ANALYSIS OF EOQ MODEL UNDER IMPERFECT COMPETITION

S. S. MISHRA* and P. P. MISHRA*

Abstract. *This paper deals with and an important aspect of econo-operations research in which a critical economic analysis is carried out to the inventory model of operations research. Dynamics of market economy is closely influenced not only by the price elasticity of demand but also by the nature of the market structure. The market structures include perfect competitions, imperfect competitions, oligopoly and monopoly etc. In this paper, an attempt has been made to analyze economic order quantity under the market structure of imperfect competition and different analyses have been presented under the specific imperfect structures. Finally, numerical computing of the model has also been added to elucidate the use of the model.*

1. Introduction

The problem of price determination for an EOQ model under imperfect competition is of central importance in the field of inventory control and management, especially such kind of models which study the dynamics of the market economy over the time. Since perfect competition is an ideal situation of market structure and generally is not found in real life. For this reason, imperfect market structure imperatively needs to be investigated to meet the demands of real existing marketing system, for example, vide Robinson [8] and Chamberlin [2] Imperfect structure is a glaring reality in the economy of marketing system and is unanimously described by eminent economists with following features:

- This is the structure where marginal revenue does not equal to price of an item.
- Demand is imperfectly elastic.
- There should not be large number of buyers and retailers in the market for that particular item.
- Firms are not free from joining and escaping from the market.

* Department of Mathematics and Statistics Dr. R. M. L. Avadh University, Faizabad – 224001, UP, India

- There should not be full knowledge of marketing by buyers and retailers.
- Market should not be near to the production system.
- Production system should not be fully dynamic.
- Only one producer or only one seller of product should be in the market.
- There are no close substitutes of product present in the market.
- More Constraints and many difficulties should be present in the entrance of industry.

There are some particular type of imperfect competitions such as monopolistic competition in which price is higher as compared with that of perfect competition. Monopoly is said to exist when there is only one producer or seller of a product which has no close substitutes or competitive. But when there exists some producers or sellers of same product in the market, this market structure is known as oligopoly. For example, there is only one company manufacturing the toothpaste of Binaca in India but it is not called monopoly because other substitutes are available in the market like Colgate, Pepsodent and Forhans etc. An imperfectly competitive firm cannot sell as much as it wants at the going price. It must recognize that its demand curve slopes down and that its output price will depend upon the quantity of the goods produced and sold. An oligopoly is a structure with only a few producers, each recognizing that its own price depends not merely on its own output but also on the actions of its important competitors in the industry. An industry with monopolistic competition has many sellers producing products that are closed substitutes for one another. Each firm has only limited ability to affect its output price which is briefly presented as

	Perfect Competition	**Imperfect Competition**		
Characteristic		**Monopolistic**	**Oligopoly**	**Monopoly**
No. of firms	Many	Many	Few	One
Ability to affect price	None	Limited	Some	Considerable
Entry barriers	None	None	Some	Complete
Example	Fruit stalls in Covent Garden	Corner grocer	Cars	De Beers

There are few researchers such as Bonanno and Giacomo [1] analyzed general equilibrium condition under imperfect competition and Hommes and Cars [3] gave the case of the cobweb and illustrated adaptive learning and roads to chaos. Goeree et.al [4] explained about the heterogeneous beliefs and gave new concept about cobweb in the non-linear form of price. Hommes [5] focused on the dynamics of the cobweb model is considered under adaptive expectations and nonlinear supply and demand. Hommes [6] again worked on cobweb dynamics under bounded rationality. Hommes et.al [7] discussed the expectation driven price volatility in an experimental economy under cobweb phenomenon. Maskin et.al [9] defined a theory of oligopoly considering its dynamic nature. Mishra and Mishra [10] emphasized on the price for an EOQ for deteriorating items with the dependency of stock as well as number of selling points in the market under perfect competition. Mishra et.al [11] elaborated a fuzzified deteriorating inventory model with breakdown of machine and shortage cost. Peter [12] performed the testing for an unstable root in conditional and structural error correction in his model.

Steven [13] provided a deep insight of monopolistic competition with outside goods. Schinkel et.al [14] presented exhaustive investigations on imperfect competition laws. Srinivasan et.al [15] in their work suggested the applications of equilibria as well as price normalization in International trade under imperfect market structure. Sonnemans et.al [16] worked on the instability of a heterogeneous cobweb economy in which a strategy experiment on expectation formation. Tuins [17] gave brief concept of oligopoly dynamics with their models and tools and further Baumol [18] also emphasized on the operation analysis and its economic theory. In this paper, an attempt has been made to determine the EOQ under imperfect competition and its specific cases such as cobweb phenomenon, price and production monopoly, Lerner's measure of monopoly power and wage determination under monopsony have been analyzed. Marginal revenue and marginal cost along with price elasticity approach have been employed to determine the EOQ and total optimal cost of the model. Numerical computing and its graphical representation have been presented to gain the deeper perspective of the model. The present work is presumably believed to provide better insight to the inventory managers and economists engaged in this field.

