),$ *j* = 0, 1. Applying Pythagorean theorem we get:

$$\|\Delta X\|^2 = (\Delta X_1)^2 + (\Delta X_2)^2 \Rightarrow \|\Delta X\| = \sqrt{(\Delta X_1)^2 + (\Delta X_2)^2} \,. \qquad (6)$$

Now, during time $\Delta t = t_1 - t_0$ the economy has moved from point $(X_1(t_0), X_2(t_0))$ to point $(X_1(t_1), X_2(t_1)).$ The speed of accumulated value of production at sector *i* (the speed of movement in direction X_i) is then $\Delta X_i / \Delta t,\ \ i, i = 1, 2.$ However, the total “length” the point has moved in the space of accumulated production values during Δt is $\|\Delta X\|$. The average speed of the *“point representing the accumulated value of production in the economy”* during Δt is then:

$$\frac{\|\Delta X\|}{\Delta t} = \sqrt{\left(\frac{\Delta X_1}{\Delta t}\right)^2 + \left(\frac{\Delta X_2}{\Delta t}\right)^2}. \qquad (7)$$

Taking the limit $\Delta t \to 0$ we get the instantaneous speed of the accumulated value of production at t_0 as:

$$\| X'(t_0) \| = \sqrt{(X_1'(t_0))^2 + (X_2'(t_0))^2}, \qquad (8)$$

which corresponds to (5) with *n* = 2.

Acceleration of accumulated value of production has unit $€/y^2$. This is seen as:

$$Q_k'(t) = \lim_{\Delta t \to 0} \frac{\Delta Q_k}{\Delta t} = \lim_{t-(t-y) \to 0} \frac{Q_k(t) - Q_k(t-y)}{t-(t-y)} = \lim_{y \to 0} \frac{Q_k(t) - Q_k(t-y)}{y}$$

where $Q_k(t) - Q_k(t-y),\ \ k = 1, \ldots, n,$ are measured in units $€/y.$

Economy level nominal *VA* with unit $€/y$ is calculated by adding the sectoral value additions Q_k. In coordinates (1), *VA*-measure is the absolute value norm $\| X'(t) \|_1 = \sum_{k=1}^{n} |Q_k(t)|$ of vector (4) because the components of (4) are non-negative. Thus the Euclidean norm (5) of vector (4) measures the same quantity as the *VA*-measure because they are two norms of the same vector.

Using *VA* we get a measure for growth in the aggregate value in production as:

$$\Delta VA(t) = VA(t) - VA(t-y) = \sum_{k=1}^{n} Q_k(t) - \sum_{k=1}^{n} Q_k(t-y) = \sum_{k=1}^{n} \Delta Q_k(t).$$

Using *Speed* we get another measure for growth in the aggregate value flow of production:

$$\Delta Speed(t) = Speed(t) - Speed(t-y) = \left(\sum_{k=1}^{n} Q_k^2(t) \right)^{1/2} - \left(\sum_{k=1}^{n} Q_k^2(t-y) \right)^{1/2}.$$

Now, *VA* and *Speed* both measure the movement of a point in coordinates (1), and the reason for defining *Speed* is that it shows the connection to the theory of particle motion that is applied here as the general framework. The empirical values for *VA* and *Speed,* and those for ΔVA and $\Delta Speed$ are highly correlated, see Section 7. This shows that the two measures for the aggregate flow of accumulated production quantify the same matter. Using value data these measures are diagrammed in Figures 7 and 8, and using volume data, in Figures 9 and 10.