2. Description of model

The theory of imperfect competition as monopoly envisages a large number of quite small firms so that each firm can neglect the possibility that its own decisions provoke any adjustment in another firm's behavior. We also assume free entry and exit from the industry in the long run. In these respects the framework resembles our earlier discussion of perfect competition. In monopolistic competition the long-run tangency equilibrium occurs where each firm's demand curve is tangent to its average cost curve at out-put level at which MC equals to MR. Each firm is maximizing profits but just breaking even. There is no further entry or exit. Imperfect competition refers to all situations in which individual firms believe that they face downward-sloping demand curves. The most important forms of imperfect competition are monopolistic competition, oligopoly and pure monopoly. Monopolistic competitors face free entry and exit to the industry, but are individually small and make similar though not identical products. Each has limited monopoly power in its special brand. In long-run equilibrium, price equals average cost but exceeds marginal revenue and marginal cost at the tangency equilibrium. Oligopolists face a tension between collusion to maximize joint profits and competition for a large share of smaller joint profits. Collusion may be formal, as in a cartel, or informal. Without creditable threats of punishment by other collusive partners, each firm is likely to face a considerable temptation to cheat. One of the most important characteristics of demand function is what is known as its elasticity. According to the law of demand, the change in price and demand are in opposite direction and it is a common experience that price changes affect the demand for different commodities in different degrees. In other words, demand for some commodities is more sensitive to price changes than is the demand for others. For example, demand for necessities decreases very little when their prices rise whereas only and slight increase in the price is known as price elasticity of demand. Price elasticity of demand is defined as the value of the ratio of the relative (or proportionate) change in the demand to the relative (or proportionate) change in the price. Mathematically, we define it as:

$$e_d = \frac{-P_t}{Q_{d_t}}\left(\frac{dQ_{d\,t}}{dP_t}\right);$$

where, negative sign shows that demand and price move in opposite direction.

Followings are important characteristics which are noteworthy in the context under consideration.

- Price elasticity of demand is always positive.
- Demand is sometimes called over elastic or under elastic according as $e > 1$ or $e < 1$ (If $e = 1$, elasticity is called normal).
- Demand for necessities or conventional necessities are inelastic or less elastic while for luxuries it is elastic.
- Demand for goods having substitutes is elastic.
- Demand for goods having several uses is elastic.
- Goods for which demand can be postponed have elastic demand.

- Demand for jointly demanded goods is comparatively less elastic.
- Elasticity of demand varies with changes in income.
- Elasticity of demand depends on levels of prices.

3. Mathematical Analysis

As we know that in case of instantaneous replenishment of inventory $Q_{d_t} = S_t$.

In this model, it is assumed that $Q_{d_t} = bP_t + a$ and supply at that time $S_t = gP_{t-1} + h.$

From above, it is clear that demand at time t is fulfilled by the supply which is decided under the view of previous price. Hence,

$$bP_t + a = gP_{t-1} + h$$
$$bP_t = gP_{t-1} + (h-a);$$
$$P_t = \frac{g}{b}P_{t-1} + \frac{(h-a)}{b}.$$

After putting $t = 1$ in above, we get:

$$P_1 = \left(\frac{g}{b}\right)P_0 + \frac{(h-a)}{(g-b)}\frac{(g-b)}{b}.$$

Upon taking iteration for $t = 2$:

$$P_2 = \left(\frac{g}{b}\right)P_1 + \frac{(h-a)}{b}$$

$$P_2 = \left(\frac{g}{b}\right)\left\{\left(\frac{g}{b}\right)P_0 + \frac{(h-a)}{(g-b)}\left(\frac{g-b}{b}\right)\right\} + \left(\frac{(h-a)}{(g-b)}\right)\left(\frac{(g-b)}{b}\right)$$

$$P_2 = \left(\frac{g}{b}\right)^2 P_0 + \left(\frac{g}{b}\right)\left(\frac{(h-a)}{(g-b)}\right)\left(\frac{g-b}{b}\right) + \left(\frac{(h-a)}{(g-b)}\right)\left(\frac{(g-b)}{b}\right)$$

For, $t = 3$:

$$P_3 = \left(\frac{g}{b}\right)P_2 + \left(\frac{(h-a)}{b}\right);$$

$$P_3 = \left(\frac{g}{b}\right)\left\{\left(\frac{g}{b}\right)^2 P_0 + \left(\frac{g}{b}\right)\frac{(h-a)}{(g-b)}\frac{(g-b)}{b} + \frac{(h-a)}{(g-b)}\frac{(g-b)}{b}\right\} + \frac{(h-a)}{(g-b)}\frac{(g-b)}{b}$$

$$P_2 = \left(\frac{g}{b}\right)^3 P_0 + \left(\frac{g}{b}\right)^2 \frac{(h-a)}{(g-b)}\frac{(g-b)}{b} +$$

$$+\left(\frac{g}{b}\right)\left(\frac{(h-a)}{(g-b)}\frac{(g-b)}{b}\right) + \frac{(h-a)}{(g-b)}\frac{(g-b)}{b}.$$

Upon taking iteration till t, we get:

$$P_t = \left(\frac{g}{b}\right)^t P_0 + \left(\frac{(h-a)}{(g-b)}\right)\left(\frac{(g-b)}{b}\right)\left\{\left(\frac{g}{b}\right)^{t-1} + \left(\frac{g}{b}\right)^{t-2} + \left(\frac{g}{b}\right)^{t-3} + ...+1\right\}.$$