4. Measurement of structural change

The production structure of an economy stays constant if the *VA* – share of every sector stays constant. If one sector increases or decreases its share during time unit *y*, a distance exists between the sectoral share vectors at time moments $t-y$ and *t*. According to the previous section, at time moment *t* the production structure of an economy is described by Vector-Function **V:**

$$\mathbf{V}(t) = (V_1(t), \dots, V_n(t)) = \left(\frac{Q_1(t)}{\sum_{k=1}^{n} Q_k(t)}, \dots, \frac{Q_n(t)}{\sum_{k=1}^{n} Q_k(t)} \right), \tag{9}$$

where the coordinate functions are sectoral shares of the total value of production. Structural change is measured by the change of function (9) with time. The derivative vector of (9) is:

$$V'(t) = (V_1'(t), \ldots, V_n'(t)) =$$

$$= \left(\frac{Q_1'(t)\left(\sum_{k=1}^{n} Q_k(t)\right) - Q_1(t)\left(\sum_{k=1}^{n} Q_k'(t)\right)}{\left(\sum_{k=1}^{n} Q_k(t)\right)^2}, \ldots \right.$$

$$\left. \ldots, \frac{Q_n'(t)\left(\sum_{k=1}^{n} Q_k(t)\right) - Q_n(t)\left(\sum_{k=1}^{n} Q_k'(t)\right)}{\left(\sum_{k=1}^{n} Q_k(t)\right)^2} \right), \qquad (10)$$

and its Euclidean norm is:

$$v_n(t) = \| V'(t) \| = (V_1'^2(t) + \ldots + V_n'^2(t))^{1/2}. \qquad (11)$$

Measure v_n calculated from the 9 main sectors of the Finnish economy is displayed in Figure 11. We can notice that v_n gives a positive value for structural change also when the structure reverts to what it has previously been. Cyclical behavior may thus cause v_n to overestimate the long-term structural change. To eliminate this problem, the corresponding long-term measure can be calculated e.g. from five-year averages of annual production values.

Eq. (10) implies that the structural change of an economy depends on sectoral price and volume changes, and on the “sizes” of the sectors. If we write $Q_j(t) = p_j(t)\, q_j(t),$ we get:

$$V_j'(t) = \frac{p_j(t)\, q_j}{\sum_{k=1}^{n} p_k(t)\, q_k(t)} + \frac{p_j(t)\, q_j'(t)}{\sum_{k=1}^{n} p_k(t)\, q_k(t)} - \frac{p_j(t)\, q_j(t) \sum_{k=1}^{n} p_k'(t) q_k(t)}{\left(\sum_{k=1}^{n} p_k(t)\, q_k(t)\right)^2} -$$

$$- \frac{p_j(t)\, q_j(t) \sum_{k=1}^{n} p_k(t) q_k'(t)}{\left(\sum_{k=1}^{n} p_k(t)\, q_k(t)\right)^2}. \qquad (12)$$

Thus the share of sector j increases if its price and flow of production increase, and decreases if prices and flows of production increase at other sectors.

If we calculate (11) from sectoral shares of total volume flow of production, we get a measure for structural change in production volumes. This measure v_r from the 9 main sectors of the Finnish economy is displayed in Figure 13. Measures v_n, v_r are calculated for Finnish manufacturing too with data 1975-2008. In this case Vector-Function (9) displays the shares of the 13 manufacturing sectors of total manufacturing production, and measures v_{nm}, v_{rm} are constructed analogously. These measures are presented in Figures 17 and 19.

5. Non-linearities in production path

Here we study the production structure in another way by measuring whether the movement of the economy in the space of accumulated sectoral productions has been linear or not, that is, whether the structure has remained constant. To illustrate this framework, see Figures 1 and 2. The non-linearity of these space curves reveals a change in the production structure. We calculate the instantaneous curvature of the curve corresponding to Function (2). If this space curve is linear, the structure of the economy stays constant or changes in a linear way. We use the extent of non-linearity of Function (2) in measuring the change in the production structure.