Let $(g/b) < 1$:

$$P_t = \left(\frac{g}{b}\right)^t P_0 + \left(\frac{(h-a)}{(g-b)}\right)\left(\frac{(g-b)}{b}\right)\left\{\frac{1-\left(\frac{g}{b}\right)^t}{1-\left(\frac{g}{b}\right)}\right\}$$

$$P_t = \left(\frac{g}{b}\right)^t P_0 - \left(\frac{(h-a)}{(g-b)}\right)\left(1-\left(\frac{g}{b}\right)^t\right)$$

$$P_t = \left(\frac{g}{b}\right)^t \left\{P_0 + \left(\frac{(h-a)}{(g-b)}\right)\right\} - \left(\frac{(h-a)}{(g-b)}\right)$$

$$Q_{d_t} = b\,P_t + a = b\left[\left(\frac{g}{b}\right)^t \left\{P_0 + \frac{(h-a)}{(g-b)}\right\} - \frac{(h-a)}{(g-b)}\right] + a$$

$$S_t = g\,P_{t-1} + h = g\left[\left(\frac{g}{b}\right)^{t-1} \left\{P_0 + \frac{(h-a)}{(g-b)}\right\} - \frac{(h-a)}{(g-b)}\right] + h$$

$$\frac{dQ_{d_t}}{dt} = b\left\{P_0 + \frac{(h-a)}{(g-b)}\right\}\left(\frac{g}{b}\right)^t \log\left(\frac{g}{b}\right).$$

Since, under cobweb phenomena, demanded quantity is decreasing function with respect to time. So, its first derivative with respect to time will be negative:

$$\frac{dQ}{dt} = -b\left\{P_0 + \frac{(h-a)}{(g-b)}\right\}\left(\frac{g}{b}\right)^t \log\left(\frac{g}{b}\right);$$

$$\frac{dP_t}{dt} = \left\{P_0 + \frac{(h-a)}{(g-b)}\right\}\left(\left(\frac{g}{b}\right)^t \log\left(\frac{g}{b}\right)\right).$$

Since, under cobweb phenomena, price of item is increasing function with respect to time. So, its first derivative with respect to time will be positive as given above:

$$\frac{dQ_{d_t}}{dP_t} = \frac{dQ_{d_t}}{dt} \times \frac{dt}{dP_t} =$$

$$= \frac{-b\left\{P_0 + \frac{(h-a)}{(g-b)}\right\}\left(\frac{g}{b}\right)^t \log\left(\frac{g}{b}\right)}{\left\{P_0 + \frac{(h-a)}{(g-b)}\right\}\left(\left(\frac{g}{b}\right)^t \log\left(\frac{g}{b}\right)\right)} = -b; \quad \frac{dQ_{d_t}}{dP_t} = -b\,.$$

We can express the following as:

$$\frac{1}{e_d} = -\frac{Q_{d_t}}{P_t}\frac{dP_t}{dQ_{d_t}} = -\frac{b\left[\left(\frac{g}{b}\right)^t\left\{P_0 + \frac{(h-a)}{(g-b)}\right\} - \frac{(h-a)}{(g-b)}\right] + a}{\left[\left(\frac{g}{b}\right)^t\left\{P_0 + \left(\frac{(h-a)}{(g-b)}\right)\right\} - \left(\frac{(h-a)}{(g-b)}\right)\right]} \times \frac{-1}{b}$$

$$e_d = \left\{1 + \frac{a}{b\left[\left(\frac{g}{b}\right)^t\left\{P_0 + \left(\frac{(h-a)}{(g-b)}\right)\right\} - \left(\frac{(h-a)}{(g-b)}\right)\right]}\right\}.$$

It is obvious to write that:

$$MR = \frac{\partial TR}{\partial Q_{d_t}} = \frac{\partial(P_t \times Q_{d_t})}{\partial Q_{d_t}} = P_t + Q_{d_t}\frac{\partial P_t}{\partial Q_{d_t}}. \tag{1}$$

Total turn over or total outlay for the commodity is given as:

$$F(P_t) = P_t \times Q_{d_t} = TR\,;$$

$$\partial TR = P_t\,\partial Q_{d_t} + Q_{d_t}\,\partial P_t.$$

Since in case of monopoly ∂P is negative, ∂Q is positive, it is clear from eqn (1) marginal revenue is less than the price of an item.

From equation (2), we get:

$$\frac{\partial TR}{\partial P_t} = P_t\frac{\partial Q_{d_t}}{\partial P_t} + Q_{d_t}\,;$$

$$\frac{\partial TR}{\partial P_t} = Q_{d_t}\left[\frac{P_t \, \partial Q_{d_t}}{Q_{d_t} \partial P_t} + 1\right] =$$

$$= Q_{d_t}\left[1 - e_d\right] = Q_{d_t}\left[1 - \frac{b\left[\left(\frac{g}{b}\right)^t \left\{P_0 + \left(\frac{(h-a)}{(g-b)}\right)\right\} - \left(\frac{(h-a)}{(g-b)}\right)\right]}{b\left[\left(\frac{g}{b}\right)^t \left\{P_0 + \left(\frac{(h-a)}{(g-b)}\right)\right\} - \left(\frac{(h-a)}{(g-b)}\right)\right] + a}\right]$$

$$\frac{\partial TR}{\partial P_t} = Q_{d_t} \frac{a}{b\left[\left(\frac{g}{b}\right)^t \left\{P_0 + \left(\frac{(h-a)}{(g-b)}\right)\right\} - \left(\frac{(h-a)}{(g-b)}\right)\right] + a} = a;$$

$\frac{\partial TR}{\partial P_t} = a = \frac{\partial F(P_t)}{\partial P_t};$ If $a > 0$ then $e < 1$ and $\frac{\partial F(P_t)}{\partial P_t} > 0.$ $F(P_t)$ is increase-ing function of P_t; i.e., the money value of the turnover increases as the price rises.

But if $a < 0$ then $e > 1$ and $\frac{\partial F(P_t)}{\partial P_t} < 0 \Rightarrow F(P_T)$ is decreasing function of P_t, i.e. the money value of the turnover decreases as the price rises.

Further, we can express as:

$$MR = \frac{\partial TR}{\partial Q_{d_t}} = \frac{\partial (P_t \times Q_{d_t})}{\partial Q_{d_t}} = P_t + Q_{d_t} \frac{\partial P_t}{\partial Q_{d_t}}.$$

Price elasticity of demand:

$$e_d = -\frac{P_t dQ_{d_t}}{Q_{d_t} dP_t}.$$

Since $\frac{\partial Q_{d_t}}{\partial P_t} < 0;$ in case of monopoly, we have:

$$MR = P_t\left(1 - \frac{1}{e_{d_t}}\right);$$

$$MR = P_t\left(1-\frac{b\left[\left(\frac{g}{b}\right)^t\left\{P_0+\frac{(h-a)}{(g-b)}\right\}-\frac{(h-a)}{(g-b)}\right]+a}{b\left[\left(\frac{g}{b}\right)^t\left\{P_0+\left(\frac{(h-a)}{(g-b)}\right)\right\}-\left(\frac{(h-a)}{(g-b)}\right)\right]}\right);$$

$$MR = \left[\left(\frac{g}{b}\right)^t\left\{P_0+\left(\frac{(h-a)}{(g-b)}\right)\right\}-\left(\frac{(h-a)}{(g-b)}\right)\right]\times$$

$$\times\left[\frac{-a}{b\left[\left(\frac{g}{b}\right)^t\left\{P_0+\frac{(h-a)}{(g-b)}\right\}-\frac{(h-a)}{(g-b)}\right]}\right]MR = \left(\frac{-a}{b}\right).$$

Moreover, we can have:

$$S_t = gP_{t-1}+h = g\left\{\left(\frac{g}{b}\right)^{t-1}\left(P_0+\frac{(h-a)}{(g-b)}\right)-\left(\frac{(h-a)}{(g-h)}\right)\right\}+h;$$

$$\frac{\partial S_t}{\partial t} = g\left\{\left(P_0+\frac{(h-a)}{(g-b)}\right)\right\}\left(\frac{g}{b}\right)^{t-1}\log\left(\frac{g}{b}\right).$$

Since, under cobweb phenomena, supply is increasing function with respect to time. So, its first derivative with respect to time will be positive as shown above. We further have:

$$\frac{\partial P_{t-1}}{\partial t} = \left\{P_0+\frac{(h-a)}{(g-b)}\right\}\left(\frac{g}{b}\right)^{t-1}\log\left(\frac{g}{b}\right)$$

$$\frac{\partial S_t}{\partial P_{t-1}} = \frac{\partial S_t}{\partial t}\times\frac{\partial t}{\partial P_{t-1}} = \frac{g\left\{\left(P_0+\frac{(h-a)}{(g-b)}\right)\right\}\left(\frac{g}{b}\right)^{t-1}\log\left(\frac{g}{b}\right)}{\left\{P_0+\frac{(h-a)}{(g-b)}\right\}\left(\frac{g}{b}\right)^{t-1}\log\left(\frac{g}{b}\right)} = g.$$

Hence, price elasticity of supply can be given as:

$$\frac{P_{t-1}}{S_t}\frac{\partial S_t}{\partial P_{t-1}} = \frac{\left(\frac{g}{b}\right)^{t-1}\left\{P_0+\left(\frac{(h-a)}{(g-b)}\right)\right\}-\left(\frac{(h-a)}{(g-b)}\right)}{g\left\{\left(\frac{g}{b}\right)^{t-1}\left(P_0+\frac{(h-a)}{(g-b)}\right)-\left(\frac{(h-a)}{(g-h)}\right)\right\}+h}\times g$$

$$e_s = \frac{\left(\frac{g}{b}\right)^{t-1}\left\{P_0 + \left(\frac{(h-a)}{(g-b)}\right)\right\} - \left(\frac{(h-a)}{(g-b)}\right)}{\left\{\left(\frac{g}{b}\right)^{t-1}\left(P_0 + \frac{(h-a)}{(g-b)}\right) - \left(\frac{(h-a)}{(g-h)}\right)\right\} + \left(\frac{h}{g}\right)}.$$

4. Inventory model under imperfect competition with cobweb phenomenon

In this case, supply and demand at any time t are equal, quantity of supply at time t is decided at time $t-1$ which is based on the price at time $t-1$.