The extent of non-linearity of a curve can be measured by its curvature. In our case:

$$k_n(t) = \frac{\| T'(t) \|}{\| X'(t) \|} \tag{13}$$

is the formula for curvature, and the velocity vector is:

$$X'(t) = (X_1'(t), \ldots, X_n'(t)) = Q_n(t)),$$

and:

$$T(t) = \frac{X'(t)}{\| X'(t) \|} = \left(\frac{X_1'(t)}{\| X'(t) \|}, \ldots, \frac{X_n'(t)}{\| X'(t) \|} \right) \tag{14}$$

is the unit tangent vector of the curve when $\| X'(t) \|$ is nonzero. Then:

$$T'(t) = \left(\frac{X_1''(t) \| X'(t) \| - X_1'(t) \dfrac{X'(t) \cdot X''(t)}{\| X'(t) \|}}{\| X'(t) \|^2}, \ldots \right.$$

$$\left. \ldots, \frac{X_n''(t) \| X'(t) \| - X_n'(t) \dfrac{X'(t) \cdot X''(t)}{\| X'(t) \|}}{\| X'(t) \|^2} \right),$$

where is the dot product of two vectors. The curvature of a curve is non-negative, and a linear curve has zero curvature, see [3] p. 537.

The instantaneous curvature of Function (2) measures change in the production structure. Measure k_n calculated from the value data of 9 main sectors is displayed in Figure 12, and measure k_r from the volume data of 9 main sectors in Figure 14. The corresponding measures for manufacturing, k_{nm}, k_{rm}, are in Figures 18 and 20. The difference in these measures can be seen by writing k_n in an expanded form:

$$k_n(t) = \left(\left(\frac{Q_1'(t) \left(\sum_{k=1}^{n} Q_k^2(t) \right) - Q_1(t) \sum_{k=1}^{n} Q_k(t)\, Q_k'(t)}{\left(\sum_{k=1}^{n} Q_k^2(t) \right)^2} \right)^2 + \right.$$

$$\left. + \left(\frac{Q_n'(t) \left(\sum_{k=1}^{n} Q_k^2(t) \right) - Q_n(t) \sum_{k=1}^{n} Q_k(t)\, Q_k'(t)}{\left(\sum_{k=1}^{n} Q_k^2(t) \right)^2} \right)^2 \right)^{1/2}.$$

Although v_n and k_n scaled at equal level in Figures 15 and 16 look similar, they measure different things; v_n has unit 1/*y* while k_n has 1/€, and the values of k_n are much smaller.

6. Modeling the production process

We estimate the following model for accumulated volume of production at sector k, $X_{kr}(unit)$:

$$X_{kr}(t) = \int_0^t q_k(s)\,\mathrm{d}s = a_k + b_k t + \frac{c_k}{2} t^2, \ \ k = 1, 2, \ldots, \tag{15}$$

where $t$is time with unit y, $q_k(unit/y)$ the flow of volume of production at sector (industry) k, and a_k, b_k and c_k are constants with units: *unit*, *unit/y*, and $unit/y^2$, respectively; $y = year$. These constants can be interpreted as: $q_k(t) = b_k + c_k t$ and $q'_k(t) = c_k$; thus c_k measures acceleration of the accumulated volume of production at sector k, see Section 3.

The estimation results for model (15) for primary X_{1r} and secondary production X_{2r} and private and public services X_{3r} are in Table 1; *D-W* is the Durbin-Watson statistic.

Table 1

Estimated models for accumulated production volumes at sectors X_1, X_2 and X_3

Sector	Constant (*T*-stat.)	Time (*T*-stat.)	Time^2 (*T*-stat.)	R^2	*D-W*
X_{1r}	-6.9×10^7 (–6.9)	65065.0 (6.5)	−15.3 (−6.1)	0.999	0.28
X_{2r}	1.2×10^9 (20.4)	-1.3×10^6 (−20.8)	322.7 (21.3)	0.999	0.25
X_{3r}	2.4×10^9 (21.9)	-2.5×10^6 (−22.4)	640.8 (22.9)	0.999	0.22

Table 1 shows that the acceleration of production volume has been statistically significantly negative in Sector X_1, and significantly positive in other sectors. The assumption in the neo-classical framework, that firms produce an equilibrium amount in a time unit, is thus rejected in every case. The DW-statistic shows that the models have a positive auto-correlation problem in residuals, which implies that a cyclical term is missing. However, we did not get rid of the problem by adding cyclical terms in the model, and so we report the results as such. The estimated models are displayed in Figures 3-5 and they are combined to model the accumulated aggregate production in Figure 2.