Let $Q_{d_t}, Q_{d_{t+1}}, Q_{d_{t+2}}, \ldots\ldots\ldots\ldots Q_{d_{t+n}}$ are the demands at time instants, $t,\ t+1,\ t+2, \ldots\ldots t+n$ respectively, quantity demanded at time t is equal to quantity supplied at that time and thus different demands at different times are fulfilled.

Total demand in T time horizon is given as:

$$D = \sum_{i=0}^{n} Q_{d_{t+i}} = \sum_{i=0}^{n} S_{t+i} = \sum_{i=0}^{n} g\left\{\left(\frac{g}{b}\right)^{t+i-1}\left(P_0 + \frac{(h-a)}{(g-b)}\right) - \left(\frac{(h-a)}{(g-h)}\right)\right\} + h.$$

Given time horizon, we can find the following costs as:

If q is an EOQ then

Carrying cost = $\frac{C_s qT}{2}$; Ordering cost = $C_0\left(\frac{D}{q}\right)$;

Total Cost = Carrying cost + Ordering cost +Raw Material cost;

$$\text{TVC} = \frac{C_s qT}{2} + \frac{C_0 D}{q} + C_r q;$$

Total fixed cost is sum of setup cost, Labor cost etc.

$$TFC = C_{stp}T + C_w T\ ;\ TC = \frac{C_s qT}{2} + \frac{C_0 D}{q} + C_{stp}T + C_r q + C_w T;$$

From profit optimization, we get:

$$\frac{\partial TC}{\partial q} = MC = MR.$$

This implies that:

$$\frac{C_s T}{2} - \frac{C_0 D}{q^2} + C_r = \left(\frac{-a}{b}\right);$$

where $a < 0$

$$\frac{C_0 D}{q^2} = \frac{C_s T}{2} + C_r + \left(\frac{a}{b}\right).$$

This gives us:

$$q = \sqrt{\frac{C_0 D}{\frac{C_s T}{2} + C_r + \frac{a}{b}}}, \quad \text{and} \quad q = \sqrt{\frac{2C_0 D\, b}{bC_s T + 2bC_r + 2a}}.$$

This can be finally written as:

$$q = \sqrt{\frac{2C_0 D\, b}{bC_s T + 2bC_r + 2a}}.$$

Hence, optimal quantity (q*) is given as:

$$q^* = \sqrt{\frac{2C_0 b \sum_{i=0}^{n} Q_{d_{t+i}}}{bC_s T + 2bC_r + 2a}}$$

$$q^* = \sqrt{\frac{2bC_0\left[\sum_{i=0}^{n} g\left\{P_0 + \frac{(h-a)}{(b-g)}\right\}\left(\frac{g}{b}\right)^{t+i-1} - \sum_{i=0}^{n} g\frac{(h-a)}{(b-g)} + \sum_{i=0}^{n} h\right]}{bC_s T + 2bC_r + 2a}}$$

$$EOQ = \sqrt{\frac{2bC_0\left[g\left(P_0 + \frac{(h-a)}{(b-g)}\right)\sum_{i=0}^{n}\left(\frac{g}{b}\right)^{t+i-1} - \frac{gn(h-a)}{(b-g)} + hn\right]}{bC_s T + 2bC_0 + 2a}}$$

$$EOQ =$$

$$= \sqrt{\frac{2bC_0\left[\left\{g\left(P_0 + \frac{(h-a)}{(b-g)}\right)\left(\frac{g}{b}\right)^{t-1}\left(1 + \left(\frac{g}{b}\right) + \left(\frac{g}{b}\right)^2 + \ldots.. + \left(\frac{g}{b}\right)^n\right)\right\} - \frac{g\, n\,(h-a)}{(b-g)} + hn\right]}{bC_s T + 2bC_r + 2a}}$$

$$EOQ=\sqrt{\frac{2bC_0\left[g\left\{P_0+\frac{(h-a)}{(b-g)}\right\}\left(\frac{g}{b}\right)^{t-1}\left(\frac{1-\left(\frac{g}{b}\right)^{n+1}}{1-\left(\frac{g}{b}\right)}\right)-\frac{g\,n(h-a)}{(b-g)}+hn\right]}{bC_sT+2bC_r+2a}};$$

$$EOQ=\sqrt{\frac{2bC_0\left[g\left\{P_0+\frac{(h-a)}{(b-g)}\right\}\left(\frac{g}{b}\right)^{t-1}\left(\frac{b^{n+1}-g^{n+1}}{b-g}\right)\left(\frac{1}{b^n}\right)-\frac{gn(h-a)}{(b-g)}+hn\right]}{bC_sT+2bC_r+2a}};$$

$$EOQ=\sqrt{\frac{2bC_0[g^t\left(P_0(b-g)(b^{n+1}-g^{n+1})\right)+g^t(b^{n+1}-g^{n+1})(h-a)-}{b^{n+t-1}(b-g)^2\times}}$$

$$\frac{-gnb^{t+n-1}(h-a)(b-g)+hnb^{n+t-1}(b-g)^2]}{\times(bC_sT+2C_rb+2a)}$$

$$EOQ=\sqrt{\frac{2C_0[g^t(b^{n+1}-g^{n+1})\{P_0(b-g)+(h-a)\}-}{b^{n+t-2}(b-g)^2\times}}$$

$$\frac{-gnb^{t+n-1}(h-a)(b-g)+hnb^{n+t-1}(b-g)^2]}{\times(bC_sT+2C_rb+2a)}.$$

Hence, total optimal variance and fixed costs are:

$$TVC^*=\frac{C_sq^*{}_tT}{2}+\frac{C_0D}{q_t^*}+C_rq_t^*$$

$$TFC=TFC^*=C_{stp}T+C_wT.$$

Finally, total optimal cost is given as:

$$TC^*=\frac{C_sq_t^*\,T}{2}+\frac{C_0q_t^*\,T}{2}+\frac{C_0D}{q_t^*}+C_rq_t^*+C_{stp}T+C_wT.$$

5. Lerner's Measure of Monopoly Power

We define the Index of Monopoly Power as:

$$IMP=\left(\frac{P_t-MC}{P_t}\right);\ 0\le IMP\le 1.$$

IMP shows the power of monopolistic firm at time t. IMP is nothing but it is inverse of price elasticity. So, it is defined as below:

$$IMP = \frac{\left(\frac{g}{b}\right)^t \left\{P_0 + \frac{(h-a)}{(g-b)}\right\} - \frac{(h-a)}{(g-b)} - \left(\frac{-a}{b}\right)}{\left[\left(\frac{g}{b}\right)^t \left\{P_0 + \frac{(h-a)}{(g-b)}\right\} - \frac{(h-a)}{(g-b)}\right]} =$$

$$= \frac{\left(\frac{g}{b}\right)^t \left\{P_0 + \frac{(h-a)}{(g-b)}\right\} - \frac{(h-a)}{(g-b)} + \left(\frac{a}{b}\right)}{\left[\left(\frac{g}{b}\right)^t \left\{P_0 + \frac{(h-a)}{(g-b)}\right\} - \frac{(h-a)}{(g-b)}\right]}.$$

This obviously falls in between o and 1 and which is given as:

$$0 \le \frac{\left(\frac{g}{b}\right)^t \left\{P_0 + \frac{(h-a)}{(g-b)}\right\} - \frac{(h-a)}{(g-b)} + \left(\frac{a}{b}\right)}{\left[\left(\frac{g}{b}\right)^t \left\{P_0 + \frac{(h-a)}{(g-b)}\right\} - \frac{(h-a)}{(g-b)}\right]} \le 1.$$

This further implies that:

$$\left(\frac{g}{b}\right)^t \left\{P_0 + \frac{(h-a)}{(g-b)}\right\} - \frac{(h-a)}{(g-b)} + \left(\frac{a}{b}\right) \le$$

$$\le \left[\left(\frac{g}{b}\right)^t \left\{P_0 + \frac{(h-a)}{(g-b)}\right\} - \frac{(h-a)}{(g-b)}\right].$$

And hence $(a/b) \le 0$ and $a \le 0$ or $b \le 0$.

It shows that monopoly of firm exists under the above condition and in another condition when $a \ge 0$ the marginal revenue will become negative so it will be not suitable for monopoly of the firm.

6. Determination of wage rate under imperfect competition

When imperfect competition in product market and perfect competition in Labor Market are satisfied then:

$MRP = MPP \times MR$

$VMP = MPP \times$ Price of an object.

Since:

$MR <$ Price of an object.

$MRP < VMP$

Under conditions of monopoly or imperfect competition in product market, assuming perfect competition in the labor market, the labor will get wage less than the value of its marginal product, we find that:

$$C_w = MRP_L \ or \quad C_w = MR \times MPP_L \quad and \ C_w = \left(1 - \frac{1}{e_d}\right) P_t \times MPP_L .$$

Since $P_t \times MPP_L = VMP_L$, we have:

$$C_w = \left(1 - \frac{1}{|\, e_d|}\right) VMP_L = \frac{-aVMP_L}{b\left[\left(\frac{g}{b}\right)^t \left\{P_0 + \frac{(h-a)}{(g-b)}\right\} - \frac{(h-a)}{(g-b)}\right]}$$

But whenever in product market, there exists an imperfect competition or monopoly then the price elasticity is less than one, so in this condition $\left(1 - \frac{1}{e_d}\right) < 1$; hence, $C_w < VMP$.

Thus,

$$TFC^* = C_{stp}T + C_w T$$

$$TFC^* = C_{stp}T + \left(\frac{-aVMP_L}{b\left[\left(\frac{g}{b}\right)^t \left\{P_0 + \frac{(h-a)}{(g-b)}\right\} - \frac{(h-a)}{(g-b)}\right]}\right)$$

$$TC^* = \frac{C_s q^*{}_t T}{2} + \frac{C_0 D}{q_t^*} + C_r q^*{}_t + C_{stp}T \ \text{ or}$$

$$TC^* = \frac{C_s q_t^* T}{2} + \frac{C_0 D}{q^*{}_t} + C_r q_t^* + C_{stp}T .$$

Finally:

$$TC^* = \frac{-aVMP_L T}{b\left[\left(\frac{g}{b}\right)^t \left\{P_0 + \frac{(h-a)}{(g-b)}\right\} - \frac{(h-a)}{(g-b)}\right]} .$$

7. Wage determination under monopsony

Monopsony means the monopoly to hire the labor or only one individual employer or firm to hire the labor. At the equilibrium state, it is important to know that MFC is equal to both *VMP* or *MRP* and thus
$TFC = C_w T$.