7. Empirical results for the structural change measures

Measures *VA* and *Speed* displayed in Figures 7 and 9 in nominal and real terms show a clear similarity. The correlations between the production and growth measures are in Tables 2 and 3. The correlation between the nominal flow of production measures is 0.999, and that between the real measures is 0.997. The correlations between ΔVA and $\Delta Speed$ in nominal and real terms are 0.950 and 0.978, respectively. Thus both measures contain the same information.

Table 2
Correlations between the "flow of production value and volume" measures

	VA_n	$Speed_n$	VA_r	$Speed_r$
VA_n	1.000			
$Speed_n$	0.999	1.000		
VA_r	0.985	0.986	1.000	
$Speed_r$	0.987	0.999	0.997	1.000

Figures 11-14 reveal the change in the production structure in Finland in nominal and real terms. The highest peak in the figures can be explained by the Finnish depression at years 1990-92 (see Figures 7-10), which was caused by the rapid liberalization of the Finnish financial markets and the collapse of the Soviet Union. Soviet Union covered roughly 25% of Finnish exports, and this share decreased close to zero during these years. Figures 11-14 imply that the structural change process has been slightly decreasing during the period excluding the depression years.

Table 3
Correlations between the "growth of production value and volume" measures

	ΔVA_n	$\Delta Speed_n$	ΔVA_r	$\Delta Speed_r$
ΔVA_n	1.000			
$\Delta Speed_n$	0.950	1.000		
ΔVA_r	0.852	0.833	1.000	
$\Delta Speed_r$	0.852	0.871	0.978	1.000

Figures 15 and 16 show a remarkable similarity between measures v and k when scaled at equal levels. The correlation matrix of all variables measuring structural change is in Table 7. Correlation 0.898 between v_r and k_r shows strong dependency. However, measured from values this dependency is weaker; the correlation between v_n and k_n is only 0.494.

Table 4

Correlations between the structural change measures

	v_n	k_n	v_r	k_r
v_n	1.000			
k_n	0.494	1.000		
v_r	0.767	0.446	1.000	
k_r	0.628	0.699	0.898	1.000

The structural change in Finnish manufacturing does not seem to have been as great during the depression years 1990-92 as in the whole economy, see Figures 17-20. The change in the composition of volume flows in manufacturing at 1990-92 is seen as a peak in the structural change process, but even a greater peak occurred at other depression years 1999-2001, and also at 2005. On the other hand, in value flows in manufacturing no such peak is seen at 1990-92.

8. Conclusions

We used the theory of particle motion and kinematics in classical mechanics in measuring production and its growth, and the structural change in production in a multi-sector economy. The speed of production and the aggregate Value Added were shown to be two norms of the same velocity vector of accumulated production. When the flow of production of the Finnish economy was measured by means of these two concepts, their correlation was roughly 0.99. Thus both measures were shown to quantify the same matter. A vector of sectoral production shares was used to describe the production structure of an economy, and the change of this vector-function was used as a measure for structural change. The curvature of the space curve describing the motion of an economy in the coordinate system of accumulated sectoral productions was used as another measure for structural change. These measures were calculated using Finnish data.

Appendix

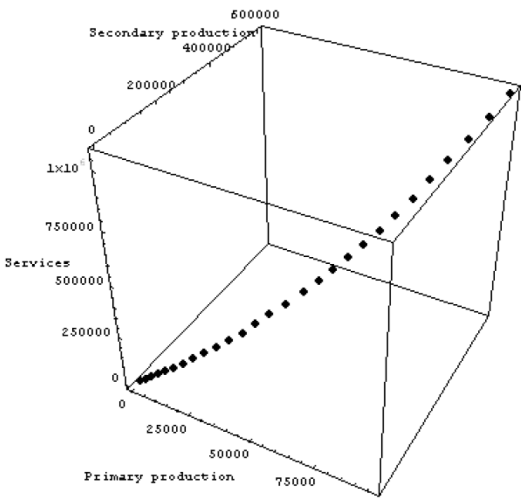

Figure 1. Accumulated production values in Finland at 1975-2002.