8. Numerical computing and graphical representation of the model

With the help of hypothetical data and using simple computer programming in C++, we present the following numerical computations in the form of tables and corresponding graphs are also given in Anexes (vide the tables and graphs at last).

9. Conclusion

We conclude that whole paper has been focused on the various aspects of the imperfect competition under the cobweb nature of the production and its consumption.

Important results have been drawn by using optimization technique of marginal revenue and marginal cost. These include EOQ of inventory item at time instant *t*, price elasticity of demand, supply elasticity of demand, Lerner's measure of monopoly power, determination of wage rate under imperfect competition and wage determination under monopsony. These results are very useful instruments in controlling the inventory and its management.

Tables and Graphs

Table 1

Computation of TOC and EOQ

For given parameters P_0 = Rs.50, n = 8, a = – 50, b = 100, g = 200, h = 100, $C_s = Rs.20$, Co= Rs.25, $C_{stp} = Rs.10$, T = 4 year, Cr = Rs.20 VMP = Rs.20;

t	P_t	e_d	EOQ	C_w	TOC (in10^8)
4	822.5	1.0006	4081.98	0.012158	3.29488
3	410.5	1.0012	2886.51	0.024361	1.64545
2	204.5	1.0025	2041.23	0.048900	8.20439
1	101.5	1.0045	1443.6	0.098522	4.07725
0.5	71.33	1.00706	1214.08	0.14019	0.286779

Table 2

Computation of TOC, EOQ and MR

For given parameters P_0 = Rs.50, t = 3, n = 8, b = 100, g = 200, h = 100, Ch = Rs.20, Co = Rs.25, Cstp = Rs.10, T = 4 year, Cr = Rs.20 VMP = Rs.20

a	P_t	e_d	EOQ	C_w	TOC(in10^8)	e_s (in10^{-5})	MR
–60	411.2	1.0015	2885.97	0.0291829	1.9772	2.48069	0.6
–80	412.6	1.0019	2884.89	0.0387785	2.64408	2.47836	0.8
–100	414	1.0024	2883.79	0.0483092	3.3154	2.47605	1
–150	417.5	1.0036	2881.03	0.0718563	5.01341	2.47036	1.5
–200	421	1.0047	2878.22	0.0950119	6.7394	2.46477	2

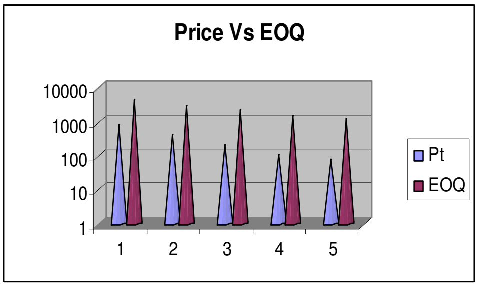
Table 3

Computation of EOQ, Cw, MR and TOC

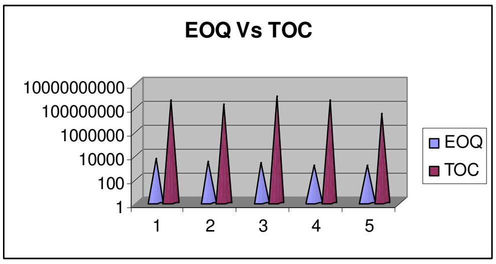
For given parameters a = – 200, t = 3, n = 8, b = 100, g = 200, h = 100, Ch = Rs.20, Co = Rs.25, $C_{stp} = Rs.10$, T = 4 year, Cr = Rs.20, VMP = Rs.20;

P_0	P_t	e_d	EOQ	C_w	e_s (in10^{-5})	MR	TOC (in10^9)
100	821	1.00244	4134.55	0.0487211	2.4187	2	1.31409
150	1221	1.00164	5089.68	0.03276	2.4878	2	1.5420
200	1621	1.00124	5891.96	0.0246761	2.4908	2	2.5943
250	2021	1.00099	6597.39	0.0197922	2.49262	2	3.2343
300	2421	1.00083	7234.35	0.016522	2.49384	2	3.8744