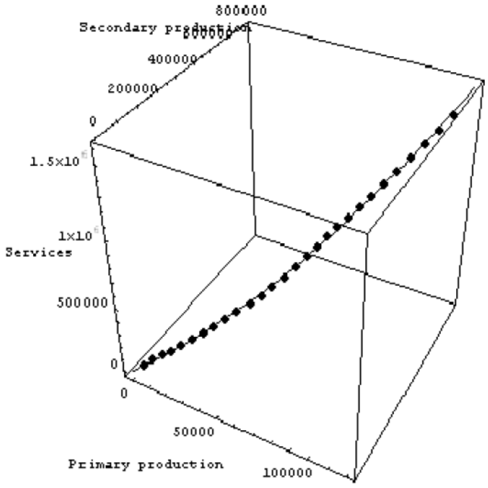

Figure 2. Accumulated and fitted production volumes in Finland at 1975-2002.

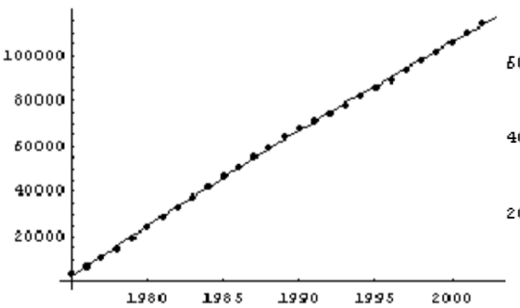

Figure 3. Accumulated and fitted volume of secondary production.

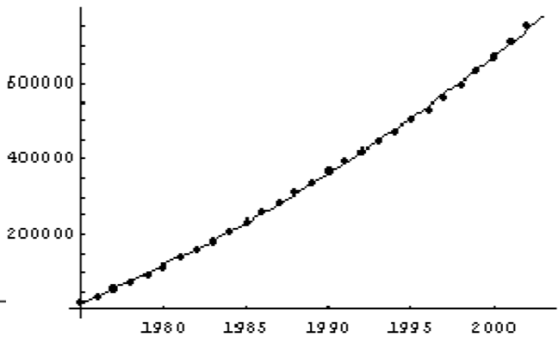

Figure 4. Accumulated and fitted volume of primary production.

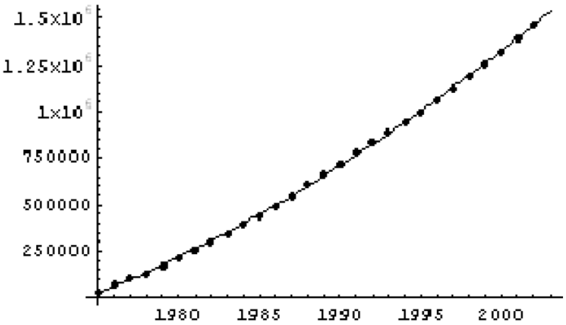

Figure 5. Accumulated and fitted volume of primary production in a and public services.

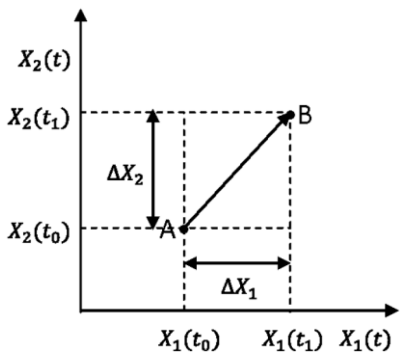

Figure 6. The speed of value of production in a and public services two-good system.

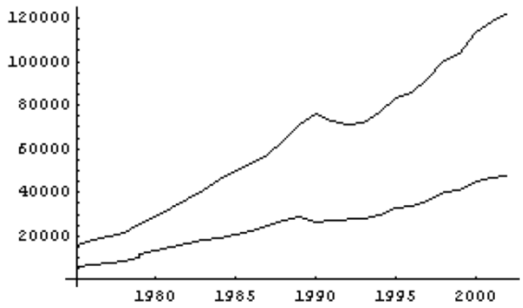

Figure 7. VA_n and $Speed_n$.

Figure 8. ΔVA_n and $\Delta Speed_n$.

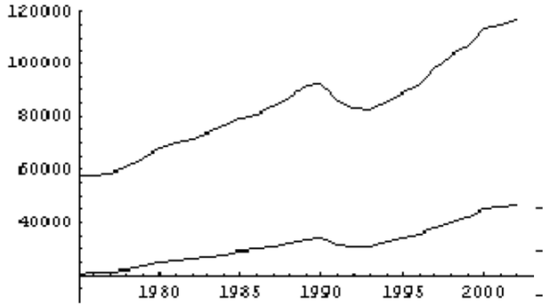

Figure 9. VA_r and $Speed_r$.

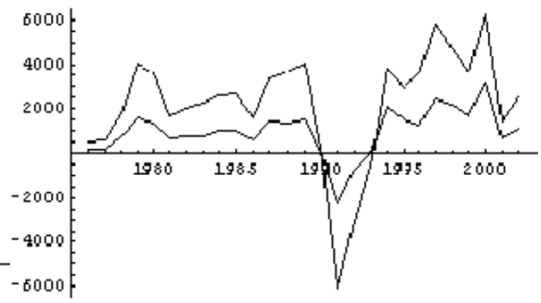

Figure 10. ΔVA_r and $\Delta Speed_r$.

Figure 11. Measure υ_n.

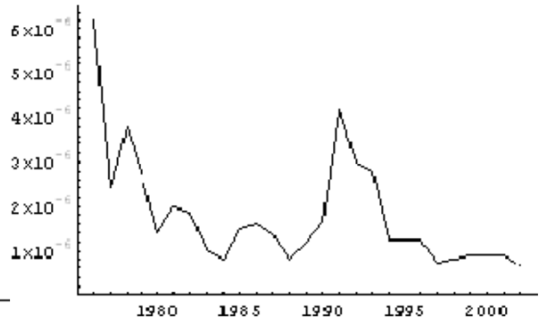

Figure 12. Measure k_n.

Figure 13. Measure υ_r..

Figure 14. Measure k_r.

Figure 15. Measures υ_n and k_n scaled at equal level.

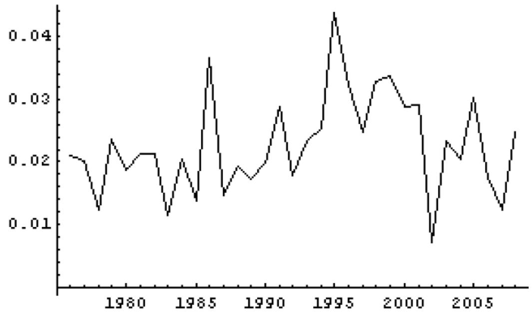

Figure 16. Measures υ_n and k_r scaled at equal level.

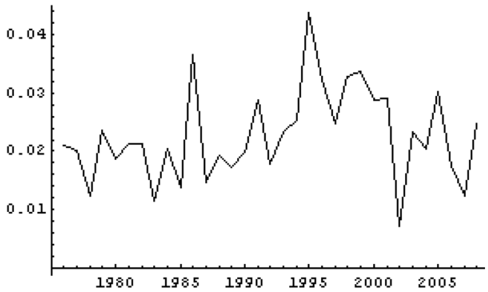

Figure 17. Measure v_{nm} from Finnish manufacturing.

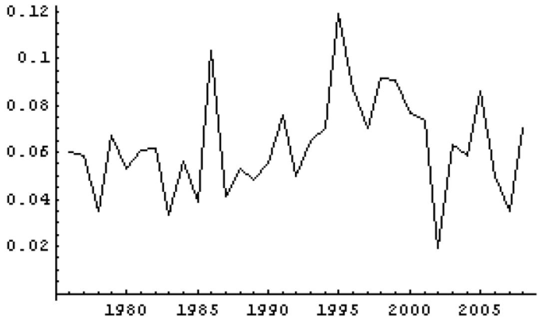

Figure 18. Measure k_{nm} from Finnish manufacturing.