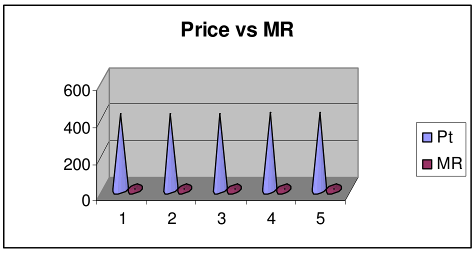
Graph 1.1



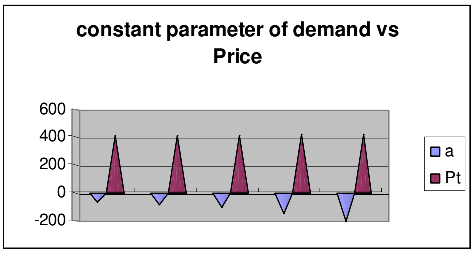
Graphs 1.2



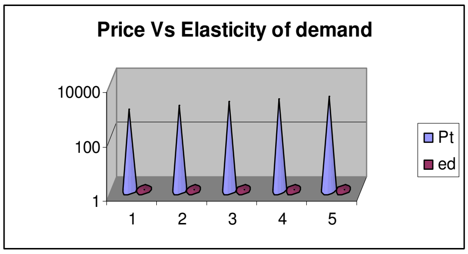
Graph 2.1



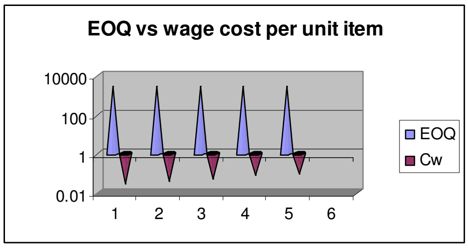
Graph 2.2



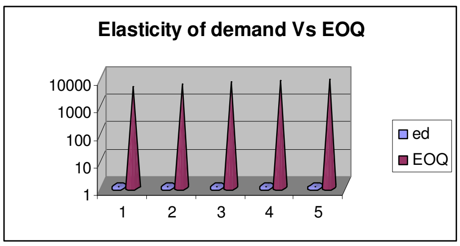
Graph 2.3



Graph 3.1



Graph 3.2



REFERENCES

[1] Bonanno and Giacomo (1990), *General Equilibrium Theory with Imperfect Competition*, Journal of Economic Surveys, Blackwell Publishing, vol. **4**(4), pp. 297-328.

[2] E. H. Chamberlin (1930), *Theory of Monopolistic Competition* (6th edition), pp. 79-80.

[3] Goeree, Jacob K. & Hommes, Cars H. (2000), *Heterogeneous beliefs and the non-linear cobweb model*, Journal of Economic Dynamics and Control, Elsevier, vol. **24**(5-7), pp. 761-798.

[4] Hommes, Cars H. (1991), *Adaptive learning and roads to chaos: The case of the cobweb*, Economics Letters, Elsevier, vol. **36**(2), pp. 127-132.

[5] Hommes, Cars H. (1994), *Dynamics of the cobweb model with adaptive expectations and nonlinear supply and demand*, Journal of Economic Behavior & Organization, Elsevier, vol. **24**(3), pp. 315-335.

[6] Hommes, C.H. (1999), *Cobweb Dynamics under Bounded Rationality*, CeNDEF Working Papers 99-05, Universiteit van Amsterdam, Center for Nonlinear Dynamics in Economics and Finance.

[7] Hommes, C.H. & Sonnemans, J. & Tuinstra, J. & van de Velden, H. (1999), *Expectation Driven Price Volatility in an Experimental Cobweb Economy*, CeNDEF Working Papers 99-07, Universiteit van Amsterdam, Center for Nonlinear Dynamics in Economics and Finance.

[8] Joan Robinsion (1930), *The Economics of Imperfect Competition*, pp. 86.

[9] Maskin, Eric & Tirole, Jean (1987), "A theory of dynamic oligopoly, III: Cournot competition.

[10] Mishra, S. S. and Mishra, P. P. (2008), *Price determination for an EOQ model for deteriorating items under perfect competition*, Int. J. Computers and Mathematics with Applications (Article in press) CAMWA: 4216, 1-2.

[11] Mishra, S. S., Pandey, H. and Singh, R. S. (2004), *A fuzzified deteriorating inventory model with breakdown of machine and shortage cost.* International Journal of Mathematical Sciences, 3, pp. 241-255.

[12] Peter Boswijk, H. (1994), *Testing for an unstable root in conditional and structural error correction models*, Journal of Econometrics, Elsevier, vol. **63**(1), pp. 3.

[13] Steven C. Salop (1979), *Monopolistic Competition with Outside Goods*, Bell Journal of Economics, the RAND Corporation, vol. **10**(1), pp.141-156, Spring.

[14] Schinkel, Maarten Pieter & Tuinstra (2006), *Imperfect competition law enforcement*, International Journal of Industrial Organization, Elsevier, vol. 24(6), pp. 1267-1297.

[15] Srinivasan, T. N. & Kletzer, K. (1994), *Price Normalization and Equilibria in General Equilibrium Models of International Trade Under Imperfect Competition*, Papers 710, Yale – Economic Growth Center.

[16] Sonnemans, Joep & Hommes, Cars & Tuinstra, Jan & van de Velden, Henk (2004), *The instability of a heterogeneous cobweb economy: a strategy experiment on expectation formation*, Journal of Economic Behavior & Organization, Elsevier, vol. **54**(4), pp. 453-481.

[17] Tuinstra, Jan (2004), *Oligopoly Dynamics: Models and Tools*: Journal of Economic Behavior & Organization, Elsevier, vol. **54**(4), pp. 611-614.

[18] W. J. Baumol (2003), *Economic Theory and Operations Analysis*, 2nd Edition, pp. 342.