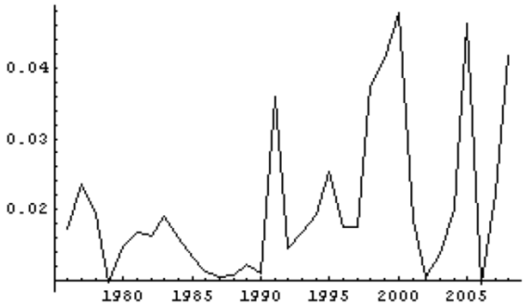

Figure 19. Measure v_{rm} from Finnish manufacturing.

Figure 20. Measure k_{rm} from Finnish manufacturing.

REFERENCES

[1] H. C. Ohanian, *Physics, Second Edition Expanded*. USA: W. W. Norton & Company, Inc. 1989.

[2] G. R. Fowles, *Analytical Mechanics.* USA: Holt, Rinehart and Winston, 1970.

[3] T. Apostol, *Calculus, Volume I, Second Edition*. USA: John Wiley & Sons, 1967.

[4] L. Bonatti and G. Felice, Endogenous growth and changing sectoral composition in advanced economies. *Structural Change and Economic Dynamics*, **19**, 109-31 (2008).

[5] N. Palan & C. Schmiedeberg, Structural Convergence of European countries. *Structural Change and Economic Dynamics,* **21**, 85-100 (2010).

[6] L. Pasinetti, *Structural Change and Economic Growth. A Theoretical Essay on the Dynamics of the Wealth of Nations*. Cambridge University Press, 1981.

[7] J. Fagerberg, Technological progress, structural change and productivity growth: a comparative study. *Structural Change and Economic Dynamics*, **11**, 393-411 (2000).

[8] M. P. Timmer and A. Szirmai, Productivity in Asian manufacturing: the unimportance of structural change. *Structural Change and Economic Dynamics*, **11**, 371-392 (2000).

[9] E. G. Silva and A. A. C. Teixeira, Surveying Structural Change: Seminal contributions and a bibliometric account. *Structural Change and Economic Dynamics* **4**, December, 273-300 (2008).

[10] A. Smith, *An Inquiry into the Nature and Causes of the Wealth of Nations (*1776*)*. Reprinted by Adamant Media Corporation, 2006.

[11] J. A. Schumpeter, *Business Cycles: A Theoretical, Historical, and Statistical Analysis of the Capitalist Process*. McGraw-Hill, New York and London, 1939.

[12] P. Sraffa, *Production of Commodities by Means of Commodities: Prelude to a Critique of Economic Theory.* Cambridge University Press, 1960.

[13] G. Dosi, Sources, procedures, and microeconomic effects of innovation. *Journal of Economic Literature,* **26**, 1120-71 (1988).

[14] G. Dosi, C. Freeman, R. Nelson, G. Silverberg, and L. Soete, *Technical Change and Economic Theory.* Pinter, London, 1988.

[15] C. Freeman, A hard landing for the new economy? Information technology and the United States National System of Innovation. *Structural Change and Economic Dynamics,* **12**, 115-39 (2001).

[16] J. H. Moore, A Measure of Structural Change in Output. *Review of Income and Wealth,* **24**(1) (1978).

[17] D. J. Poirier, *The Econometrics of Structural Change*. North-Holland Publishing Company, 1976.

[18] J-M. Dufour, Generalized Chow Tests for Structural Change: A Coordinate Free Approach. *International Economic Review*, **23**(3) (1982).

[19] W. Ploberger and W. Kramer, On Studentizing a Test for Structural Change. *Economics Letters,* **20**, 341-344 (1986).

[20] *National Accounts*. Statistics Finland, Helsinki. See http://www.stat.fi, 2003.